Cyclical Fluctuations, Financial Frictions, and Productivity Differences across Firms*

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Abstract

We bridge two literatures that have largely developed in parallel: the endogenous growth literature, which uses tractable representative-agent models to study the evolution of aggregate TFP, and the misallocation literature, which studies why financial frictions prevent capital from flowing to its most productive uses across heterogeneous firms. We build a tractable model that imports key features of the misallocation literature into a representative-agent framework. Credit rationing arising from adverse selection and moral hazard prevents capital from flowing efficiently to the most productive firms. As credit rationing becomes more or less intense, aggregate TFP acquires an endogenous component, accounting for about 30 percent of the variance of TFP at business cycle frequencies. We show that our tractable model can match key features of the observed distribution of productivity across firms and its evolution over the business cycle.

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1 Introduction

Two strands of the macroeconomics literature have developed largely in parallel without much cross-fertilization. The first is the endogenous growth literature, which uses representative-agent frameworks to study the low-frequency evolution of aggregate total factor productivity (TFP) based on foundational contributions by Lucas (1988), Romer (1990), and Aghion and Howitt (1992).[1] More recently, Comin and Gertler (2006) showed how endogenous technology adoption and R&D generate medium-frequency TFP dynamics that persist well beyond the business cycle. These models are tractable enough to be estimated and embedded in larger quantitative frameworks, but they abstract from the cross-sectional distribution of productivity across firms.[2]

The second is the misallocation literature, which studies why capital and labor are not efficiently allocated across heterogeneous firms and what this misallocation implies for aggregate productivity. A large body of work documents that productivity dispersion across firms within narrowly defined industries is substantial and persistent, and shows that financial frictions are a leading culprit: credit constraints, information asymmetries, and moral hazard prevent resources from flowing to their most productive uses.[3] The theoretical models that can account for these patterns in full generality are necessarily complex—they require tracking distributions of firm-level state variables and are difficult to embed in standard representative-agent business cycle frameworks. Yet bridging these literatures has direct macroeconomic stakes. The aggregate Solow residual is not a primitive of the economy: it aggregates up from the allocation of inputs across firms with different productivity levels, so when financial frictions cause that allocation to vary with credit conditions, measured TFP inherits a component that is endogenous to macroeconomic developments rather than

[1] Lucas (1988), Romer (1990), and Aghion and Howitt (1992) established how human capital accumulation, R&D, and innovation endogenously determine the level and growth rate of productivity. King and Rebelo (1990) and Rebelo (1991) embedded these mechanisms in calibrated representative-agent frameworks, showing how endogenous growth alters the predictions of RBC models with implications for policy analysis.

[2] For further examples in this tradition, see Jones et al. (2005) and Dang et al. (2012) on endogenous growth with business cycle implications, and Gornemann et al. (2025) for a quantitative open-economy application with endogenous productivity.

[3] Foundational contributions include Foster et al. (2008), Restuccia and Rogerson (2008), Hsieh and Klenow (2009), Midrigan and Xu (2014), Moll (2014), Gopinath et al. (2017), and David and Venkateswaran (2019).

reflecting technological change alone.

This paper bridges these two literatures. Our central contribution is to show how key features of the misallocation literature, such as endogenous productivity dispersion driven by financial frictions, can be imported into a tractable representative-agent setting. In our model, each firm is endowed with two technologies: a production technology, whose idiosyncratic efficiency is private information, and a financial intermediation technology, which is available to all firms on equal terms. Because firm-level production efficiency is not directly observed by investors and the claims of firms are not credible, this informational friction results in all firms being financed with an aliquot share of household savings, regardless of productivity. In a secondary market for funds, where the intermediation technology of a firm allows a little more screening of productivity to be performed, firms compare the returns from their two technologies and sort themselves into lenders, firms deploying their intermediation technology; borrowers that default strategically without producing; and producers. This sorting depends on the realization of the production-technology draw, and all we need to do is characterize the productivity points that demarcate different groups of firms. As these cutoff points respond to aggregate macroeconomic conditions, the resulting model delivers endogenous dynamics for aggregate TFP akin to those in the endogenous growth literature.

But our contribution extends beyond tractability. The framework captures strategic default as an equilibrium outcome, not a knife-edge case ruled out by contract design, and the equilibrium variation in the mass of defaulting firms is quantitatively important: it accounts for roughly one-third of the endogenous TFP component, over and above what a single-cutoff model without strategic default can generate. Strategic default thus serves a dual purpose: it amplifies the endogenous TFP component by concentrating production among high-productivity firms, and in our simulations also stabilizes the dynamic path of that component—when aggregate conditions shift, firms near the default threshold adjust first, distributing the reallocation burden across two margins rather than concentrating it in a single threshold, which yields a smoother and more persistent endogenous TFP response than the no-default variant produces. We also provide an explicit decomposition of the Solow residual into an exogenous technology component and an endogenous reallocation component: roughly 30 percent of Solow TFP variance at business cycle frequencies is endogenously related to shifts in financial conditions, with 10 percent of this variation directly linked to the default margin. Certain shocks, preference shocks for instance, produce TFP movements that are entirely endogenous.

Our approach avoids a classic criticism of the endogenous growth literature. Models of endogenous growth can lead to strong predictions and policy recommendations based on long-run growth effects that are difficult to ascertain empirically. In our model, the endogenous growth effects are not permanent. The tractable structure allows us to deploy standard methods for an empirical verification. While the endogenous growth literature has generated deep insights about long-run forces shaping aggregate TFP, empirically isolating those forces has proven difficult: Aggregate time series data cannot readily distinguish neoclassical from endogenous growth models (Pack, 1994), and the low-frequency statistical models underlying many empirical exercises are nearly indistinguishable from one another at typical macroeconomic sample lengths (Müller & Watson, 2008). By contrast, we can use data on the dispersion of productivity for firms within narrowly defined industries in support of the micro-foundations of our model.

As in Stiglitz and Weiss (1981), SW henceforth, credit rationing is at the center of our model, but it arises from some additional incentives built into our model. In SW, default arises involuntarily when project returns fall short of the promised repayment; the interest rate affects bank returns by changing the composition of applicants (adverse selection) and the riskiness of projects chosen (incentive effect/moral hazard). Our model adds a different channel: firms with sufficiently low productivity find it profitable to divert borrowed funds to an outside option rather than produce, making default a strategic, equilibrium choice.[4] This moral hazard margin, different from the one in SW, generates a two-tier sorting structure. In our model interest rates influence both the strategic default rate and the average productivity of firms that choose to borrow, produce, and repay their loans.[5]

[4]There are other important differences between our analysis and the one in Stiglitz and Weiss (1981) including. We develop a general equilibrium model whereas theirs is a static partial equilibrium model. It has no capital accumulation, no goods market, no aggregate TFP, and no business cycle dynamics. The bank's interest rate is a parameter, not an endogenous price that co-moves with output, investment, and consumption as in our model.

[5]In an entrepreneurship model with multiple sources of financial frictions, Paulson et al. (2006) showed—via structural estimation on Thai data—that moral hazard due to information asymmetry is the dominant source.

In our model, misallocation is not static. As aggregate conditions change, the thresholds governing which firms lend, borrow, and default fluctuate, and with them the dispersion of productivity across firms and aggregate TFP. The model thus encompasses two sources of aggregate TFP: a standard exogenous component and an endogenous component driven by changes in the distribution of productive activity across firms. Under our baseline calibration, this endogenous channel generates meaningful differences in impulse responses to TFP shocks relative to models without financial frictions and productivity dispersion. In the efficient limit of our model, all credit flows to the most productive firm and the economy collapses to a standard RBC model, a benchmark against which we calibrate and evaluate the model.

Despite its relative simplicity, the model is sufficiently flexible to match key features of the data. We calibrate the model to standard targets for credit market conditions and productivity dispersion, and find that it comes close to replicating several untargeted moments. In particular, the model replicates the countercyclical behavior of within-industry productivity dispersion: regressions of dispersion on GDP growth yield coefficients of similar sign and magnitude in model-simulated and observed data, with overlapping confidence intervals. The model also reproduces the sign of the relationship between productivity dispersion and default rates, although the coefficient is not statistically significant in simulated data.

2 Literature Review

Our work has many antecedents. It is related first to papers that showcase models with costly state verification, such as Carlstrom and Fuerst (1997), Bernanke et al. (1999), and Christiano et al. (2014). These papers established that the agency costs of financial intermediation can substantially amplify and propagate aggregate shocks, a foundational insight that allows us to go further. A common moodelling device for these papers is the assumption that credit needs to be allacated to firms with heterogeneous productivity. However, the productivity dispersion is static and cannot respond to economic conditions. Default in those models is also not strategic: it stems from the inability to repay after uncertainty resolves unfavorably. By contrast, in our model default is strategic, varies with economic conditions, and feeds back into productivity dispersion and aggregate TFP.

Our work is also related to papers that include endogenously evolving productivity dispersion. Khan and Thomas (2013) were the first to explore the endogenous TFP channel in a quantitative DSGE setting where reallocation of capital across heterogeneous firms determines aggregate TFP. Their model works through the interaction of two frictions—collateralized borrowing constraints and partial capital irreversibility—that together distort capital allocation and generate persistent endogenous TFP movements. Our model captures the same channel through a more parsimonious mechanism: strategic default sorts firms by productivity and generates credit rationing without requiring capital-level heterogeneity or (S,s) adjustment rules. And unlike their model, which is designed to rule out default in equilibrium, default is an equilibrium outcome in ours. Khan and Thomas's rich framework allows them to match detailed microeconomic moments on firm investment behavior that our model does not target; the tradeoff is that their solution algorithm requires tracking a three-dimensional firm distribution. By contrast, the simplicity of our framework allows us to embed it in many variants of an RBC model, and we view it as a building block that others can readily reuse or extend.

Our emphasis on tractability connects us to Buera and Moll (2015), who developed a model where heterogeneous firms face collateral constraints and shocks to those constraints open productivity wedges. In their benchmark model, idiosyncratic firm productivity is independently and identically distributed (i.i.d.) across entrepreneurs and over time, so the cutoff governing which firms produce is determined exogenously by the collateral constraint rather than by equilibrium sorting. When they allow for persistent idiosyncratic shocks—treated in their online appendix under logarithmic utility to preserve tractability—the conclusions were the same as in the i.i.d. case. Under logarithmic preferences, moreover, a tightening of collateral constraints is exactly isomorphic to an exogenous TFP shock, and no other shocks are considered; the endogenous TFP component therefore cannot be separately identified from the exogenous one. Our model delivers an endogenous TFP component even in the i.i.d. limit, and the strategic-default margin generates an additional channel that does not arise in their framework.

Our work is also related to Liu and Wang (2014) and Dong et al. (2025), who studied heterogeneous firms under collateral constraints in similar settings. In Liu and Wang (2014), borrowing constraints are governed by an exogenous parameter rather than by the equilibrium sorting of firms across lending, production, and default. By design, no firm defaults in equilibrium in their framework; default is instead an equilibrium outcome in ours, and it is precisely the endogenous variation in the mass of defaulting firms that drives an important part of the endogenous TFP response. A further difference is that the endogenous TFP channel in our model operates through equilibrium sorting at the default margin, without relying on entry and exit costs. Dong et al. (2025) is a targeted extension of Liu and Wang (2014) that focuses on the effects of turbulence shocks. In their model, misallocation arises from the joint presence of credit constraints and production distortions; turbulence operates by tightening the borrowing constraints of high-productivity firms and amplifying pre-existing misallocation. In our model, misallocation arises endogenously from financial frictions alone, without requiring pre-existing production distortions."

The misallocation literature more broadly provides important context for our work. Restuccia and Rogerson (2008) and Hsieh and Klenow (2009) established that policy distortions and wedges between marginal products account for large aggregate productivity losses. Midrigan and Xu (2014) and Moll (2014) emphasized financial frictions as a central driver of misallocation and productivity losses. Moll (2014) established an important benchmark: With serially persistent idiosyncratic productivity, entrepreneurs can gradually self-finance out of credit constraints, which reduces misallocation and allows TFP to recover over time. Without serial correlation, however, that particular self-financing channel disappears and TFP is fully driven by exogenous factors. Our paper goes further by demonstrating that endogenous TFP dynamics arise generically even when idiosyncratic productivity is i.i.d. First, an equilibrium-selection channel makes the cutoffs for lending, borrowing, and default responsive to aggregate conditions, producing endogenous TFP variation independent of wealth dynamics. Second, explicitly modeling strategic default creates an additional, quantitatively relevant source of TFP fluctuations through firms' default choices and their general-equilibrium effects. Together these mechanisms substantially broaden the range of financial-contracting features that can drive aggregate misallocation and TFP dynamics beyond the scope of Moll (2014). David and Venkateswaran (2019) and Gopinath et al. (2017) documented the importance of capital misallocation both in the United States and in Southern Europe. On the firm dynamics side, Clementi and Hopenhayn (2006) and Cooley and Quadrini (2001) developed theories of financing constraints and firm dynamics that inform our modeling of credit frictions.

Our paper also contributes to a body of work exploring heterogeneous firms, financial frictions, and default. Gilchrist et al. (2014) studied credit spreads under uncertainty shocks in a model with default. Arellano et al. (2019) examined the role of uncertainty shocks in a model with noncontingent debt and equilibrium default. Gomes and Schmid (2021) developed a model with endogenous default where firms vary in leverage and studied the implications for credit spreads.[6]

Finally, our paper is related to an extensive literature explaining productivity differences across firms. Beyond financial frictions, alternative explanations include slow learning of average productivity in the face of noise shocks, as in Jovanovic (1982); the coexistence of older and newer technology vintages, as in Caballero and Hammour (1994), Caballero and Hammour (1996), and Caballero and Hammour (1998); search and matching frictions between workers and firms, as in Barlevy (2002); and imperfect product substitutability, as in Bernard et al. (2003) and Melitz (2003).

3 Data Motivation

We document that there are sizable and pervasive within-industry productivity differences in the U.S. manufacturing sector. Then we show that the dispersion of TFP across establishments in the same industry is negatively correlated with GDP growth. We also document that the dispersion in productivity levels is negatively associated with the default rate on bank loans and the real interest rate. The summary statistics that we construct are the same ones that we will use to assess our theoretical model.

[6] Jo et al. (2021) and Ottonello and Winberry (2020) also introduced financial frictions and firm default, but our model is more tractable and intuitive by incorporating moral hazard and strategic default; they focused on size and leverage distributions, whereas our model emphasizes consistency with productivity dispersion. Guo et al. (2025) discussed business cycles with firms' private information and equity financing.

3.1 Data Description

We draw dispersion in productivity across 86 four-digit NAICS manufacturing industries from experimental productivity dispersion statistics, Dispersion Statistics on Productivity (DiSP), derived from Census Bureau microdata.[7] The most recent release of DiSP covers the years 1987–2021 on an annual basis. Our analysis focuses on the second-moment measure of establishment-level total factor productivity.

Figure 1 shows the evolution of TFP dispersion over time. The solid line denotes the average within-industry TFP dispersion. This average is weighted by each industry's contribution to GDP.[8] The top panel considers the percentage difference in productivity between the establishment at the 90^{th} percentile of the TFP distribution and the 10^{th} percentile establishment in the same industry. To interpret the values on the vertical axis consider that one hundred percent means that the plant at the 90^{th} percentile of the productivity distribution makes twice as much output with the same measured inputs as the 10^{th} percentile plant. As shown, the average TFP gap has been sizable and relatively stable, starting at around 95 percent in 1987 and moving gradually up to a little over 120 percent in 2021.

The dot-dashed bands capture the dispersion of the same productivity gap across industries. In a point-wise fashion (year by year), we consider the 90-10 gap for each industry and highlight the gap for the industry at the 15^{th} percentile and at the 85^{th} percentile. The range spanned by these percentiles is fairly symmetric. Moreover, the within-industry productivity gap is still sizable, even at the lower range across industries. The takeaway is that the average TFP gap across all industries is not swayed by industry outliers. Large within-industry productivity differences are pervasive.

For robustness, the bottom panel repeats the same analysis for the interquartile range (IQR). We conclude that within-industry productivity differences are important quantitatively for the outer segments of the distribution as well as for segments closer to the center of the distribution.

Turning to cyclical variation, the shaded areas in Figure 1 show recessions as dated by the

[7]This dataset is described in Cunningham et al. (2023).

[8]The industry value-added data used to construct the weighted average TFP dispersion shown in Figure 1 is from the Bureau of Economic Analysis.

National Bureau of Economic Research. Focusing on the average within-industry gaps, no clear pattern is readily apparent. Regardless of whether we focus on the TFP gap between the 90^{th} and 10^{th} percentile plant, or the gap in the IQR, TFP dispersion rises in some recessions and falls on others. To check for systematic patterns taking into account the full dataset, not just average TFP gaps, we turn to some regressions next.

3.2 Regression Results

Absent a visually discernible pattern, we fall back on regression analysis to assess how within-industry productivity dispersion is related to key macroeconomic indicators. Our modest goal is to construct a set of moments that we can use to judge the empirical relevance of our model. We do not have loftier goals of establishing causal relationships. Our simple framework is the following:

$$\gamma_t = c + \beta x_t + \varepsilon_t, \tag{1}$$

where γ_t is a weighted average of the productivity gap (expressed in percent) between the 90^{th} and 10^{th} percentile plants. For weights, we use again industry value-added shares. The term x_t denotes alternative cyclical indicators (possibly included jointly), and ε_t is an error term.[9]

Table 2 shows our regression results for alternative choices of the independent variable, x_t. The first set of results in the table is for the annual growth rate of real GDP, expressed as a percentage.[10] The relationship is negative, implying that within-industry TFP dispersion is countercyclical. This finding is consistent with Cunningham et al. (2023), Bloom et al. (2018), and Kehrig (2011), among others.

In our second regression, as shown in Table 2, the independent variable is a short-term real interest rate, which we take as a simple indicator of financial conditions.[11] In this

[9] To construct industry weights, we aggregate the value added of multiple NAICS manufacturing industries that correspond to the same BLS code.

[10] We use an annual chain-type quantity index of real gross domestic product from the National Income and Products Account (NIPA) of the U.S. Bureau of Economic Analysis.

[11] We use an ex-ante real interest rate on a one-year Treasury bill from the Cleveland Fed, available from FRED, the data platform of the Federal Reserve Bank of St. Louis. This real rate is calculated with a model that uses Treasury yields, inflation data, inflation swaps, and survey-based measures of inflation expectations.

case, the estimate of the slope coefficient β is negative, implying that less costly financing is associated with weaker selection pressure and, by extension, lower allocative efficiency.

In our third specification of Table 2, the independent variable is the delinquency rate of business loans at commercial banks.[12] The coefficient is negative, implying that elevated delinquency rates are associated with lower within-industry TFP dispersion.

Different explanations are consistent with this relationship. One possible explanation for this association is that delinquency rates tend to rise before outright defaults. To the extent that lower productivity firms are more likely to default, one can expect TFP dispersion to be reduced by the default of unproductive firms.[13] But causality could also flow in the other direction.

Across specifications, the regression coefficients are significant at the 1 percent level. We progressively extend the regression specification to include all three variables jointly. Although some of the slope coefficients go down in magnitude in some of the joint regressions, their signs are unchanged. As an additional robustness check, we verify that an increase in the charge-off rates on commercial and industrial bank loans is associated with a decrease in within-industry productivity dispersion—much the same as for delinquency rates on the same type of bank loans.

4 Model

Our analysis focuses on the key role of credit markets in reallocating capital among heterogeneous firms. Each firm in the goods sector is endowed with two technologies. The first is a production technology that combines capital and labor using an idiosyncratic efficiency parameter ω drawn from a distribution on the unit interval; this draw is transitory and privately observed by the firm. The second is a financial intermediation technology, available

[12] The data on delinquency rates on business loans are from the data release Charge-Offs and Delinquency Rates on Business Loans and Leases at Commercial Banks of the Board of Governors of the Federal Reserve System. This release is based on data from the FFIEC Consolidated Report on Conditions and Income. Delinquent loans and leases are defined as those at least thirty days past due. We constructed annual averages from quarterly data.

[13] The BLS Dispersion Statistics on Productivity do not allow us to separate whether differences in TFP across quantiles of the within-industry distribution change because the lower quantiles rise or because the upper quantiles fall.

on equal terms to every firm, which allows it to lend funds to other firms at the prevailing inter-firm rate. Firms also experience persistent aggregate technology shocks that shift the productivity of the production technology for all firms simultaneously. Because the idiosyncratic efficiency of each firm's production technology is private information, households allocate an aliquot share of their savings to each firm without conditioning on ω. We will show that firms with sufficiently high realizations of ω borrow from firms with low realizations to purchase additional capital, while the latter find it optimal to deploy their financial intermediation technology instead of producing.

Unlike in a frictionless credit market, moral hazard due to asymmetric information results in misallocation of capital and reduced productivity in our framework. Moral hazard limits the borrowing capacity of the most efficient firm. Asymmetric information arises from uncertainty about borrower quality, as the production-technology efficiency ω above some threshold level is private and unobservable. Whereas the most productive firms could offer to pay a higher rate to borrow more funds, this strategy lacks credibility. Words are cheap, and all eligible firms could claim high productivity. Thus in our model, credit is rationed, as in Stiglitz and Weiss (1981).

Our interpretation of inter-firm lending reflects the broader role of credit markets in allocating credit. As Bernanke and Gertler (1990) note, this lending can be viewed as intermediated by competitive financial institutions that neither use resources nor earn profits in equilibrium. Variation in the cutoff points we just discussed will also lead to endogenous changes in the size of the intermediation sector, as the boundary between firms deploying their production technology and those deploying their financial intermediation technology shifts with aggregate conditions.

Aside from the endogenous financial intermediation, the other main innovation of the model consists of tracking the dispersion of productivity across goods-producing firms in a tractable way. Relative to a standard model, the complications will consist of characterizing three additional equilibrium objects. The first two objects are a couple of cutoff points within the distribution of firm productivity. A lower cutoff point will separate firms with such low productivity that lending to more productive firms will be more profitable than producing. Just on the other side of this cutoff will be more-productive firms that borrow in the inter-firm market. An upper cutoff point will demarcate firms that borrow and default from those that do not default. The third additional object is the price at which funds are lent across firms.

As these cutoff points and the cost of credit fluctuate, productivity differences across firms will also fluctuate, as will default rates, together with the overall efficiency of the economy. We will show that despite the simplicity of our model, key characteristics of these fluctuations are well aligned with U.S. data.

Firms are at the center of our model, but we start with households for ease of exposition.

4.1 Households

There is an infinitely-lived representative household that has preferences over consumption and labor, respectively C_t and H_t, in line with King et al. (1988). The household solves the following problem:

$$\max_{\left\{A_{t+\tau}, C_{t+\tau}, H_{t+\tau}, B^H_{t+\tau}\right\}_{\tau=0}^{\infty}} E_t \sum_{\tau=0}^{\infty} \beta^{\tau} \left[\ln(C_{t+\tau} - \nu_{ct+\tau}) - \frac{\vartheta}{1+\nu} (H_{t+\tau})^{1+\nu} \right], \tag{2}$$

subject to

$$C_{t+\tau} + A_{t+\tau} + B^H_{t+\tau} = R^A_{t+\tau} A_{t+\tau-1} + W_{t+\tau} H_{t+\tau} + R^B_{t+\tau-1} B^H_{t+\tau-1} + \Pi_{t+\tau} + T_{t+\tau} + \Xi_{t+\tau}. \tag{3}$$

In period $t+\tau$, the household chooses how much to consume, $C_{t+\tau}$, and work, $H_{t+\tau}$. The term $\nu_{ct+\tau}$ is an exogenous shock to consumption, which follows an autoregressive process of order one:

$$\nu_{ct} = \rho_{\nu} \nu_{ct-1} + \varepsilon_{\nu t}, \qquad \varepsilon_{\nu t} \sim \mathcal{N}(0, \sigma^2_{\nu}). \tag{4}$$

The parameter β discounts future utility. The parameter ϑ captures the disutility from working, and the parameter ν governs the Frisch elasticity of labor supply. The household also chooses the amount of assets $A_{t+\tau}$ and a government bond $B^H_{t+\tau}$. The household enters period $t+\tau$ with assets $A_{t+\tau-1}$ which carry a state-contingent return $R^A_{t+\tau}$ from ownership of the banking sector and receives a non-state contingent return $R^B_{t+\tau-1}$ from its holdings of the government bond $B^H_{t+\tau-1}$. It also supplies $H_{t+\tau}$ units of labor at the wage rate $W_{t+\tau}$.

The term $\Pi_{t+\tau}$ captures profits from ownership of goods-producing and capital-producing firms (the latter could be non-zero in equilibrium), and the term $T_{t+\tau}$ represents a lump-sum transfer from the government, which runs a balanced budget, period by period. Finally, the term Ξ_t refers to transfers that the household receives because firms that take the outside option, which is described below, are subject to a haircut on the returns from the outside option.

The first-order conditions for assets, consumption, labor, and government bonds are derived in Appendix B.1; see equations (B.1)–(B.4), where λ_{ct} is the Lagrange multiplier attached to the budget constraint (3).

4.2 Production and Financial Intermediation

Firms in the goods sector are at the heart of our model. The firms in this sector follow a two-period overlapping structure from period t to $t + 1$. This device is familiar from decentralizations of the RBC model that rely on equity contracts to allocate household savings to firms. Table 1 provides a roadmap to the sequence of choices made and actions taken by firms within each period. We highlight with a **boldface** font the choices or actions that are specific to our model. Lowercase letters will denote variables of individual firms; we shall reserve uppercase letters for aggregate variables.

Firms operate in a perfectly competitive market, producing a homogeneous good. Ex ante, each firm is endowed with two technologies: a production technology and a financial intermediation technology. An idiosyncratic efficiency parameter $\omega \in [0, 1]$, drawn from an identical distribution at the start of each period, governs how productive the firm would be if it chose to deploy its production technology. The financial intermediation technology, which consists of lending funds in the inter-firm market, is identical across firms. Depending on the realization of ω, firms endogenously choose which technology to deploy: those with high ω find production more profitable and borrow to expand their capital stock, while those with low ω find it optimal to act as financial intermediaries and lend their funds to higher-productivity firms.

In a slight abuse of notation, the term ω will do double duty by denoting the idiosyncratic production efficiency and by serving as an index for other firm-specific variables. Since households have no visibility into the idiosyncratic production efficiency of firms, each firm receives an aliquot share of the households' savings, a_t. So, notice that a_t does not depend on ω in equilibrium.

Starting from the production function, the output of an individual firm in period $t+1$, $y_{t+1}(\omega)$, is governed by

$$y_{t+1}(\omega) = \omega Z_{t+1} k_t(\omega)^{\alpha} h_{t+1}(\omega)^{1-\alpha}. \tag{5}$$

The terms $k_t(\omega)$ and $h_{t+1}(\omega)$ denote the levels of capital and labor inputs used by the firm. The idiosyncratic productivity ω follows the cumulative distribution function $\mu(\omega)$ on the interval $[0,1]$, satisfying $\mu(0)=0$, $\mu(1)=1$, and $\mu'(\omega)>0$. The aggregate technology shock Z_{t+1} evolves according to

$$\log Z_{t+1} = \rho_z \log Z_t + \varepsilon^z_{t+1}, \qquad \varepsilon_{zt} \sim \mathcal{N}(0, \sigma_z^2), \tag{6}$$

where the parameter ρ_z governs the persistence of the shock process.

Firms make plans in period t to produce in period $t+1$. After their idiosyncratic productivity ω is known, the intermediation market opens. Depending on their productivity, some firms borrow while others lend at the predetermined rate ρ_t. Firms that borrow decide whether to use an outside option by purchasing the government bond or produce.[14] If they choose to produce, they purchase physical capital.

Firms can walk away from loans and default. If a firm decides to default, it can retain a fraction $\Theta_t(\omega)$ of the funds borrowed from other firms $b_t(\omega)$. We assume that $\Theta_t(\omega)$ increases in ω. For simplicity, we make the average fraction of funds that can be diverted equal to θ, so

$$\theta = \int_{\overline{\omega}_t}^{\overline{\overline{\omega}}_t} \Theta_t(\omega) \frac{\mu'(\omega)}{\mu(\overline{\overline{\omega}}_t) - \mu(\overline{\omega}_t)} \mathrm{d}\omega, \tag{7}$$

where, as will be shown later, $\overline{\omega}_t$ and $\overline{\overline{\omega}}_t$ define the boundaries of defaulting firms, and $\frac{\mu'(\omega)}{\mu(\overline{\overline{\omega}}_t) - \mu(\overline{\omega}_t)}$ gives the corresponding probability density over that interval.

Creditor firms are assumed not to be able to go after any funds set aside for the outside option. The diverted assets are placed with the government bond; taking this outside option

[14] Notice that lending firms can always decide to take the outside option, which pins down ρ_t.

incurs a haircut ξ on the bond return, which is rebated in lump-sum fashion to households. The precise outside-option payoff and the role of ξ are set out below when we characterize the optimization problem of defaulting firms.

Whereas the idiosyncratic productivity of firms is not directly observed by households, we assume that firms have a bit more information. While still far from perfect, a screening technology allows lending firms to tell whether borrowers can be expected to make more than the outside option by producing. This technology prevents firms with the lowest levels of production efficiency from borrowing and defaulting, thereby supporting a mass of lenders among firms with $\omega \leq \overline{\omega}_t$.[15] Firms whose private productivity ω falls between $\overline{\omega}_t$ and $\overline{\overline{\omega}}_t$ find it advantageous to divert all available funds towards the outside option. Only firms that can make higher returns by producing will produce.

With constant returns to scale production, the economy would attain the first-best allocation if the most efficient firm, the one with $\omega = 1$, could borrow all financing from other firms, allowing it to be the only firm to produce. This allocation is hindered by the presence of moral hazard due to asymmetric information. By limiting the borrowing capacity of the most efficient firm, moral hazard gives less efficient firms room to borrow from financial intermediaries. Asymmetric information refers to uncertainty about the quality of borrowers. The productivity level above $\overline{\omega}_t$ is private information that is not observed by other firms. Producing firms may seek more funds by offering to borrow above the prevailing lending rate. However this offer lacks credibility as all firms above a certain threshold could claim high productivity. The result is credit rationing, as in Stiglitz and Weiss (1981). Consequently, all inter-firm financial contracts are identical and do not depend on ω in equilibrium.

In the next three sections, we will consider three distinct optimization problems spanning the possible combinations of actions that firms can take. Firms can borrow from other firms and produce, borrow and default, or lend. We will show that firms sort themselves in each group depending on their idiosyncratic productivity, ω. For now, take this result as a hypothesis. The proof of the hypothesis is set up in Section 5 with details pushed to Appendix A.

[15] A fully accurate screening technology would deliver the first-best allocation with only the most productive firm producing. The imperfect screening we assume guarantees instead that low-efficiency firms choose to lend rather than switch to borrowing and defaulting.

4.2.1 Firms Choosing to Produce ($\omega \geq \overline{\overline{\omega}}_t$)

Firms acquire the capital stock required for production in period $t+1$, by combining funds from households $a_t(\omega)$ with a loan $b_t(\omega)$ from the mass of lending firms; consequently,

$$b_t^{tot}(\omega) = a_t(\omega) + b_t(\omega), \tag{8}$$

where $b_t^{tot}(\omega)$ denotes the total borrowing of the firm. By contractual agreement, the funds borrowed can only be used to purchase capital; so,

$$b_t^{tot}(\omega) = Q_t k_t(\omega), \tag{9}$$

where Q_t is the price of capital in terms of consumption. Let $R_{t+1}(\omega)$ denote the rate of return on capital ownership and $\pi_{t+1}(\omega)$ describe the profit of the firm. The revenue of the firm includes the proceeds from the sale of output as well as from the sale of the undepreciated fraction of capital. The expenses encompass commitments associated with loan servicing as well as remuneration for labor services. Thus,

$$\pi_{t+1}(\omega) = z_{t+1}\omega k_t(\omega)^\alpha h_{t+1}(\omega)^{1-\alpha} + (1-\delta)Q_{t+1}k_t(\omega) - R_{t+1}(\omega)b_t^{tot}(\omega) - W_{t+1}h_{t+1}(\omega), \tag{10}$$

where δ is the depreciation rate.

At time t, the problem of this producer of goods is to choose $b_t^{tot}(\omega)$ and $k_t(\omega)$ to maximize expected profits in period $t+1$, knowing that the firm will be able to choose the optimal amount of labor in that period. This maximization problem can be expressed as:

$$\max_{k_t(\omega), b_t^{tot}(\omega)} E_t \left\{ \max_{h_{t+1}(\omega)} \beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \pi_{t+1}(\omega) \right\}, \tag{11}$$

where $\beta \frac{\lambda_{ct+1}}{\lambda_{ct}}$ is the stochastic discount factor coming from the household's problem. This maximization is subject to (9).

Under constant returns to scale, both labor-to-capital and output-to-labor ratios are equalized across firms with different productivity levels and do not depend on ω. Thus, they coincide with the corresponding ratios of the aggregate variables. The first-order conditions imply:

$$W_t = (1-\alpha)\,\frac{Y_t}{H_t}, \tag{12}$$

$$R_{t+1}(\omega) = \frac{1}{Q_t}\alpha\ Z_{t+1}\left(\frac{H_{t+1}}{K_t}\right)^{1-\alpha}\omega + \frac{(1-\delta)}{Q_t}Q_{t+1} \tag{13}$$

under every state of nature. Equation (12) describes the demand for labor of the individual firm. Equation (13) is the zero-profit condition that can be interpreted as follows: $\frac{1}{Q_t}$ is the capital gained from one unit of consumption. This capital $\frac{1}{Q_t}$ earns a rental rate $\alpha Z_{t+1}\left(\frac{H_{t+1}}{K_t}\right)^{1-\alpha}\omega$ equal to the marginal product of capital. After production, the undepreciated part of capital can be resold at the price Q_{t+1}, so capital gains equal $\frac{(1-\delta)}{Q_t}Q_{t+1}$.

Upon observing ω, the producing firm determines the demand for inter-firm loans by solving the following problem:

$$\max_{b_t(\omega)|\omega} E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\Big(R_{t+1}(\omega)\,(a_t(\omega)+b_t(\omega)) - R^A_{t+1}(\omega)a_t(\omega) - \rho_t b_t(\omega)\Big)\right]. \tag{14}$$

It generates returns from owning capital by utilizing borrowed funds and distributes these returns to meet equity commitments of households and repay loans from other firms.

Note that firms that produce, given constant returns to scale technology, would choose not to borrow in the inter-firm market if the returns to production were lower than the cost of inter-firm funding. The inter-firm rate is such that $\rho_t \leq E_t R_{t+1}$. Thus, this maximization problem implies that firms that borrow with the intent of producing will be interested in borrowing as much as possible. Accordingly, supply conditions will have to determine how much these firms can borrow.

We use the zero-profit condition that holds under every state of nature to size the return paid to households by firms in this segment, i.e.

$$R^A_{t+1}(\omega) = \frac{R_{t+1}(\omega)\,(a_t(\omega)+b_t(\omega)) - \rho_t b_t(\omega)}{a_t(\omega)}. \tag{15}$$

4.2.2 Firms Choosing to Borrow and Default ($\overline{\omega}_t < \omega < \overline{\overline{\omega}}_t$)

The problem of the firm that diverts the borrowed funds by choosing the outside option can be described as follows:

$$\max_{l_t(\omega)|\omega} E_t \left[\beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \Big(\left(R_t^B - \xi\right) \left(a_t(\omega) + \Theta_t(\omega) b_t(\omega)\right) - R_{t+1}^A(\omega) a_t(\omega) \Big) \right]. \tag{16}$$

The firm earns returns by investing borrowed funds in the government bond and uses these to pay the returns on equity to households. When taking the outside option, a haircut ξ is applied to the yield R_t^B on the government bond. The term $\Theta_t(\omega)$ reflects that only a fraction $\Theta_t(\omega) > 0$ of the funds borrowed in the inter-firm market can be retained by the borrower when diverting the funds. Notice that this firm will be glad to borrow as much as possible, just as long as government bonds minus a haircut ξ pay a positive return. Therefore, similar to the previous situation, the supply conditions dictate the borrowing capacity of the firms in this segment.

We use the zero-profit condition that holds under every state of nature to size the return paid to households by firms in this segment, i.e.

$$R_{t+1}^A(\omega) = \frac{\left(R_t^B - \xi\right)\left(a_t(\omega) + \Theta_t(\omega) b_t(\omega)\right)}{a_t(\omega)}. \tag{17}$$

4.2.3 Firms Choosing to Lend ($\omega \leq \overline{\omega}_t$)

The problem of the firm that lends in the inter-firm market collapses to

$$\max_{l_t(\omega)|\omega} E_t \left[\beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \left(\rho_t \frac{1 - \mu(\overline{\overline{\omega}}_t)}{1 - \mu(\overline{\omega}_t)} l_t(\omega) + \frac{\mu(\overline{\overline{\omega}}_t) - \mu(\overline{\omega}_t)}{1 - \mu(\overline{\omega}_t)} (1 - \theta) l_t(\omega) - R_{t+1}^A(\omega) a_t(\omega) \right) \right], \tag{18}$$

subject to the constraint that $l_t(\omega) \leq a_t(\omega)$.

The term $\frac{1 - \mu(\overline{\overline{\omega}}_t)}{1 - \mu(\overline{\omega}_t)}$ represents the share of loans to firms that choose to produce, yielding the return ρ_t. The term $\frac{\mu(\overline{\overline{\omega}}_t) - \mu(\overline{\omega}_t)}{1 - \mu(\overline{\omega}_t)}$ represents the share of loans to firms that choose to divert the borrowed funds, yielding the average fraction that can be recovered, $1 - \theta$.[16] The

[16]Proposition 2 establishes that $\overline{\overline{\omega}}_t > \overline{\omega}_t$, ensuring that the share of loans to firms that choose to produce and the share of loans to firms that choose to divert the borrowed funds are non-negative and not greater than 1.

firm pays the household the return on assets $R^A_{t+1}(\omega)$.

Notice that the revenues of the firm are increasing in the amount of loans it supplies. Thus, the budget constraint will hold with equality:

$$l_t(\omega) = a_t(\omega). \tag{19}$$

The first-order condition with respect to $l_t(\omega)$ is

$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(\rho_t\frac{1-\mu(\overline{\overline{\omega}}_t)}{1-\mu(\overline{\omega}_t)} + \frac{\mu(\overline{\overline{\omega}}_t)-\mu(\overline{\omega}_t)}{1-\mu(\overline{\omega}_t)}(1-\theta) - R^A_{t+1}(\omega)\right)\right] = 0. \tag{20}$$

Notice that this first-order condition is implied by the stronger zero-profit condition that holds under every state of nature and that we use to size the return paid to households by firms in this segment, i.e.

$$R^A_{t+1}(\omega) = \rho_t\frac{1-\mu(\overline{\overline{\omega}}_t)}{1-\mu(\overline{\omega}_t)} + \frac{\mu(\overline{\overline{\omega}}_t)-\mu(\overline{\omega}_t)}{1-\mu(\overline{\omega}_t)}(1-\theta). \tag{21}$$

4.3 Capital-Producing Firms

In period t, competitive capital-producing firms buy capital from the goods-producing firms, repair depreciated capital and build new capital. They sell both the new and refurbished capital next period.

Let I^g_t denote aggregate gross investment expenditures. We introduce quadratic adjustment costs measured in units of investment such that the supply of investment goods is given by:

$$I^n_t = Z_{It}\left[1-\frac{\phi}{2}\left(\frac{I^g_t}{I^g_{t-1}}-1\right)^2\right]I^g_t. \tag{22}$$

The parameter ϕ governs investment adjustment costs for current production relative to past production. The term Z_{it} is a shock process driven by

$$\ln(Z_{It}) = \rho_I\ln(Z_{It-1}) + \varepsilon_{It}, \qquad \varepsilon_{It}\sim\mathcal{N}(0,\sigma^2_I). \tag{23}$$

The aggregate capital stock evolves according to:

$$K_t = I_t^n + (1-\delta)K_{t-1}, \tag{24}$$

where K_t is the amount of capital allocated to the goods-producing firms.

The capital producing firms are owned by households, and solve the problem

$$\max_{I^g_{t+i}} E_t \sum_{i=0}^{\infty} \beta^i \frac{\lambda_{ct+i}}{\lambda_{ct}} \left\{ Q_{t+i} Z_{It+i} \left[1 - \frac{\phi}{2} \left(\frac{I^g_{t+i}}{I^g_{t+i-1}} - 1 \right)^2 \right] I^g_{t+i} - I^g_{t+i} \right\}. \tag{25}$$

The first-order condition, derived in Appendix B.2 (equation (B.5)), determines the equilibrium price of capital Q_t via a standard Tobin's q condition.

Competition will ensure zero profits for the goods-producing firms. Accordingly, the profits rebated to households as aliquot shares will be given by

$$\Pi_t = Q_t Z_{It} \left[1 - \frac{\phi}{2} \left(\frac{I^g_t}{I^g_{t-1}} - 1 \right)^2 \right] I^g_t - I^g_t. \tag{26}$$

4.4 The Government

The government finances its transfers T_t by issuing government bonds B^G_t to balance its budget period by period:

$$T_t = B^G_t - R^B_{t-1} B^G_{t-1}. \tag{27}$$

The household and firms can buy government bonds, so

$$B^G_t = B^H_t + D_t, \tag{28}$$

where D_t denotes firms' holdings of government bonds. We assume that the government does not make bonds available to households but sells them only to firms, so that

$$B^H_t = 0 \tag{29}$$

and only use the households' stochastic discount factor to price the government bond. It implies that the government budget constraint can be written as:

$$T_t = D_t - r^B_{t-1} D_{t-1}. \tag{30}$$

Since firms that take the outside option are subject to a haircut cost ξ on their investment in government bonds in the previous period, the amount of transfers rebated to the household to ensure that there are no deadweight losses in the economy is equal to

$$\Xi_t = \xi D_{t-1}. \tag{31}$$

5 Analytical Characterization of the Equilibrium

This section characterizes the equilibrium analyticially and derives its key implications for aggregate output and measured TFP. Section 5.1 provides conditions that pin down the loan rate ρ_t and two cutoff points $\overline{\omega}_t$ and $\overline{\overline{\omega}}_t$. These cutoff points sort firms into three segments depending on the realization of the idiosyncratic productivity shock: 1) firms with $\omega \leq \overline{\omega}_t$ lend; 2) firms with $\overline{\omega}_t < \omega < \overline{\overline{\omega}}_t$ borrow and default; 3) firms with $\omega \geq \overline{\overline{\omega}}_t$ borrow and produce. Our strategy is to posit a solution that satisfies the first-order conditions of the intermediate goods sector and then verify that no firm can or has an incentive to switch to a different segment. To this purpose, we formulate three propositions. The aggregation of individual firm decisions and the formal definition of competitive equilibrium are left for Appendix B.3. Section 5.2 then integrates individual firm outputs across the producing segment to obtain aggregate output, and shows how the equilibrium cutoff $\overline{\overline{\omega}}_t$ generates an endogenous component of the measured Solow residual.

5.1 The Loan Rate and Cutoff Points

Here we clarify the importance of the screening technology that allows lenders to tell whether borrowers can be expected to make more than the outside option. This technology prevents the firms with the lowest firm-specific technology levels from borrowing.

Assume (and then verify) that if the firms that have a firm-specific productivity $\omega < \overline{\omega}_t$ will choose to lend, then the following condition holds:

$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(\frac{1}{Q_t}\alpha\ Z_{t+1}\left(\frac{H_{t+1}}{K_t}\right)^{1-\alpha}\overline{\omega}_t+\frac{(1-\delta)}{Q_t}Q_{t+1}\right)\right]=E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R^B_t-\xi\right)\right]. \tag{32}$$

This condition imposes that for a firm with productivity right at the cutoff point between borrowing and lending, $\omega = \overline{\omega}_t$, the expected return from producing $R_{t+1}(\overline{\omega}_t)$ equals the expected return of the outside option $R^B_t - \xi$.

We also need to impose an additional condition to induce a firm to lend. We need to compare the return from lending to the return from diverting funds. Firms that lend will only lend as long as the return from lending, the lending rate net of the losses expected from firms diverting funds, matches the returns from borrowing and defaulting:

$$\begin{aligned}&E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(\rho_t\frac{1-\mu(\overline{\overline{\omega}}_t)}{1-\mu(\overline{\omega}_t)}+(1-\theta)\frac{\mu(\overline{\overline{\omega}}_t)-\mu(\overline{\omega}_t)}{1-\mu(\overline{\omega}_t)}\right)a_t\right]=\\&E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R^B_t-\xi\right)\left(a_t+\Theta_t(\overline{\omega}_t)b_t\right)\right].\end{aligned} \tag{33}$$

Notice there is no dependence of b_t on ω because $b_t = b_t(\omega)$ for all ω due to asymmetric information. At this point, we have spelled out all the elements that are needed for our first proposition.

Proposition 1. *Given that $\Theta_t(\omega)$ is non-negative and increasing in ω, firms with idiosyncratic productivity ω less than the cutoff point $\overline{\omega}_t$ will have no incentive to deviate from lending.*

A proof of this proposition is offered in Appendix A. The proof relies primarily on equations (32) and (33).

For the next step of pinning down $\overline{\overline{\omega}}_t$, consider the following condition:

$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R^B_t-\xi\right)\left(a_t+\Theta_t(\overline{\overline{\omega}}^*_t)b_t\right)\right]=E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_{t+1}(\overline{\overline{\omega}}^*_t)(a_t+b_t)-\rho_t b_t\right)\right]. \tag{34}$$

It imposes that the marginal firm with productivity level $\overline{\overline{\omega}}^*_t$ will be indifferent between diverting funds and producing.

Taking the first derivative with respect to $\overline{\overline{\omega}}_t^*$ of both sides of equation (34), we will also need to impose the following condition on these derivatives:

$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_t^B-\xi\right)\Theta_t'(\overline{\overline{\omega}}_t^*)b_t\right]<E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_{t+1}'(\overline{\overline{\omega}}_t^*)(a_t+b_t)\right)\right]. \tag{35}$$

In other words, for a marginal firm with the productivity level $\overline{\overline{\omega}}_t^*$ the expected returns from diverting funds grow less steeply than the expected returns from producing. This condition is really an additional condition on the choice of the function $\Theta_t(\omega)$. It implies that a firm with a slightly higher productivity than $\overline{\overline{\omega}}_t^*$ will prefer producing to diverting funds, while a firm with a slightly lower productivity than $\overline{\overline{\omega}}_t^*$ will prefer diverting funds to producing. Note that it is a local condition in the sense that it applies to a marginal firm with the productivity level $\overline{\overline{\omega}}_t^*$.

We define $\overline{\overline{\omega}}_t=\max(\overline{\omega}_t,\overline{\overline{\omega}}_t^*)$. This definition ensures that terms that represent shares in the expression of the return from lending, for instance, $\frac{\mu(\overline{\overline{\omega}}_t)-\mu(\overline{\omega}_t)}{1-\mu(\overline{\omega}_t)}$ in equation (33), are well-defined. But we can do more and establish that $\overline{\overline{\omega}}_t^*$ is never less than $\overline{\omega}_t$, in line with the following proposition.

Proposition 2. *Take $\overline{\overline{\omega}}_t^*$ to be the productivity of a firm indifferent between diverting funds and producing. Given that $\Theta_t(\omega)$ is non-negative, then it must be that $\overline{\overline{\omega}}_t^*\geq\overline{\omega}_t$.*

Notice that, by definition of $\overline{\overline{\omega}}_t$, a corollary of Proposition 2 is that $\overline{\overline{\omega}}_t=\overline{\overline{\omega}}_t^*$.

Before moving to our final proposition, we need to introduce one more condition on the function $\Theta(\omega)$. Assuming the function is convex, we need to choose a function such that

$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_t^B-\xi\right)\Theta_t'(1)b_t\right]<E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(\frac{1}{Q_t}\alpha\ Z_{t+1}\left(\frac{H_{t+1}}{K_t}\right)^{1-\alpha}\right)(a_t+b_t)\right]. \tag{36}$$

The left-hand side of the inequality is the maximum bound of the first derivative of the returns to borrowing and defaulting, whereas the right-hand side is the first derivative of the returns of firms that produce. Given that this condition holds for the most efficient firm with $\omega=1$ that has the largest marginal benefits from diversion, it will also hold for all firms with $\omega<1$ whose marginal benefit from diversion is smaller (including $\omega=\overline{\overline{\omega}}_t$). Accordingly, this global condition implies the local condition (35). We can use this global condition to prove that no diverting firm has an incentive to deviate by producing, and no producing firm has an incentive to switch to diverting. With this condition specified, we can state our final proposition.

Proposition 3. *Given that $\Theta_t(\omega)$ is non-negative, convex and increasing in ω, equations (32), (33), and (34), together with the slope condition under (36) are sufficient to ensure that, depending on the realization of their idiosyncratic productivity ω, firms sort themselves into three groups:*

1. *firms with $\omega \leq \overline{\omega}_t$ lend;*

2. *firms with $\overline{\omega}_t < \omega < \overline{\overline{\omega}}_t$ borrow and default;*

3. *firms with $\omega \geq \overline{\overline{\omega}}_t$ borrow and produce.*

A proof of this proposition is in Appendix A.

5.2 Aggregate Output and the TFP Decomposition

Once the propositions above pin down the equilibrium cutoffs, individual producing-firm outputs can be integrated across the mass of producing firms. The full derivation is in Appendix B; the key step uses the fact that under constant returns to scale every producing firm employs the same capital–labor ratio, so aggregate output is proportional to the integral of idiosyncratic productivity ω over the producing segment:

$$Y_t = \frac{Z_t\, K_{t-1}^{\alpha}\, H_t^{1-\alpha}}{1 - \mu(\overline{\overline{\omega}}_{t-1})} \int_{\overline{\overline{\omega}}_{t-1}}^{1} \omega\, \mu'(\omega)\, \mathrm{d}\omega. \tag{37}$$

This equation already reveals the central insight: measured TFP—the Solow residual defined by $Y_t/(K_{t-1}^{\alpha} H_t^{1-\alpha})$—equals the exogenous shock Z_t multiplied by the conditional expectation of ω among producing firms, $E[\omega \mid \omega > \overline{\overline{\omega}}_{t-1}]$. Because the cutoff $\overline{\overline{\omega}}_{t-1}$ responds to financial conditions, the Solow residual contains an endogenous component even when Z_t is held fixed. Evaluating the integral in closed form requires a distributional assumption on μ.

5.2.1 Functional Form for the Distribution of Idiosyncratic Productivity

We choose the Beta distribution to govern draws of the idiosyncratic productivity ω. With parameters η_1 and η_2, the probability density function is

$$\mu'_{\eta_1,\eta_2}(\omega) = \frac{\omega^{\eta_1-1}(1-\omega)^{\eta_2-1}}{B(\eta_1,\eta_2)}, \tag{38}$$

where $B(\eta_1,\eta_2) = \Gamma(\eta_1)\Gamma(\eta_2)/\Gamma(\eta_1+\eta_2)$ and Γ is the Gamma function. This distribution affords the flexibility to match large differences in productivity across firms that choose to produce and borrow, while capturing that financial intermediaries account for a large fraction of the financing available to producing firms.

We also need to specify the diversion function $\Theta_t(\omega)$, which governs the fraction of borrowed funds $b_t(\omega)$ that a firm can divert. We choose

$$\Theta_t(\omega) = \omega^{\psi} F_t, \tag{39}$$

where ψ is a parameter and F_t is determined endogenously to satisfy

$$\theta = \int_{\overline{\omega}_t}^{\overline{\overline{\omega}}_t} \omega^{\psi} F_t \frac{\mu'(\omega)}{\mu(\overline{\overline{\omega}}_t) - \mu(\overline{\omega}_t)} \mathrm{d}\omega. \tag{40}$$

This functional form ensures the slope condition (36) holds, which is used in the proof that ω uniquely determines each firm's segment. We verify numerically that the condition is satisfied for all shock realizations; setting $\psi = 2$ has not produced a violation.

5.2.2 Closed-form Decomposition of TFP into Endogenous and Exogenous Components

With the Beta distribution, the integral in equation (37) evaluates in closed form. Specifically, by using the recurrence relation $\Gamma(z+1) = z\,\Gamma(z)$ to rewrite $\omega \cdot \mu'_{\eta_1,\eta_2}(\omega)$ as a scalar multiple of the Beta(η_1+1,η_2) density. The full derivation is provided in Appendix B, equation (B.45). Aggregate output then simplifies to

$$Y_t = \Phi_t \cdot Z_t \cdot K_{t-1}^{\alpha} H_t^{1-\alpha}, \tag{41}$$

where the endogenous TFP prefactor is

$$\Phi_t \;\equiv\; \frac{\eta_1}{\eta_1+\eta_2}\frac{1-\mu_{\eta_1+1,\eta_2}(\overline{\overline{\omega}}_{t-1})}{1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_{t-1})}. \tag{42}$$

The prefactor Φ_t equals the conditional expectation of ω among producing firms under the Beta(η_1, η_2) distribution, which in turn equals $\eta_1/(\eta_1 + \eta_2)$ times the ratio of upper-tail survival probabilities of a Beta($\eta_1 + 1, \eta_2$) to a Beta(η_1, η_2) distribution evaluated at $\overline{\overline{\omega}}_{t-1}$.

Equations (41)–(42) give the TFP decomposition a precise quantitative form. The measured Solow residual is

$$\text{TFP}_t \;\equiv\; \frac{Y_t}{K_{t-1}^{\alpha} H_t^{1-\alpha}} \;=\; \underbrace{\Phi_t}_{\text{endogenous}} \cdot \underbrace{Z_t}_{\text{exogenous}} .$$

In a frictionless RBC model, $\overline{\overline{\omega}}_t = 1$ for all t, so Φ_t is constant and the Solow residual moves one-for-one with Z_t. In our model, $\overline{\overline{\omega}}_{t-1}$ responds endogenously to financial conditions, causing Φ_t to fluctuate. A tightening that raises the default cutoff concentrates production among higher-productivity firms and raises Φ_t; a loosening has the opposite effect. The Solow residual therefore inherits variation from both the exogenous technology process and the endogenous reallocation of production across firms. Section 7.1 and the quantitative analysis document the empirical importance of this channel.

6 Parameter Choices and Model Solution

We now turn to the quantitative implementation of the model. This section is organized as follows. Section 6.1 describes the choice of parameter values, combining steady-state calibration targets with a simulated method of moments estimation of the shock processes and adjustment-cost parameter. Section 6.2 outlines the second-order perturbation solution method. Section 6.3 reports the targeted and untargeted moments and assesses the overall fit of the model.

6.1 Parameters

We choose model parameters with a mix of estimation and calibration. Specifically, we estimate model-specific parameters using the simulated method of moments (SMM) on second moments. We pin down other parameters by matching first moments. Finally, we set a few parameters to standard values from the literature.

We use the simulated method of moments (SMM) to size the shock processes, total factor productivity, Z_t, investment-specific technology, Z_{It}, and the consumption shock ν_{ct}. We model each shock as an AR(1) process. We size the persistence parameters and standard deviations of the innovations. Along with sizing these shocks, we also estimate by SMM the investment adjustment cost parameter, ϕ.

For the implementation of the SMM estimation, we minimize a quadratic objective function based on variances, correlations, and autocorrelations of real GDP, real consumption, the relative price of investment, and the average delinquency rate on business loans at commercial banks.[17]

Our data run from the first quarter of 1987 through the fourth quarter of 2021, matching the sample period in Section 3, a choice dictated by the availability of data on the dispersion of productivity. The data moments for the SMM exercise are computed after HP-filtering the data (using a value for the smoothing parameter of 1,600 as standard for quarterly data). We take the same approach for observed and simulated data. For the SMM objective function, we employ a modified optimal weighting matrix with model moments from the average of 10 simulated samples of the same length as the observed data.[18]

We choose selected parameters to match steady-state targets for key variables, using first

[17]For GDP and relative prices we used data from the NIPA Release of the U.S. Bureau of Economic analysis. We use chain-type indexes. The relative price of investment is the ratio of the price index for gross private domestic investment to the price index for personal consumption excluding food and energy. The delinquency rate on business loans at commercial banks is from the data release Charge-Off and Delinquency Rates on Loans and Leases at Commercial Banks of the Board of Governors of the Federal Reserve System. The series for real GDP and delinquency rates are the same as those used in Section 3. In our simple model, there is no distinction between delinquency rates, charge-off rates, and the outright default rate of borrowers.

[18]Specifically, in our modification of the SMM optimal weighting matrix, we replace the weight on the real GDP-default rate correlation with the matrix's maximum diagonal element, the weights on the investment price variance and default rate variance each with half that maximum diagonal element.

moments from the same sample period as the SMM estimation. We size the two parameters of the Beta distribution η_1 and η_2, the haircut ξ, and the diversion share θ at values that jointly match the 1) within-industry dispersion of productivity as captured by the weighted average gap in TFP between the 90^{th} and 10^{th} percentile firm, 2) the average spread between the return on assets for households and the risk-free rate of the outside option of firms, 3) the average share of bank credit, and 4) the average delinquency rate on business loans.[19] The idea behind these choices of the parameters is the following. The shape of the productivity distribution is important to match that the establishment at the 90^{th} percentile is 2.08 as productive as the one at the 10^{th} percentile. We compute the target ratio of weighted-average within-industry dispersion over the sample period by taking the mean of the data points underlying the solid line in the top panel of Figure 1. Appendix D provides formulas to calculate this target ratio from the model. We map lending firms into stylized banks. The parameter ξ is relevant for matching that banks provide about 47% of private credit in the United States.[20] We equate this value to our model counterpart expression $\frac{L_t}{B_t^{tot}}$ evaluated in the steady state. We use the same data on delinquent loans as in Section 3 to size the average delinquency rate at 2.6%. We map it to the share of borrowing firms diverting funds $\frac{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t)-\mu_{\eta_1,\eta_2}(\overline{\omega}_t)}{1-\mu_{\eta_1,\eta_2}(\overline{\omega}_t)}$ to calibrate θ. We match the average spread between the bank prime loan rate and a one-year Treasury bill with the spread between the return to assets for households and the risk-free rate for the outside option of firms. The prime rate is a good match for R_t^A, since it is an unsecured (defaultable) rate.[21]

Before discussing SMM-estimated parameters, we first outline the calibration of the remaining parameters. Calibrated values appear at the top of Table 3 and follow guidance from prior literature: the discount rate $\beta = 0.9925$, the capital share $\alpha = 0.3$, the depreciation rate $\delta = 0.01$, the inverse Frisch elasticity of labor supply $\nu = 2$. Under KPR preferences, the utility function features log utility in consumption, so $\sigma = 1$ is imposed implicitly and

[19] The shape of the productivity distribution affects all the described targets, so the word "*jointly*" follows. For exact identification, we consider four variables to match four targets.

[20] We construct our measure using data from the Z.1 Financial Accounts of the United States. We sum non-financial corporate and non-financial non-corporate business loans, subtract non-financial business other loans and advances, divide by total credit to non-financial corporations from the Bank for International Settlements, and take the average.

[21] The data for the bank prime loan rate and the return on one-year Treasury bills are from the Federal Reserve Board's H.15 Release.

is not a free parameter.

We set ϑ at 0.84, a value that ensures that aggregate labor takes a value of 1 in the steady state.

6.2 Model Solution

We solve the model with an exact second-order perturbation method using the pruning algorithm of Kim et al. (2008). When drawing data samples from the model, we found that a first-order perturbation method was not sufficiently accurate to avoid the upper cutoff point $\overline{\overline{\omega}}_t$ falling below the lower cutoff point $\overline{\omega}_t$, a theoretical impossibility. By contrast, a second-order perturbation solution is sufficiently accurate to avoid this problem. We provide further accuracy checks in Section G of the online appendix.

6.3 Simulated Method of Moments, Results

Table 4 reports confidence intervals for both the data moments and the model moments alongside the point estimates. Beyond summarising sampling uncertainty, the width of the data confidence intervals also reflects the weight each moment receives in the SMM objective: because the estimation uses the inverse of the long-run variance of the sample moments as its weighting matrix, a wider data interval indicates a noisier moment that is correspondingly downweighted. The model matches most of the targeted second moments well: the data and model confidence intervals overlap for 10 of the 14 moments. The four variances are all matched within sampling uncertainty, as are the correlations involving GDP, consumption, and the default rate, and the autocorrelations of GDP and consumption. For four moments—the correlation between GDP and the investment price, the correlation between the investment price and the default rate, and the first-order autocorrelations of the investment price and the default rate—the confidence intervals do not overlap, indicating a quantitatively significant discrepancy. Nonetheless, for every moment in the table, including these four, the model correctly matches the sign.

Table 5 reports the relative contributions of the shocks in our calibration to fluctuations in variables targeted by the SMM procedure and other key variables. Aggregate TFP shocks are the primary drivers of fluctuations in GDP, but we find that consumption shocks are the most important driver of TFP dispersion. The table also shows that ISP shocks are less important for our model to hit the moments targeted in the SMM estimation procedure.

7 Dynamic Responses to Shocks and Model Assessment

Before turning to an assessment of the model by checking untargeted moments, Section 7.1 discusses the effects of an aggregate TFP shock. We compare our model to a frictionless RBC model, a special case of our model in which the most efficient firm is the only one to produce and attracts all household savings.[22] The analysis in this section also lays the groundwork for Section 8, which asks how much of the endogenous TFP response depends specifically on the strategic-default margin by contrasting the baseline model with a variant in which that margin is absent.

7.1 Effects of a TFP Shock

Figure 2 shows the responses of key aggregate variables to a one-standard-deviation expansionary TFP shock, ε_t^z. The size of the shock process and its persistence follow the estimates from Section 6. In each panel, the solid lines show responses from the baseline model with financial frictions and endogenous TFP dispersion. The dashed lines show responses in a frictionless RBC model, a special case of our model in which all production is carried out by the most efficient firm.

On impact (period 1 in the figure), the positive productivity shock increases the marginal product of capital, which in turn raises the average return on capital. It is this standard exogenous productivity that pushes up aggregate output on impact. An endogenous TFP channel kicks in from the second period.

Since the shock is autocorrelated, today's TFP shock reduces the marginal product of capital expected for the next period. In line with this effect, the demand for capital af-

[22] Section E of the appendix proves that when firms can credibly disclose their productivity, our model collapses to a standard RBC model.

ter initially expanding starts declining. This expected decline brings down the expected price of capital. Accordingly, the average return to capital falls. This return is averaged across firms with different idiosyncratic productivity, and it includes a term that reflects the marginal product of capital and another term that captures gains or losses from the resale of undepreciated capital. It is this second term that pushes down the average return on capital.

As a reminder, Equation (34) pins down $\overline{\overline{\omega}}_t$.[23] From this equation, we can see that, all else equal, a decrease in the average return of capital pushes up $\overline{\overline{\omega}}_t$. While many of the variables entering equation (34) do change, the decrease in the average return of capital is the most important quantitatively. Intuitively, more productive firms are better able to withstand a decrease in the average return to capital. As $\overline{\overline{\omega}}_t$ rises, the productivity dispersion declines, as captured by a reduction in the gap between the productivity of the 90^{th} and 10^{th} percentile firms.

As the aggregate technology shock is expected to gradually wane, the risk-free interest rate declines. This decline makes the outside option less attractive. Accordingly, the default rate goes down. In turn, the returns to lending increase. As more firms find it attractive to lend, the productivity of a firm indifferent between lending and borrowing, $\overline{\omega}$, increases.

Notice that as $\overline{\overline{\omega}}_t$ does not rise quite as much as $\overline{\omega}_t$, the default rate goes down. Accordingly, default rates move countercyclically. More generally, expansionary shocks can lead to an increase or a decrease in the default rate depending on whether the effects on the expected returns dominate or the effects through the outside option dominate.

It is useful to keep the structural roles of the two cutoffs distinct, in anticipation of the comparison developed in Section 8. The upper cutoff $\overline{\overline{\omega}}_{t-1}$ enters aggregate production directly: it is the threshold below which firms choose to default rather than repay, so only firms with $\omega \geq \overline{\overline{\omega}}_{t-1}$ contribute to output in period t. It is therefore $\overline{\overline{\omega}}_{t-1}$ that governs the endogenous prefactor Φ_t and shapes measured TFP. The lower cutoff $\overline{\omega}_t$, by contrast, is purely forward-looking: it marks the boundary between lenders and producers and responds to current and expected-future conditions, but it does not appear directly in the aggregate production function. The two cutoffs co-move in equilibrium, yet their separate determination allows the production-relevant margin ($\overline{\overline{\omega}}$) and the lending-incentive margin ($\overline{\omega}$) to

[23]The aggregate counterpart of equation (34) is given in Section B of the online appendix.

absorb shocks with some degree of independence. This independence is precisely what is lost in the no-default model, where a single cutoff must play both roles simultaneously.

Due to a fall in the productivity dispersion, investment rises more than in a frictionless RBC model. Total output also increases by more. On impact, consumption rises relatively less compared to a frictionless RBC model as consumption is crowded out by more productive investments. Subsequently, starting from the next period, consumption rises relatively more due to higher total output.

Summing up, financial frictions and firm dispersion lead to persistent output and interest rate dynamics.

7.2 Effects of Other Shocks

Figure 3 shows the responses of key aggregate variables to a one-standard-deviation shock to consumption preferences, $\varepsilon_{\nu t}$, leading to an expansion in consumption. Again, the size of the shock process and its persistence follow the estimates from Section 6.

The differences between our model and the canonical RBC model are more dramatic for this shock. Starting from the risk-free rate, its rise is supercharged relative to the rise in the RBC model. Two forces cumulate to produce this difference, a substitution and a wealth effect. By construction, the shock has a pronounced substitution effect, which results in a rise in the risk-free rate in both models. In a closed-economy setting which precludes the possibility of borrowing from abroad, the rise is necessary to push down investment leaving room to consume more today for any level of labor input. But in our model with financial frictions, this rise in the risk free rate also affects the outside option for defaulting firms. There is a rise in borrowing costs and an increase in defaults. The rise in borrowing costs also contributes to concentrating production in higher productivity firms—the ones more able to withstand increases in borrowing costs. As a result, the productivity dispersion falls. This fall in dispersion is the extra endogenous kick to productivity which is the main determinant of the difference between the evolution of ouput across the two models. But consider that this effect generates a wealth effect that boosts consumption by more, hence the need for a much larger rise in the risk-free interest rate in our model relative to the frictionless model.

Although the differences between the two models are quite persistent, we have confirmed that both models are stationary. The model with financial frictions does return to its steady state at horizons well beyond the one for the figure.

Figure 4 shows the responses of key aggregate variables to an expansionary one-standard-deviation shock to investment technology ε_{It}. And once again, the size of the shock process and its persistence follow the estimates from Section 6.

Investment-specific technology shocks generate strong substitution effects as the benefits of higher productivity can only be accrued by boosting investment. Accordingly very similar considerations apply as for consumption shocks. The real interest rate needs to rise to curb consumption in this case. Default rates rise, as the rise in borrowing costs is the dominant force, dispersion decreases, and total factor productivity rises endogenously, giving rise to a sizable wealth effect (relative to the size of the shock). As Section 8 makes clear, the strategic-default margin plays a dual role throughout: it acts as an amplifier by concentrating production among high-productivity firms, and simultaneously as a stabilizer that prevents the oscillatory TFP dynamics that emerge when a single cutoff must govern both the selection of producers and the lending incentive.

7.3 Untargeted Moments

Having illustrated the response of the model to the shocks we used for SMM estimation, we can now turn to an assessment of unconditional moments. To check whether the model is consistent with the regression results discussed in Section 3, we draw 1,000 data samples, each of the same length as the observed data.[24] For each of the 1,000 samples, we estimate regressions analogous to those of Section 3. Table 6 allows comparisons between slope coefficients of regressions on model-simulated data (averaged over 1,000 samples) and observed data.

As shown in Table 6, the match between model and data results is particularly good for the coefficient on GDP growth. The sign is matched across model and data columns and the results are significantly different from zero in both cases. Moreover, the confidence intervals for the regressions on model and observed data overlap. The slope coefficient on

[24]Since the model is calibrated at quarterly frequency while observed data are annual, we lower the data frequency to yearly by averaging the quarterly model-simulated data.

the real interest rate is positive for model generated data, but it is insignificant. Finally, the coefficients on the delinquency rate for the regressions on model and observed data are an exact match, although the coefficient is insignificant for the regression on model data.

In sum, given that no second moments involving the productivity dispersion measure were directly targeted in the model calibration, it is remarkable that the match between regressions on model and observed data is this good.

8 Endogenous TFP Dynamics With and Without Strategic Default

To sharpen the intuition for how financial frictions shape measured TFP, we contrast the baseline model with a simplified variant in which strategic default is ruled out by assumption. The comparison between these two model variants allows us to showcase the important influence of endogenous selection through default in driving the endogenous component of TFP.

In the *no-default model*, the outside option is unavailable, so no firm has an incentive to divert borrowed funds; the two-cutoff structure of the baseline collapses to a single cutoff $\overline{\omega}_t$ that separates lenders from producers. All other model blocks—the household problem, capital-producing firms, and the shock processes—are unchanged. Because no funds are diverted, the government sector drops out entirely, and the inter-firm loan rate is pinned directly by the risk-free rate less the intermediation spread. Appendix F develops this variant in full detail.

Figure 5 presents a side-by-side comparison of the TFP decomposition for the two models across all three shocks keeping the model parameters unchanged across models. In each panel, the stacked areas display the two components of the Solow residual introduced in Section 5.2: the blue area is the exogenous component Z_t, the orange area is the endogenous prefactor Φ_t, and the top of the stack traces total measured TFP. The left column shows the baseline (two-cutoff) model; the right column shows the no-default (one-cutoff) model. The y-axis scale is shared within each row so that magnitudes are directly comparable across the two models.

For the TFP shock (top row), both models generate a positive TFP response, with the endogenous component amplifying the exogenous shock. The amplification is stronger in the baseline model: strategic default provides an additional margin through which the upper cutoff $\overline{\overline{\omega}}_t$ responds to aggregate conditions, concentrating production among high-productivity firms and lifting the TFP prefactor further than the single-cutoff model can. For the preference and IST shocks (middle and bottom rows), Z_t is exactly zero—these shocks do not move the exogenous productivity process—so the entire TFP response is endogenous and confined to Φ_t. The contrast between the two models is striking here: the baseline generates a smooth, monotonically decaying Φ_t response, while the no-default model generates a pronounced oscillatory pattern that alternates sign period by period.

8.1 Results for the No-Default Model

The oscillatory behavior in the no-default model arises from a feedback loop between the single cutoff $\overline{\omega}_t$ and aggregate output in which the sign of the selection channel reverses across consecutive periods. Two features of the model interact to produce it.

The first feature is the role of the cutoff as a lagged state variable in production. As shown in equation (F.20) of Appendix F, aggregate output in period t depends on the cutoff from the *previous* period:

$$Y_t = E[\,\omega \mid \omega > \overline{\omega}_{t-1}] \cdot Z_t\, K_{t-1}^{\alpha}\, H_t^{1-\alpha}. \tag{43}$$

The mechanism is straightforward. Because capital is allocated equally across all producing firms, the aggregate capital–labor ratio is identical for every producer. The only source of heterogeneity in individual output is idiosyncratic productivity ω; summing across producers, aggregate output is therefore proportional to the conditional mean of ω among firms with $\omega > \overline{\omega}_{t-1}$. This is the *selection effect*: when the cutoff rises, the producing cohort is leaner and average productivity rises; when the cutoff falls, lower-productivity firms enter the cohort and drag down average productivity.

The second feature is the forward-looking determination of $\overline{\omega}_t$. The indifference condition (F.9) equates the expected discounted return from lending today with the expected discounted return from producing, so the cutoff is a function of current and expected future variables, not of past conditions. This creates a fundamental tension: $\overline{\omega}_t$ responds on impact to current shocks, but its effect on production materialises only one period later, through equation (43).

To fix ideas, consider a positive TFP shock; analogous dynamics set in for the preference and IST shocks, where the initiating impulse comes from the demand side rather than from a direct shift in Z_t. The feedback loop can be traced period by period.

- *Period t.* The positive TFP shock raises the current marginal product of capital. More firms find production attractive; the indifference condition shifts so that $\overline{\omega}_t$ *falls*, admitting additional lower-productivity producers into the active cohort.

- *Period $t+1$.* The lower $\overline{\omega}_t$ reduces $E[\omega \mid \omega > \overline{\omega}_t]$, so effective TFP is *lower* than it would have been at the steady-state cutoff. Output falls relative to what a higher cutoff would have delivered; consumption falls with it; and the bond Euler equation then requires R^B_{t+1} to *rise* to restore household indifference between consuming today and tomorrow. The higher risk-free rate tightens the lending threshold, pushing $\overline{\omega}_{t+1}$ *up*.

- *Period $t+2$.* The higher $\overline{\omega}_{t+1}$ raises $E[\omega \mid \omega > \overline{\omega}_{t+1}]$; effective TFP is now *higher*. Output recovers; consumption rises; R^B_{t+2} *falls*; and $\overline{\omega}_{t+2}$ *falls* again.

- *Subsequent periods.* The cycle repeats with alternating sign, decaying at a rate of approximately 0.69 per period.

The root cause of the oscillation is a sign mismatch between the cutoff's forward-looking response and the lagged selection effect it triggers. The cutoff $\overline{\omega}_t$ and the TFP factor $E[\omega \mid \omega > \overline{\omega}_t]$ always move in the same direction—a higher cutoff selects a more productive cohort. But the cutoff's forward-looking response to a positive shock is to *fall* on impact, and a lower cutoff then depresses effective TFP in the following period. This delayed negative feedback is what generates the negative autoregressive coefficient in the law of motion for $\overline{\omega}$ and the eigenvalue of -0.69 that characterizes the oscillatory dynamics in the right column of Figure 5.

8.2 Comparison of the No-default and Baseline Models

In the baseline model, aggregate output depends on the *upper* cutoff $\overline{\overline{\omega}}_{t-1}$, not on $\overline{\omega}_{t-1}$. As explained in Appendix F, the forward-looking condition that pins down the lower cutoff is unchanged and given by equation (32), but $\overline{\omega}_t$ does not enter the aggregate output equation directly in the baseline model. The two cutoffs are linked through the share of defaulting firms $\frac{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t)-\mu_{\eta_1,\eta_2}(\overline{\omega}_t)}{1-\mu_{\eta_1,\eta_2}(\overline{\omega}_t)}$, but they respond with a degree of independence to aggregate shocks.

Two additional stabilising forces are present in the baseline that are absent in the no-default model. First, in the baseline, when aggregate conditions change, the mass of defaulting firms can expand or contract almost immediately, absorbing much of the shock before it has the opportunity to feed back through the production cutoff. Second, the aggregate equity return equation in the baseline contains diversion-correction terms—involving the diversion scaling factor F_{t-1}, the aggregate diversion D_{t-1}, and the diversion function evaluated at the upper cutoff $\Theta(\overline{\overline{\omega}}_{t-1})$—that create a positive feedback channel from past defaulter activity to current returns. This positive channel offsets the negative selection channel described above, with the net result that the dominant eigenvalue for $\overline{\overline{\omega}}_t$ remains positive. The oscillatory mode is suppressed, and the IRFs in the left column of Figure 5 decay smoothly toward zero.

8.3 How Much TFP Variance is Endogenous?

The impulse responses in Figure 5 are informative about the dynamic structure of the endogenous TFP component, but they are computed shock by shock. To obtain an unconditional, model-wide assessment we simulate each model and ask what fraction of the variance of the Solow residual is attributable to the exogenous process Z_t alone.

The calculation proceeds as follows. We draw 50 simulated samples of 500 quarters each from the second-order perturbation solution of each model, using all three shocks simultaneously. All series are HP-filtered with smoothing parameter $\lambda = 1{,}600$ before moments are computed. Because production in period t uses the cutoff determined in period $t-1$ as shown in equation (F.20), the measured Solow residual decomposes as

$$\log \mathrm{TFP}_t = \log \Phi_{t-1} + \log Z_t, \tag{44}$$

where Φ_{t-1} is the endogenous prefactor evaluated at the lagged cutoff. The variance of $\log \mathrm{TFP}_t$ therefore satisfies

$$\mathrm{Var}(\log \mathrm{TFP}_t) = \mathrm{Var}(\log Z_t) + \mathrm{Var}(\log \Phi_{t-1}) + 2\,\mathrm{Cov}(\log \Phi_{t-1},\, \log Z_t). \tag{45}$$

The headline statistic is the ratio $\mathrm{Var}(\log Z_t)/\mathrm{Var}(\log \mathrm{TFP}_t)$, which equals one in a frictionless RBC economy where Φ_t is constant, and falls below one whenever the endogenous component adds variance to the Solow residual. The gap relative to the RBC benchmark,

$$1 - \frac{\mathrm{Var}(\log Z_t)}{\mathrm{Var}(\log \mathrm{TFP}_t)}, \tag{46}$$

measures the share of Solow TFP variance that cannot be accounted for by the exogenous process alone.

Table 7 reports results averaged across the 50 simulations.

In the baseline model, roughly 30 percent of the variance of the Solow TFP cannot be explained by exogenous productivity fluctuations alone; this 30 percent is accounted for by the endogenous mechanism. The no-default model also amplifies Solow TFP variance beyond what Z_t alone can generate, but to a lesser extent: the gap is roughly 20 percent, about three-quarters of the baseline figure. The difference between the two models reflects the additional default margin present in the baseline. When conditions change, the mass of defaulting firms adjusts rapidly, concentrating production in the most productive firms more aggressively than the single-cutoff mechanism can, and thereby generating a larger endogenous component of measured TFP.

9 Conclusion

We have developed a model in which informational frictions give rise to credit misallocation. In our model, as in Stiglitz and Weiss (1981), credit is rationed.

Each firm is endowed with two technologies: a production technology and a financial intermediation technology. The efficiency of the production technology is an idiosyncratic draw that is private information. Because households cannot observe this draw, they end up financing all firms equally. Depending on the realization of their production-technology draw, firms optimally choose which technology to deploy: Those with sufficiently low draws act as financial intermediaries and lend to firms with high draws. Borrowers are subject to a default decision.

We show that this tractable model can capture some key facets of the data. First and foremost, it is able to capture the average within-industry dispersion in productivity observed at the plant level in the United States. The model also does a good job in matching the correlation between productivity dispersion and GDP over the business cycle.

In our model, changes in productivity dispersion are linked to economic condition, implying that a component of aggregate TFP evolves endogenously. We have shown that this endogenous component of productivity can induce notable TFP movements for realistically sized shocks.

We have further isolated the specific contribution of strategic default to the endogenous TFP component by contrasting the baseline model with a no-default variant in which the two-cutoff structure collapses to a single lending threshold. Unconditional simulations show that roughly 30 percent of the variance of the Solow residual is endogenously generated in the baseline model, compared with roughly 20 percent in the no-default variant. Strategic default therefore accounts for a meaningful share of the endogenous TFP component—acting not merely as an amplifier that concentrates production among high-productivity firms, but also as a stabiliser of the endogenous component.

We hope that extensions of our tractable model can serve as a building block in the exploration of how alternative policies affect productivity. Extensions could allow the study of fiscal and monetary policy choices. Going beyond our current analysis, we can speculate that expansionary fiscal policies could have additional desirable effects in our model as the associated increases in real interest rates would force some unproductive firms to quit. By contrast, monetary policy would face additional challenges in our setup. The typical New Keynesian rationale to lower policy rates in a downturn of aggregate demand could inefficiently keep low-productivity firms afloat. In sum, we see our work as opening promising avenues for further research.

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Tables and Figures

Table 1: Summary of firms' decisions and actions within each period

	Period t	Period $t+1$
1	Raise equity a_t	Produce, **outside option matures**
2	**Draw productivity level** $\omega \in [0,1]$	Repay loans to other firms
3	**Lend or borrow in inter-firm market**	Pay households
4	**Some borrowing firms take the outside option and default**	
5	Purchase physical capital	

Note: The bolded entries denote actions or events specific to our model and not part of a standard RBC model.

Table 2: Regressions of within-industry TFP dispersion (percent difference between the TFP for the 90^{th} percentile firm and that for the 10^{th} percentile firm), 1987–2021

	(1) 90-10 perc.	(2) 90-10 perc.	(3) 90-10 perc.	(4) 90-10 perc.	(5) 90-10 perc.	(6) 90-10 perc.
GDP Growth	-2.85**			-2.24**	-2.87**	-3.16**
	[-4.82,-0.89]			[-3.74,-0.73]	[-4.26,-1.48]	[-4.81,-1.51]
Real interest rate		-4.33**		-3.95**	-2.40**	-3.52**
		[-6.06,-2.60]		[-5.52,-2.38]	[-4.11,-0.69]	[-5.06,-1.99]
Delinquency rate			-3.52**		-2.67**	
			[-5.48,-1.55]		[-4.39,-0.95]	
Charge-off rate						-5.47*
						[-10.48,-0.46]
r2	0.21	0.44	0.29	0.56	0.67	0.62
N	35.00	35.00	35.00	35.00	35.00	35.00

Note: 95% confidence intervals in brackets; $**$ p<0.01, $*$ p<0.05.

Table 3: Model parameters

	Value	Description	
Conventional			
β	0.9925	Discount rate	
α	0.3	Capital share in production	
δ	0.01	Depreciation rate	
ν	2	Inverse Frisch elasticity of labor supply	
Estimated to match first moments with steady-state conditions			*Targets/Explanation*
η_1	1.543	First parameter of Beta distribution	jointly to match the average 1) 90-10 within-industry productivity dispersion, 2) spread between the loan rate and interbank lending rate, 3) share of bank credit, and 4) default rate on business loans
η_2	2.735	Second parameter of Beta distribution	
ξ	0.007	Haircut on the returns from the outside option	
θ	0.0003	Average fraction of funds that can be diverted	
Estimated by simulated methods of moments			*Targets/Explanation*
ϕ	0.240	Investment adjustment costs	estimated to hit the variances, correlations, and autocorrelations of 1) real output, 2) real consumption, 3) relative investment price, and 4) delinquency rate
ρ_z	0.409	Persistence of technology shock	
σ_z	0.010	s.d. of technology shock	
ρ_I	0.990	Persistence of investment technology shock	
σ_I	0.001	s.d. of investment technology shock	
ρ_ν	0.986	Persistence of consumption shock	
σ_ν	0.021	s.d. of consumption shock	
Specific			*Explanation*
ψ	2	Parameter in the function $\Theta_t(\omega) = \omega^\psi F_t$	Ensures that the slope condition is verified
ϑ	0.84236	Disutility of labor	Supports aggregate labor = 1 in the steady state

Table 4: SMM estimation: Data moments and model counterparts, 1987:Q1–2021:Q4

	Data	Data 5th perc.	Data 95th perc.	Model	Model 5th perc.	Model 95th perc.
Var(GDP)	1.70	0.77	2.63	2.29	1.60	3.08
Corr(GDP,Consumption)	0.91	0.25	1.57	0.84	0.76	0.90
Corr(GDP,Investment price)	0.31	0.14	0.48	0.70	0.62	0.78
Corr(GDP,Default rate)	-0.58	-0.78	-0.39	-0.59	-0.70	-0.46
Var(Consumption)	2.10	0.41	3.80	1.78	1.23	2.45
Corr(Consumption,Investment price)	0.22	0.10	0.35	0.46	0.32	0.59
Corr(Consumption,Default rate)	-0.42	-0.57	-0.27	-0.27	-0.43	-0.11
Var(Investment price)	0.56	0.40	0.72	0.62	0.47	0.78
Corr(Investment price,Default rate)	-0.48	-0.71	-0.26	-0.94	-0.96	-0.92
Var(Default rate)	0.34	0.24	0.45	0.34	0.19	0.52
Autocorr(GDP)	0.64	0.46	0.82	0.60	0.49	0.71
Autocorr(Consumption)	0.57	0.31	0.84	0.63	0.51	0.74
Autocorr(Investment price)	0.90	0.64	1.17	0.22	0.08	0.36
Autocorr(Default rate)	0.95	0.65	1.24	0.24	0.07	0.39

Note: The table reports the second moments targeted in the SMM procedure for the model calibration—variances, correlations, and autocorrelations at business-cycle frequencies. The model moments are computed from the average of 1,000 simulated samples of the same length as the observed data. Both observable and model-simulated data are HP-filtered setting the smoothing parameter to 1,600 as standard for quarterly data. The data 5th and 95th percentile columns show the bounds of a 90% confidence interval constructed under the assumption of asymptotic normality, using Newey–West HAC standard errors (Bartlett kernel with optimal bandwidth) for the sample moments. The model 5th and 95th percentile columns show the corresponding empirical percentiles from 1,000 simulated samples.

Table 5: Variance decomposition (percent)

	Output	Invest. Price	Consumption	Default Rate	Real Rate	90-10 TFP disp.
TFP	73.9	87.6	28.5	68.4	74.4	44.2
Consumption	25.0	11.5	69.4	29.1	23.0	52.7
ISP	1.1	0.9	2.0	2.5	2.6	3.2

Note: "TFP" refers to the shock to total factor productivity, Z_t. "Consumption" refers to the consumption shock, ν_{ct}. "ISP" refers to the investment technology shock Z_t^I.

Table 6: Regression results: model-simulated and observed data, 1987-2021

	90-10 TFP dispersion	
	Model	Data
GDP growth	-0.68***	-2.85***
	$[-0.93; -0.50]$	$[-4.82; -0.89]$
Real interest rate	0.06	-4.33***
	$[-1.44; 0.95]$	$[-6.06; -2.60]$
Delinquency rate	-3.52	-3.52***
	$[-11.75; 1.24]$	$[-5.48; -1.55]$

Note: The results under the "model" column are based on 1000 model-simulated samples of the same length as the observed data. We use yearly means of quarterly model data to replicate the frequency of the observed data. The results under the "data" column match the relevant entries of Table 2 and are repeated here for ease of comparison. 95% confidence intervals are reported in brackets. The confidence intervals for regressions on simulated data are based on the 2.5-97.5 percentiles from the 1000 data replications.

Table 7: Variance decomposition of the Solow residual

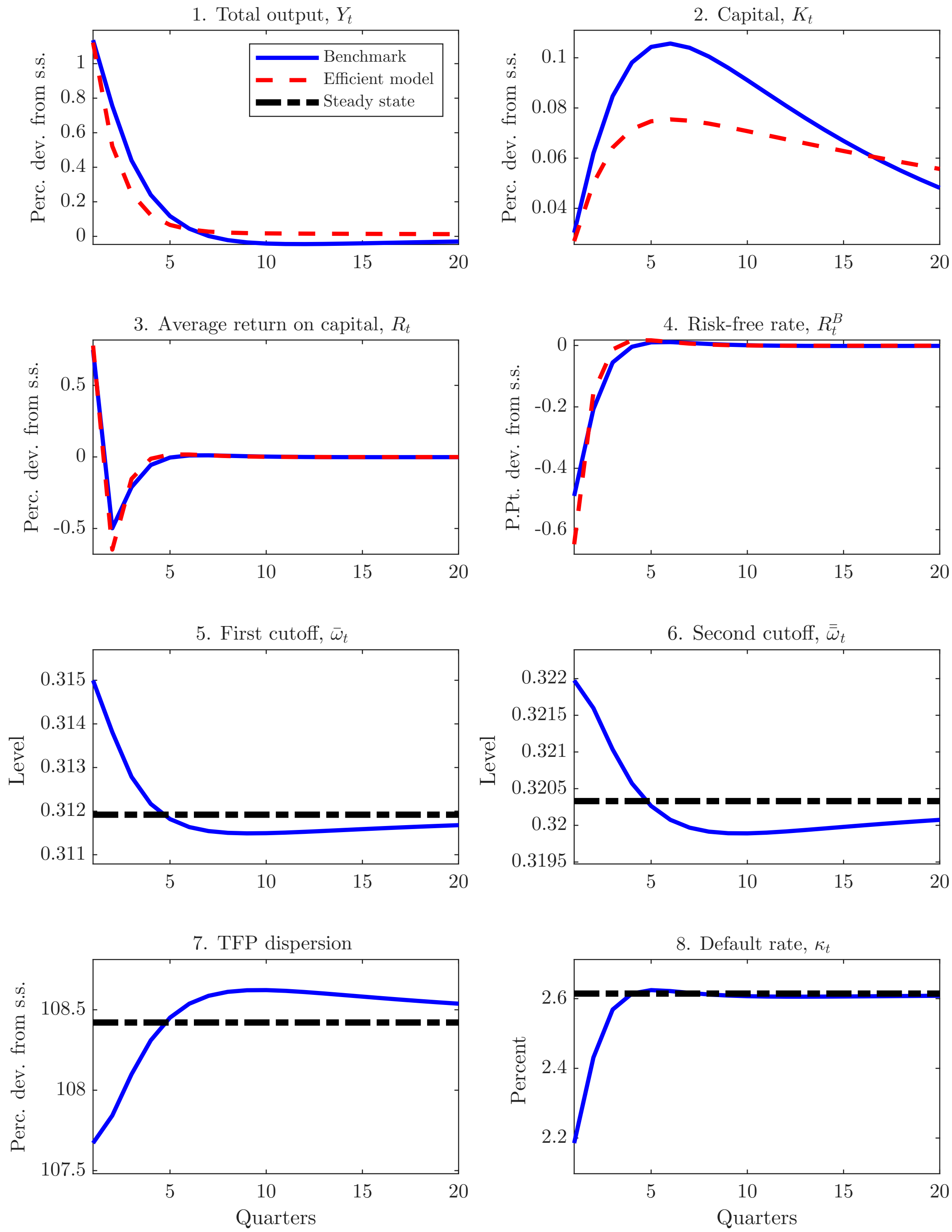
	$\text{Var}(\log Z_t)/\text{Var}(\log \text{TFP}_t)$	Gap vs. RBC (percent)
RBC benchmark	1.000	0.0
Two-cutoff model (baseline)	0.715	28.5
One-cutoff model (no strategic default)	0.788	21.2

Note: HP-filtered simulations ($\lambda = 1{,}600$), 50 draws of 500 quarters each, all three shocks active. The ratio equals one when Φ_t is constant (RBC); the gap measures the endogenous share of Solow TFP variance.

Figure 1: TFP dispersion, 1987–2021

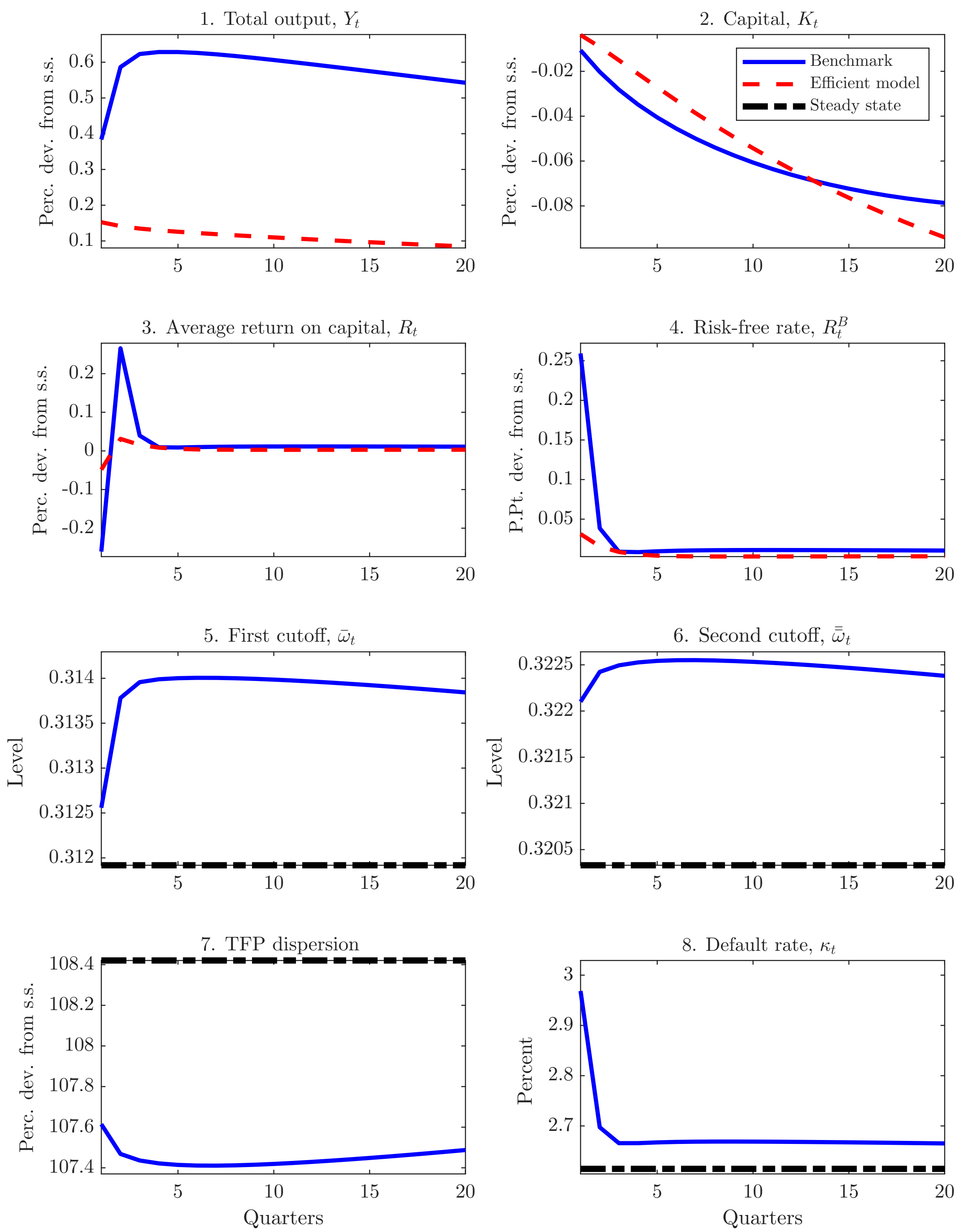
Note: Shaded areas denote recessions as dated by the NBER Business Cycle Dating Committee.
Source: Dispersion Statistics on Productivity, a dataset published jointly by the Bureau of Labor Statistics and the Census Bureau, and authors' calculations.

Figure 2: A TFP shock



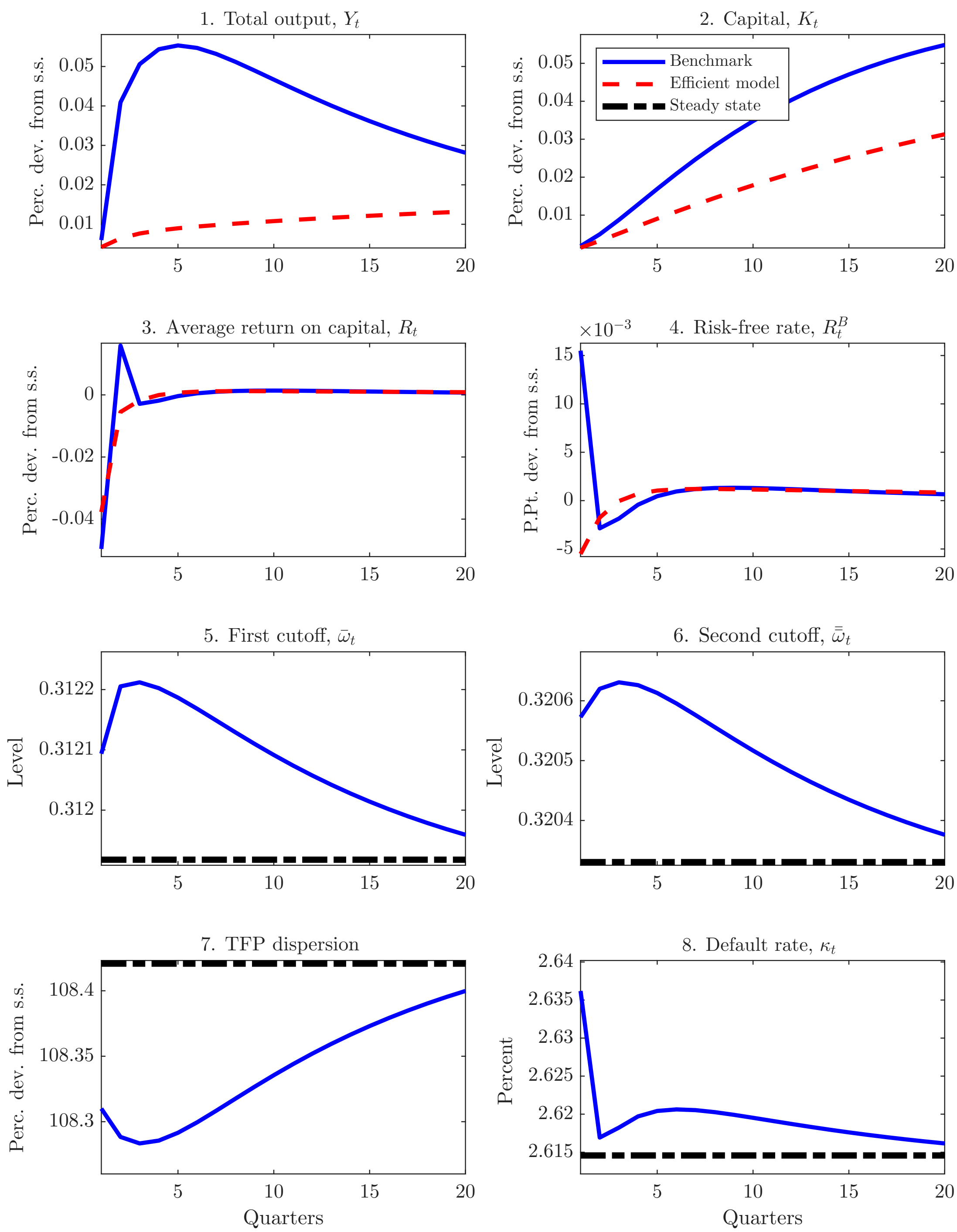
Note: "Omega bar" refers to $\overline{\omega}_t$ and "omega double bar" refers to $\overline{\overline{\omega}}_t$. The responses start from the stochastic steady state and are also shown in deviation from the stochastic steady state, where relevant.

Figure 3: A consumption shock



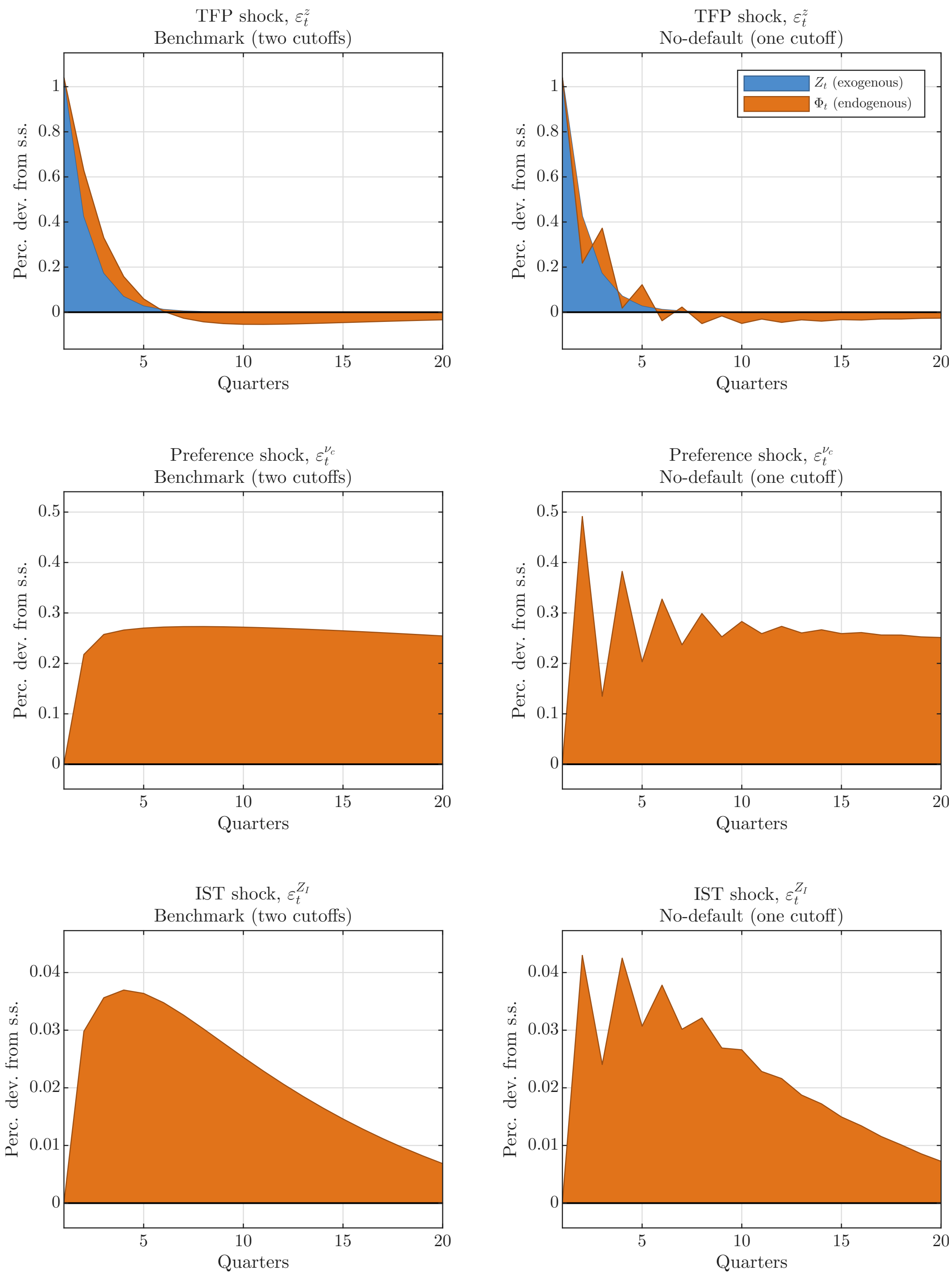
Note: "Omega bar" refers to $\overline{\omega}_t$ and "omega double bar" refers to $\overline{\overline{\omega}}_t$. The responses start from the stochastic steady state and are also shown in deviation from the stochastic steady state, where relevant.

Figure 4: An investment technology shock



Note: "Omega bar" refers to $\overline{\omega}_t$ and "omega double bar" refers to $\overline{\overline{\omega}}_t$. The responses start from the stochastic steady state and are also shown in deviation from the stochastic steady state, where relevant.

Figure 5: TFP decomposition: baseline vs. no-default model



Note: Each panel shows the impulse response of the Solow residual decomposed into its two components (Section 5.2). Blue area: exogenous TFP shock Z_t (log % deviation from steady state). Orange area: endogenous TFP, prefactor Φ_t (log % deviation from steady state). The top of the stacked area equals total measured TFP. Rows correspond to the three shocks; columns correspond to the two models. The y-axis scale is identical within each row to facilitate direct comparison. Left column: baseline model with strategic default (two-cutoff model, $\overline{\omega}_t$ and $\overline{\overline{\omega}}_t$). Right column: no-default model (one-cutoff model, $\overline{\omega}_t$ only; Appendix F).

Online Appendix

This appendix is organized as follows.

Appendix A provides formal proofs of the three propositions that establish the existence and properties of the two firm-segment cutoff points and the inter-firm loan rate in equilibrium.

Appendix B derives the first-order conditions for the household and capital-producing-firm optimization problems; provides a formal definition of competitive equilibrium; derives key model aggregates—including the aggregate equity return and the aggregate resource constraint—by integrating firm-level outcomes across the three firm segments using the Beta distribution; and lists all equilibrium conditions in compact notation (29 equations in 29 unknowns).

Appendix C solves for the non-stochastic steady state, describing an iterative algorithm that guesses the cutoff points and bond rate and verifies consistency across all equilibrium conditions.

Appendix D explains how to compute model-implied counterparts of the empirical TFP dispersion measures by rescaling the productivity distribution to the producing-firm segment.

Appendix E develops a representative-firm RBC benchmark that removes the informational friction by eliminating idiosyncratic productivity and decentralising the model via the standard household-owns-capital structure.

Appendix F develops a variant of the baseline model in which the outside option is unavailable, so that strategic default is eliminated and firms sort into only two segments (lenders and producers) rather than three; derives the single-cutoff equilibrium conditions, steady state, and connections to the baseline and RBC benchmark.

Appendix G compares alternative solution methods.

A Proofs of Propositions

We prove here the three propositions stated in Section 5.1.

A.1 Proof of Proposition 1

Equation (33) can be described as

$$\begin{aligned} &E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(\rho_t\frac{1-\mu(\overline{\overline{\omega}}_t)}{1-\mu(\overline{\omega}_t)}+(1-\theta)\frac{\mu(\overline{\overline{\omega}}_t)-\mu(\overline{\omega}_t)}{1-\mu(\overline{\omega}_t)}\right)a_t\right]= \\ &E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_t^B-\xi\right)\left(a_t+\Theta_t(\omega)b_t\right)\right] \end{aligned} \tag{A.1}$$

when $\omega=\overline{\omega}_t$. The left-hand side of equation (A.1) does not depend on ω, while the right-hand side of equation (A.1) increases in ω due to our assumption that $\Theta_t(\omega)$ is increasing in ω. Therefore, firms with $\omega<\overline{\omega}_t$ will have no incentive to deviate to the outside option because they get higher expected profits from lending than from diverting funds.

It is left to show that firms with $\omega<\overline{\omega}_t$ will have no incentive to deviate to production. We need to establish that the expected profits from lending are higher than the expected profits from producing, i.e.,

$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(\rho_t\frac{1-\mu(\overline{\overline{\omega}}_t)}{1-\mu(\overline{\omega}_t)}+(1-\theta)\frac{\mu(\overline{\overline{\omega}}_t)-\mu(\overline{\omega}_t)}{1-\mu(\overline{\omega}_t)}\right)a_t\right]>E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_{t+1}(\omega)a_t\right)\right], \tag{A.2}$$

for all firms with $\omega<\overline{\omega}_t$. Notice that these firms cannot borrow in the inter-firm market because of the screening technology.

By combining equations (13) and (32), we get

$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}R_{t+1}(\overline{\omega})\right]=E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_t^B-\xi\right)\right]. \tag{A.3}$$

Plug this result into equation (33):

$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(\rho_t\frac{1-\mu(\overline{\overline{\omega}}_t)}{1-\mu(\overline{\omega}_t)}+(1-\theta)\frac{\mu(\overline{\overline{\omega}}_t)-\mu(\overline{\omega}_t)}{1-\mu(\overline{\omega}_t)}\right)a_t\right]= E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}R_{t+1}(\overline{\omega})\left(a_t+\Theta_t(\overline{\omega}_t)b_t\right)\right]. \quad (A.4)$$

Notice that

$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}R_{t+1}(\overline{\omega})a_t\right]>E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_{t+1}(\omega)a_t\right)\right] \quad (A.5)$$

for all firms with $\omega<\overline{\omega}_t$ since $R_{t+1}(\omega)$ is increasing in ω. Combining this result with $\Theta_t(\overline{\omega}_t)b_t\geq 0$ and $\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}R_{t+1}(\overline{\omega})\geq 0$, we get

$$\begin{aligned}E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(\rho_t\frac{1-\mu(\overline{\overline{\omega}}_t)}{1-\mu(\overline{\omega}_t)}+(1-\theta)\frac{\mu(\overline{\overline{\omega}}_t)-\mu(\overline{\omega}_t)}{1-\mu(\overline{\omega}_t)}\right)a_t\right]&=\\ E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}R_{t+1}(\overline{\omega})\left(a_t+\Theta_t(\overline{\omega}_t)b_t\right)\right]&\geq\\ E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}R_{t+1}(\overline{\omega})a_t+\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}R_{t+1}(\overline{\omega})\Theta_t(\overline{\omega}_t)b_t\right]&>E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_{t+1}(\omega)a_t\right)\right], \quad (A.6)\end{aligned}$$

for all firms with $\omega<\overline{\omega}_t$. Thus, equation (A.2) is verified. □

A.2 Proof of Proposition 2

Let us prove it by contradiction. Assume that $\overline{\overline{\omega}}_t^*<\overline{\omega}_t$. Therefore, by our definition $\overline{\overline{\omega}}_t=\max(\overline{\omega}_t,\overline{\overline{\omega}}_t^*)=\overline{\omega}_t$. Plugging this result into equation (33):

$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\rho_t a_t\right]=E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_t^B-\xi\right)\left(a_t+\Theta_t(\overline{\omega}_t)b_t\right)\right]. \quad (A.7)$$

Consider the right-hand-side of equation (34) and substitute for $E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\rho_t\right]$ from equation (A.7).

$$\begin{aligned}&E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_{t+1}(\overline{\overline{\omega}}_t^*)(a_t+b_t)-\rho_t b_t\right)\right]=\\ &E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_{t+1}(\overline{\overline{\omega}}_t^*)(a_t+b_t)-\left(R_t^B-\xi\right)\left(1+\Theta_t(\overline{\omega}_t)\frac{b_t}{a_t}\right)b_t\right)\right].\end{aligned} \quad (A.8)$$

Using that $R_{t+1}(\omega) > 0$ and strictly increasing in ω, together with equation (A.3) lead to the following inequality:

$$0 < E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}R_{t+1}(\overline{\overline{\omega}}_t^*)\right] < E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}R_{t+1}(\overline{\omega}_t)\right] = E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_t^B - \xi\right)\right] \tag{A.9}$$

Plugging this inequality into equation (A.8) and collecting terms result in:

$$\begin{aligned}
&E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_{t+1}(\overline{\overline{\omega}}_t^*)(a_t+b_t) - \rho_t b_t\right)\right] = \\
&E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_{t+1}(\overline{\overline{\omega}}_t^*)(a_t+b_t) - \left(R_t^B-\xi\right)\left(1+\Theta_t(\overline{\omega}_t)\frac{b_t}{a_t}\right)b_t\right)\right] < \\
&E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(\left(R_t^B-\xi\right)(a_t+b_t) - \left(R_t^B-\xi\right)\left(1+\Theta_t(\overline{\omega}_t)\frac{b_t}{a_t}\right)b_t\right)\right] = \\
&E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(\left(R_t^B-\xi\right)\left(a_t - \Theta_t(\overline{\omega}_t)\frac{b_t^2}{a_t}\right)\right)\right] \leq E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(\left(R_t^B-\xi\right)a_t\right)\right] \leq \\
&E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_t^B-\xi\right)\left(a_t+\Theta_t(\overline{\overline{\omega}}_t^*)b_t\right)\right],
\end{aligned} \tag{A.10}$$

where we use that $\Theta_t(\overline{\omega}_t)\frac{b_t^2}{a_t} \geq 0$ and $\Theta_t(\overline{\overline{\omega}}_t^*)b_t \geq 0$. Therefore,

$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_t^B-\xi\right)\left(a_t+\Theta_t(\overline{\overline{\omega}}_t^*)b_t\right)\right] > E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_{t+1}(\overline{\overline{\omega}}_t^*)(a_t+b_t) - \rho_t b_t\right)\right]. \tag{A.11}$$

However, inequality (A.11) contradicts equation (34). Therefore, $\overline{\overline{\omega}}_t^* \geq \overline{\omega}_t$ □

A.3 Proof of Proposition 3

Note that equations (33) and (34) are constructed such that a marginal firm with productivity level $\overline{\omega}_t$ receives the same expected profits from lending and diverting funds, while a marginal firm with productivity level $\overline{\overline{\omega}}_t$ receives the same expected profits from diverting funds and producing. For this proposition, we resolve the tie by assuming these marginal firms choose to lend and produce, respectively. Since the probability distribution of ω is continuous, the probability that $\omega = \overline{\omega}_t$ or $\omega = \overline{\overline{\omega}}_t$ is zero.

Given the results of Propositions 1 and 2, we are left to show that

1. No firm with idiosyncratic productivity $\overline{\omega}_t < \omega < \overline{\overline{\omega}}_t$ has an incentive to deviate from

borrowing and defaulting.

2. No firm with idiosyncratic productivity $\omega > \overline{\overline{\omega}}_t$ has an incentive to deviate from borrowing and producing.

We will show our proof in three smaller steps.

Step 1: Firms with $\omega > \overline{\omega}_t$ have higher expected profits from borrowing and defaulting than from lending.

Proof of step 1: Equation (33) can be described as

$$\begin{aligned} & E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(\rho_t\frac{1-\mu(\overline{\overline{\omega}}_t)}{1-\mu(\overline{\omega}_t)}+(1-\theta)\frac{\mu(\overline{\overline{\omega}}_t)-\mu(\overline{\omega}_t)}{1-\mu(\overline{\omega}_t)}\right)a_t\right]= \\ & E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_t^B-\xi\right)\left(a_t+\Theta_t(\omega)b_t\right)\right]. \end{aligned} \tag{A.12}$$

when $\omega = \overline{\omega}_t$. The left-hand side of equation (A.12) does not depend on ω, while the right-hand side of equation (A.12) increases in ω due to our assumption that $\Theta_t(\omega)$ is increasing in ω. Therefore, firms with $\omega > \overline{\omega}_t$ get higher expected profits from diverting funds than from lending. □

Step 2: Firms with $\omega > \overline{\overline{\omega}}_t$ have lower expected profits from borrowing and defaulting than from producing.

Proof of step 2:

Equation (33) can be described as

$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_t^B-\xi\right)\left(a_t+\Theta_t(\omega)b_t\right)\right]=E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_{t+1}(\omega)(a_t+b_t)-\rho_t b_t\right)\right] \tag{A.13}$$

when $\omega = \overline{\overline{\omega}}_t$. Taking the first derivative of both sides of equation (A.13) with respect to ω, we need to show that

$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_t^B-\xi\right)\Theta_t'(\omega)b_t\right]<E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_{t+1}'(\omega)(a_t+b_t)\right)\right] \tag{A.14}$$

for all $\omega > \overline{\overline{\omega}}_t$. According to this inequality, the expected profits from diverting funds grow less steeply than the expected profits from producing for all firms with the productivity level

$\omega > \overline{\overline{\omega}}_t$. Together with the equalization of expected profits in equation (A.13) evaluated at $\omega = \overline{\overline{\omega}}_t$, this inequality implies that the expected profits from diverting funds are lower than the expected profits from producing for all $\omega > \overline{\overline{\omega}}_t$.

Let us show that condition (36) is sufficient to ensure inequality (A.14) for all $\omega > \overline{\overline{\omega}}_t$. Finding the derivative of $R'_{t+1}(\omega)$ from equation (13) and plugging it into equation (A.14), we get

$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_t^B-\xi\right)\Theta_t'(\omega)b_t\right] < E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(\frac{1}{Q_t}\alpha\ Z_{t+1}\left(\frac{H_{t+1}}{K_t}\right)^{1-\alpha}\right)(a_t+b_t)\right]. \quad \text{(A.15)}$$

Since $\Theta_t(\omega)$ is convex (so $\Theta_t'(\omega)$ is non-decreasing in ω), then

$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_t^B-\xi\right)\Theta_t'(\omega)b_t\right] \le E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_t^B-\xi\right)\Theta_t'(1)b_t\right] \quad \text{(A.16)}$$

for all $\overline{\overline{\omega}}_t < \omega \le 1$. Therefore, if

$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_t^B-\xi\right)\Theta_t'(1)b_t\right] < E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(\frac{1}{Q_t}\alpha\ Z_{t+1}\left(\frac{H_{t+1}}{K_t}\right)^{1-\alpha}\right)(a_t+b_t)\right] \quad \text{(A.17)}$$

holds, then it implies

$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_t^B-\xi\right)\Theta_t'(\omega)b_t\right] < E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R'_{t+1}(\omega)(a_t+b_t)\right)\right] \quad \text{(A.18)}$$

for all $\omega > \overline{\overline{\omega}}_t$. It establishes the sufficiency of the global slope condition in equation (36). □

Step 3: Firms with $\omega > \overline{\overline{\omega}}_t$ have higher expected profits from producing than from lending. *Proof of step 3:* It directly follows from the result of Proposition 2 and the two results shown in steps 1 and 2. Since $\overline{\overline{\omega}}_t \ge \overline{\omega}_t$, the result of step 1 implies that firms with $\omega > \overline{\overline{\omega}}_t$ have higher expected profits from diverting funds than from lending. Combining this implication with the result of step 2 that establishes that firms with $\omega > \overline{\overline{\omega}}_t$ have higher expected profits from producing than from diverting funds, ensures that firms with $\omega > \overline{\overline{\omega}}_t$ have higher expected profits from producing than from lending. □

B Equilibrium Definition

This appendix is organized as follows. Sections B.1 and B.2 complete the derivations of the first-order conditions from the household and capital-producing-firm optimization problems, providing the steps omitted from the main text for brevity. Section B.3 presents the aggregation of firm-level decisions into aggregate variables and states a formal definition of competitive equilibrium. Sections B.4–B.8 derive the Beta-distribution closed forms for the aggregate equity return, aggregate output, average return on capital, aggregate equity-return market clearing, and the diversion scaling factor F_t. Finally, Section B.9 lists all equilibrium conditions in compact, Beta-distribution-specific notation.

B.1 Household Problem

The representative household maximizes

$$\max_{\left\{A_{t+\tau},C_{t+\tau},H_{t+\tau},B^H_{t+\tau}\right\}_{\tau=0}^{\infty}} E_t \sum_{\tau=0}^{\infty} \beta^{\tau} \left[\ln(C_{t+\tau} - \nu_{ct+\tau}) - \frac{\vartheta}{1+\nu}(H_{t+\tau})^{1+\nu}\right],$$

subject to

$$C_{t+\tau} + A_{t+\tau} + B^H_{t+\tau} = R^A_{t+\tau} A_{t+\tau-1} + W_{t+\tau} H_{t+\tau} + R^B_{t+\tau-1} B^H_{t+\tau-1} + \Pi_{t+\tau} + T_{t+\tau} + \Xi_{t+\tau},$$

reproduced from equations (2) and (3). The first-order conditions for assets,

$$-\lambda_{ct} + \beta E_t \left\{\lambda_{ct+1} R^A_{t+1}\right\} = 0, \tag{B.1}$$

consumption,

$$\frac{1}{C_t - \nu_{ct}} = \beta E_t \left\{\lambda_{ct+1} R^B_t\right\}, \tag{B.2}$$

labor,

$$-\frac{\vartheta}{C_t - \nu_{ct}} H_t^{\nu} + \beta E_t \left\{\lambda_{ct+1} R^B_t\right\} W_t = 0, \tag{B.3}$$

and government bonds,

$$-\lambda_{ct} + \beta E_t \left\{\lambda_{ct+1} R^B_t\right\} = 0, \tag{B.4}$$

follow from the stationarity conditions of the Lagrangian, where λ_{ct} is the multiplier on the budget constraint (3).

B.2 Capital-Producing Firms

Capital-producing firms solve

$$\max_{I^g_{t+i}} E_t \sum_{i=0}^{\infty} \beta^i \frac{\lambda_{ct+i}}{\lambda_{ct}} \left\{ Q_{t+i} Z_{It+i} \left[1 - \frac{\phi}{2} \left(\frac{I^g_{t+i}}{I^g_{t+i-1}} - 1 \right)^2 \right] I^g_{t+i} - I^g_{t+i} \right\},$$

reproduced from equation (25). The first-order condition implies

$$0 = E_t \left\{ \begin{array}{c} Q_t Z_{It} \left[-\phi \left(\frac{I^g_t}{I^g_{t-1}} - 1 \right) \frac{1}{I^g_{t-1}} \right] I^g_t + Q_t Z_{It} \left[1 - \frac{\phi}{2} \left(\frac{I^g_t}{I^g_{t-1}} - 1 \right)^2 \right] - 1 \\ + \beta \frac{\lambda_{ct+1}}{\lambda_{ct}} Q_{t+1} Z_{It+1} \phi \left(\frac{I^g_{t+1}}{I^g_t} - 1 \right) \left(\frac{I^g_{t+1}}{I^g_t} \right)^2 \end{array} \right\} \tag{B.5}$$

B.3 Aggregation and Equilibrium

We proceed as follows: First, we link individual and aggregate variables, then we define a competitive equilibrium.

Since a mass $\mu(\overline{\omega}_t)$ of firms lend and the complement mass $1 - \mu(\overline{\omega}_t)$ of firms borrow, the inter-firm market clears when

$$\int_0^{\overline{\omega}_t} l_t(\omega) \mu'(\omega) \mathrm{d}\omega = \int_{\overline{\omega}_t}^1 b_t(\omega) \mu'(\omega) \mathrm{d}\omega, \tag{B.6}$$

which by defining

$$L_t = \int_0^{\overline{\omega}_t} l_t(\omega) \mu'(\omega) \mathrm{d}\omega, \tag{B.7}$$

$$B_t = \int_{\overline{\omega}_t}^1 b_t(\omega) \mu'(\omega) \mathrm{d}\omega \tag{B.8}$$

translates into:

$$L_t = B_t. \tag{B.9}$$

Since each type of firm borrows the same amount, we have $a_t(\omega) = a_t$ and $b_t(\omega) = b_t$ for each ω. Using the definition $A_t = \int_0^1 a_t(\omega)\mu'(\omega)\mathrm{d}\omega$ and equation (B.8), we can relate individual to aggregate variables as follows:

$$a_t = A_t, \tag{B.10}$$

$$b_t = \frac{B_t}{1-\mu(\overline{\omega}_t)}. \tag{B.11}$$

Next, consider the aggregation of budget constraints of firms in each segment. For lending firms, $l_t(\omega) = a_t(\omega)$ for each ω. Aggregating over the relevant mass of firms, i.e.,

$$\int_0^{\overline{\omega}_t} l_t(\omega)\mu'(\omega)\mathrm{d}\omega = \int_0^{\overline{\omega}_t} a_t(\omega)\mu'(\omega)\mathrm{d}\omega, \tag{B.12}$$

results into

$$L_t = \mu(\overline{\omega}_t)A_t. \tag{B.13}$$

Let $d_t(\omega) = a_t(\omega) + \Theta_t(\omega)b_t(\omega)$ define the amount of resources diverted and invested in the outside option by the firm of productivity ω. Aggregating over the relevant mass of firms, i.e.,

$$\int_{\overline{\omega}_t}^{\overline{\overline{\omega}}_t} \left(a_t(\omega) + \Theta_t(\omega)b_t(\omega)\right)\mu'(\omega)\mathrm{d}\omega = \int_{\overline{\omega}_t}^{\overline{\overline{\omega}}_t} d_t(\omega)\mu'(\omega)\mathrm{d}\omega, \tag{B.14}$$

and combining it with the market-clearing condition in the bond's market, i.e.,

$$D_t = \int_{\overline{\omega}_t}^{\overline{\overline{\omega}}_t} d_t(\omega)\mu'(\omega)\mathrm{d}\omega, \tag{B.15}$$

result into

$$D_t = \left(\mu(\overline{\overline{\omega}}_t) - \mu(\overline{\omega}_t)\right)A_t + \frac{\mu(\overline{\overline{\omega}}_t) - \mu(\overline{\omega}_t)}{1-\mu(\overline{\omega}_t)}\theta B_t. \tag{B.16}$$

For producing firms, $a_t(\omega) + b_t(\omega) = b_t^{tot}(\omega)$. Aggregating over the relevant mass of firms, i.e.,

$$\int_{\overline{\overline{\omega}}_t}^{1} \left(a_t(\omega) + b_t(\omega)\right) \mu'(\omega)\mathrm{d}\omega = \int_{\overline{\overline{\omega}}_t}^{1} b_t^{tot}(\omega)\mu'(\omega)\mathrm{d}\omega, \tag{B.17}$$

and defining

$$B_t^{tot} = \int_{\overline{\overline{\omega}}_t}^{1} b_t^{tot}(\omega)\mu'(\omega)\mathrm{d}\omega. \tag{B.18}$$

result into

$$B_t^{tot} = \left(1 - \mu(\overline{\overline{\omega}}_t)\right) A_t + \frac{1 - \mu(\overline{\overline{\omega}}_t)}{1 - \mu(\overline{\omega}_t)} B_t. \tag{B.19}$$

In equilibrium, all firms that produce will raise the same amount of financing. Therefore, $k_t(\omega) = \tilde{k}_t$ and, based on the properties of constant-returns-to-scale production functions, $h_{t+1}(\omega) = \tilde{h}_{t+1}$, where $\tilde{k}_t$ and $\tilde{h}_{t+1}$ are constant across firms. Accordingly, total production will be

$$Y_{t+1} = \int_{\overline{\overline{\omega}}_t}^{1} Z_{t+1}\omega \tilde{k}_t^{\alpha} \tilde{h}_{t+1}^{1-\alpha} \mu'(\omega)\mathrm{d}\omega, \tag{B.20}$$

such that the aggregate level of capital and labor are related to individual values as follows:

$$K_t = \int_{\overline{\overline{\omega}}_t}^{1} \tilde{k}_t \mu'(\omega)\mathrm{d}\omega = (1 - \mu(\overline{\overline{\omega}}_t))\tilde{k}_t, \tag{B.21}$$

$$H_{t+1} = \int_{\overline{\overline{\omega}}_t}^{1} \tilde{h}_{t+1} \mu'(\omega)\mathrm{d}\omega = (1 - \mu(\overline{\overline{\omega}}_t))\tilde{h}_{t+1}. \tag{B.22}$$

Therefore,

$$Y_{t+1} = \frac{1}{1 - \mu(\overline{\overline{\omega}}_t)} \int_{\overline{\overline{\omega}}_t}^{1} Z_{t+1}\omega K_t^{\alpha} H_{t+1}^{1-\alpha} \mu'(\omega)\mathrm{d}\omega. \tag{B.23}$$

Individual producing firms borrow to finance the purchase of capital, i.e., $b_t^{tot}(\omega) = Q_t k_t(\omega)$, so aggregating over this mass of firms results into the aggregate constraint

$$B_t^{tot} = Q_t K_t. \tag{B.24}$$

Let R_t define the average return on capital that producing firms receive. To this purpose, the relevant probability density function is $\frac{\mu'(\omega)}{1-\mu(\overline{\overline{\omega}}_{t-1})}$.

$$R_t = \int_{\overline{\overline{\omega}}_{t-1}}^{1} R_t(\omega) \frac{\mu'(\omega)}{1-\mu(\overline{\overline{\omega}}_{t-1})} \mathrm{d}\omega. \tag{B.25}$$

Substituting for $R_t(\omega)$ from equation (13):

$$R_t = \int_{\overline{\overline{\omega}}_{t-1}}^{1} \left(\frac{1}{Q_{t-1}} \alpha \; Z_t \left(\frac{H_t}{K_{t-1}} \right)^{1-\alpha} \omega + \frac{(1-\delta)}{Q_{t-1}} Q_t \right) \frac{\mu'(\omega)}{1-\mu(\overline{\overline{\omega}}_{t-1})} \mathrm{d}\omega. \tag{B.26}$$

The aggregate equity return to households is defined as

$$R_t^A = \int_0^1 R_t^A(\omega) \mu'(\omega) \mathrm{d}\omega. \tag{B.27}$$

The following subsection derives that this equation can be used to get that

$$R_t^A A_{t-1} = B_{t-1}^{tot} R_t + (R_{t-1}^B - \xi) D_{t-1} + (1-\theta) \frac{\mu(\overline{\overline{\omega}}_{t-1}) - \mu(\overline{\omega}_{t-1})}{1-\mu(\overline{\omega}_{t-1})} B_{t-1}. \tag{B.28}$$

This equation says that the funds paid to the household are equal to the returns from lending to producing firms net of intermediation costs, plus the returns from diverting funds, and including the funds that were lent to diverting firms and recovered.

The goods market clears:

$$Y_t = C_t + I_t^g. \tag{B.29}$$

Now we are ready to define a competitive equilibrium. The equilibrium is an allocation

$$\left\{ C_t, H_t, Y_t, K_t, B_t^{tot}, I_t^n, I_t^g, \Pi_t, T_t, B_t^G, B_t^H, D_t, \Xi_t, \overline{\omega}_t, \overline{\overline{\omega}}_t, Z_t, Z_{It}, \nu_{ct}, F_t, L_t, A_t, B_t \right\}_{t=0}^{\infty},$$

together with the sequence of prices $\left\{ \lambda_{ct}, R_t^A, W_t, R_t^B, Q_t, R_t, \rho_t \right\}_{t=0}^{\infty}$ satisfying equations (B.1), (B.2), (B.3), (B.4), (4), (6), (23), (12), (22), (24), (B.5), (26), (27), (28), (29), (31), (32), (33), (34), (40), (B.9), (B.13), (B.16), (B.19), (B.23), (B.24), (B.26), (B.28), and (B.29), together with the slope conditions defined in Appendix A, given initial conditions K_0, I_0^g, B_0^{tot},

B_0^G, B_0^H, D_0, $\overline{\omega}_0$, $\overline{\overline{\omega}}_0$, A_0, Z_0, $Z_{I,0}$, ν_{c0}, R_0^B, Q_0, and the exogenous processes $\{\varepsilon_t^z, \varepsilon_{It}, \varepsilon_{\nu t}\}$.[25]

B.4 Aggregate Equity Return

By definition

$$R_t^A = \int_0^{\overline{\omega}_{t-1}} R_t^A(\omega)\mu'(\omega)\mathrm{d}\omega + \int_{\overline{\omega}_{t-1}}^{\overline{\overline{\omega}}_{t-1}} R_t^A(\omega)\mu'(\omega)\mathrm{d}\omega + \int_{\overline{\overline{\omega}}_{t-1}}^{1} R_t^A(\omega)\mu'(\omega)\mathrm{d}\omega =$$
$$\int_0^{\overline{\omega}_{t-1}} \left(\rho_{t-1}\frac{1-\mu(\overline{\overline{\omega}}_{t-1})}{1-\mu(\overline{\omega}_{t-1})} + \frac{\mu(\overline{\overline{\omega}}_{t-1})-\mu(\overline{\omega}_{t-1})}{1-\mu(\overline{\omega}_{t-1})}(1-\theta)\right)\mu'(\omega)\mathrm{d}\omega +$$
$$\int_{\overline{\omega}_{t-1}}^{\overline{\overline{\omega}}_{t-1}} \frac{\left(R_{t-1}^B-\xi\right)\left(a_{t-1}(\omega)+\Theta_t(\omega)b_{t-1}(\omega)\right)}{a_{t-1}(\omega)}\mu'(\omega)\mathrm{d}\omega +$$
$$\int_{\overline{\overline{\omega}}_{t-1}}^{1} \frac{R_t(\omega)\left(a_{t-1}(\omega)+b_{t-1}(\omega)\right)-\rho_{t-1}b_{t-1}(\omega)}{a_{t-1}(\omega)}\mu'(\omega)\mathrm{d}\omega, \quad \text{(B.30)}$$

where we use equations (21), (17), and (15) to size the equity returns on each segment of firms.

[25] To express equations (33) and (34) in terms of the aggregate variables A_t, B_t, and F_t, we use equations (B.10), (B.11), and (39) to substitute for a_t, b_t, and $\Theta_t(\omega)$, respectively.

Simplifying

$$R_t^A = \rho_{t-1}\frac{1-\mu(\overline{\overline{\omega}}_{t-1})}{1-\mu(\overline{\omega}_{t-1})}\mu(\overline{\omega}_{t-1}) + \frac{\mu(\overline{\overline{\omega}}_{t-1})-\mu(\overline{\omega}_{t-1})}{1-\mu(\overline{\omega}_{t-1})}(1-\theta)\mu(\overline{\omega}_{t-1}) +$$
$$\int_{\overline{\omega}_{t-1}}^{\overline{\overline{\omega}}_{t-1}} \frac{(R_{t-1}^B-\xi)d_{t-1}(\omega)}{a_{t-1}}\mu'(\omega)\mathrm{d}\omega + \int_{\overline{\overline{\omega}}_{t-1}}^{1} \frac{R_t(\omega)(a_{t-1}+b_{t-1})}{a_{t-1}}\mu'(\omega)\mathrm{d}\omega - \int_{\overline{\overline{\omega}}_{t-1}}^{1} \frac{\rho_{t-1}b_{t-1}}{a_{t-1}}\mu'(\omega)\mathrm{d}\omega =$$
$$\rho_{t-1}\left(1-\mu(\overline{\overline{\omega}}_{t-1})\right)\frac{b_{t-1}}{a_{t-1}} + \left(\mu(\overline{\overline{\omega}}_{t-1})-\mu(\overline{\omega}_{t-1})\right)(1-\theta)\frac{b_{t-1}}{a_{t-1}} + \frac{(R_{t-1}^B-\xi)}{A_{t-1}}D_{t-1} +$$
$$\left(1-\mu(\overline{\overline{\omega}}_{t-1})\right)\frac{\left(A_{t-1}+\frac{B_{t-1}}{1-\mu(\overline{\overline{\omega}}_{t-1})}\right)}{A_{t-1}}\int_{\overline{\overline{\omega}}_{t-1}}^{1} R_t(\omega)\mu'(\omega)\mathrm{d}\omega - \rho_{t-1}\left(1-\mu(\overline{\overline{\omega}}_{t-1})\right)\frac{b_{t-1}}{a_{t-1}} =$$
$$(1-\theta)\frac{\mu(\overline{\overline{\omega}}_{t-1})-\mu(\overline{\omega}_{t-1})}{1-\mu(\overline{\omega}_{t-1})}\frac{B_{t-1}}{A_{t-1}} + \frac{(R_{t-1}^B-\xi)}{A_{t-1}}D_{t-1} +$$
$$\left(1-\mu(\overline{\overline{\omega}}_{t-1})\right)\frac{\left(A_{t-1}+\frac{B_{t-1}}{1-\mu(\overline{\overline{\omega}}_{t-1})}\right)}{A_{t-1}}\int_{\overline{\overline{\omega}}_{t-1}}^{1} R_t(\omega)\frac{\mu'(\omega)}{1-\mu(\overline{\overline{\omega}}_{t-1})}\mathrm{d}\omega =$$
$$(1-\theta)\frac{\mu(\overline{\overline{\omega}}_{t-1})-\mu(\overline{\omega}_{t-1})}{1-\mu(\overline{\omega}_{t-1})}\frac{B_{t-1}}{A_{t-1}} + \frac{(R_{t-1}^B-\xi)}{A_{t-1}}D_{t-1} + \frac{B_{t-1}^{tot}}{A_{t-1}}R_t, \quad \text{(B.31)}$$

where we use $a_{t-1}(\omega) = a_{t-1}$ and $b_{t-1}(\omega) = b_{t-1}$ for all ω and equations (B.10), (B.11), (B.14), and (B.19). Moreover, we use that

$$\frac{b_{t-1}}{a_{t-1}} = \frac{\mu_{\eta_1,\eta_2}(\overline{\omega}_{t-1})}{1-\mu_{\eta_1,\eta_2}(\overline{\omega}_{t-1})} = \frac{B_{t-1}}{A_{t-1}}\frac{1}{1-\mu_{\eta_1,\eta_2}(\overline{\omega}_{t-1})} \quad \text{(B.32)}$$

by combining equations (B.10), (B.11), and (B.9).

We can re-write equation (B.31) as follows:

$$R_t^A A_{t-1} = B_{t-1}^{tot}R_t + (R_{t-1}^B-\xi)D_{t-1} + (1-\theta)\frac{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_{t-1})-\mu_{\eta_1,\eta_2}(\overline{\omega}_{t-1})}{1-\mu(\overline{\omega}_{t-1})}B_{t-1}. \quad \text{(B.33)}$$

B.5 Beta Distribution: Aggregate Output

Plugging the functional form of the distribution into equation (B.23) and taking the integral:

$$Y_{t+1} = \frac{1}{1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t)} \int_{\overline{\overline{\omega}}_t}^{1} Z_{t+1}\omega K_t^{\alpha} H_{t+1}^{1-\alpha} \mu'(\omega) \mathrm{d}\omega =$$

$$\frac{1}{1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t)} \int_{\overline{\overline{\omega}}_t}^{1} Z_{t+1}\omega K_t^{\alpha} H_{t+1}^{1-\alpha} \frac{\omega^{\eta_1-1}(1-\omega)^{\eta_2-1}}{B(\eta_1,\eta_2)} \mathrm{d}\omega =$$

$$\frac{1}{1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t)} \int_{\overline{\overline{\omega}}_t}^{1} Z_{t+1} K_t^{\alpha} H_{t+1}^{1-\alpha} \frac{\omega^{\eta_1}(1-\omega)^{\eta_2-1}}{\Gamma(\eta_1)\Gamma(\eta_2)} \Gamma(\eta_1+\eta_2) \mathrm{d}\omega =$$

$$\frac{1}{1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t)} \int_{\overline{\overline{\omega}}_t}^{1} Z_{t+1} K_t^{\alpha} H_{t+1}^{1-\alpha} \frac{\omega^{\eta_1}(1-\omega)^{\eta_2-1}}{\Gamma(\eta_1+1)\Gamma(\eta_2)} \Gamma(\eta_1+\eta_2+1) \frac{\Gamma(\eta_1+1)}{\Gamma(\eta_1)} \frac{\Gamma(\eta_1+\eta_2)}{\Gamma(\eta_1+\eta_2+1)} \mathrm{d}\omega =$$

$$\frac{1}{1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t)} \int_{\overline{\overline{\omega}}_t}^{1} Z_{t+1} K_t^{\alpha} H_{t+1}^{1-\alpha} \frac{\omega^{\eta_1}(1-\omega)^{\eta_2-1}}{\Gamma(\eta_1+1)\Gamma(\eta_2)} \Gamma(\eta_1+\eta_2+1) \frac{\eta_1\Gamma(\eta_1)}{\Gamma(\eta_1)} \frac{\Gamma(\eta_1+\eta_2)}{(\eta_1+\eta_2)\Gamma(\eta_1+\eta_2)} \mathrm{d}\omega =$$

$$\frac{\eta_1}{\eta_1+\eta_2} \frac{1-\mu_{\eta_1+1,\eta_2}(\overline{\overline{\omega}}_t)}{1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t)} Z_{t+1} K_t^{\alpha} H_{t+1}^{1-\alpha}, \quad \text{(B.34)}$$

where we use the property of the gamma function: $\Gamma(z+1) = z\Gamma(z)$.

B.6 Beta Distribution: Average Return on Capital

Plugging the functional form of the distribution into equation (B.26) and taking the integral:

$$R_t = \int_{\overline{\overline{\omega}}_{t-1}}^{1} R_t(\omega) \frac{\mu'_{\eta_1,\eta_2}(\omega)}{1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_{t-1})} \mathrm{d}\omega =$$

$$\int_{\overline{\overline{\omega}}_{t-1}}^{1} \left(\frac{1}{Q_{t-1}} \alpha\ Z_t \left(\frac{H_t}{K_{t-1}}\right)^{1-\alpha} \omega + \frac{(1-\delta)}{Q_{t-1}} Q_t \right) \frac{\mu'_{\eta_1,\eta_2}(\omega)}{1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_{t-1})} \mathrm{d}\omega = \quad \text{(B.35)}$$

$$\frac{1}{1-\mu_{\eta_1,\eta_2}(\overline{\omega}_{t-1})}\int_{\overline{\overline{\omega}}_{t-1}}^{1}\frac{1}{Q_{t-1}}\alpha\ Z_t\left(\frac{H_t}{K_{t-1}}\right)^{1-\alpha}\omega\frac{\omega^{\eta_1-1}(1-\omega)^{\eta_2-1}}{B(\eta_1,\eta_2)}\mathrm{d}\omega+$$

$$\frac{1}{1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_{t-1})}\int_{\overline{\overline{\omega}}_{t-1}}^{1}\frac{(1-\delta)}{Q_{t-1}}Q_t\frac{\omega^{\eta_1-1}(1-\omega)^{\eta_2-1}}{B(\eta_1,\eta_2)}\mathrm{d}\omega=$$

$$\frac{\eta_1}{\eta_1+\eta_2}\frac{1-\mu_{\eta_1+1,\eta_2}(\overline{\overline{\omega}}_{t-1})}{1-\mu_{\eta_1,\eta_2}(\overline{\omega}_{t-1})}\frac{1}{Q_{t-1}}\alpha\ Z_t\left(\frac{H_t}{K_{t-1}}\right)^{1-\alpha}+\frac{(1-\delta)}{Q_{t-1}}Q_t \quad \text{(B.36)}$$

where we use the similar manipulations as before to calculate the integral.

B.7 Beta Distribution: Aggregate Equity Return

Plugging the functional form of the distribution into equation (B.30) and taking the integral:

$$R_t^A=\int_{0}^{\overline{\omega}_{t-1}}\left(\rho_{t-1}\frac{1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_{t-1})}{1-\mu_{\eta_1,\eta_2}(\overline{\omega}_{t-1})}+\frac{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_{t-1})-\mu_{\eta_1,\eta_2}(\overline{\omega}_{t-1})}{1-\mu_{\eta_1,\eta_2}(\overline{\omega}_{t-1})}(1-\theta)\right)\mu'_{\eta_1,\eta_2}(\omega)\mathrm{d}\omega+$$

$$\int_{\overline{\omega}_{t-1}}^{\overline{\overline{\omega}}_{t-1}}\frac{(R_{t-1}^B-\xi)\left(a_{t-1}(\omega)+\Theta_t(\omega)b_{t-1}(\omega)\right)}{a_{t-1}(\omega)}\mu'_{\eta_1,\eta_2}(\omega)\mathrm{d}\omega+$$

$$\int_{\overline{\overline{\omega}}_{t-1}}^{1}\frac{R_t(\omega)\left(a_{t-1}(\omega)+b_{t-1}(\omega)\right)-\rho_{t-1}b_{t-1}(\omega)}{a_{t-1}(\omega)}\mu'_{\eta_1,\eta_2}(\omega)\mathrm{d}\omega= \quad \text{(B.37)}$$

$$\rho_{t-1}\frac{1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_{t-1})}{1-\mu_{\eta_1,\eta_2}(\overline{\omega}_{t-1})}\mu_{\eta_1,\eta_2}(\overline{\omega}_{t-1})+\frac{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_{t-1})-\mu_{\eta_1,\eta_2}(\overline{\omega}_{t-1})}{1-\mu_{\eta_1,\eta_2}(\overline{\omega}_{t-1})}(1-\theta)\mu_{\eta_1,\eta_2}(\overline{\omega}_{t-1})+$$

$$\int_{\overline{\omega}_{t-1}}^{\overline{\overline{\omega}}_{t-1}}\frac{(R_{t-1}^B-\xi)d_{t-1}(\omega)}{a_{t-1}}\mu'_{\eta_1,\eta_2}(\omega)\mathrm{d}\omega+$$

$$\int_{\overline{\overline{\omega}}_{t-1}}^{1}\frac{R_t(\omega)(a_{t-1}+b_{t-1})}{a_{t-1}}\mu'_{\eta_1,\eta_2}(\omega)\mathrm{d}\omega-\int_{\overline{\overline{\omega}}_{t-1}}^{1}\frac{\rho_{t-1}b_{t-1}}{a_{t-1}}\mu'_{\eta_1,\eta_2}(\omega)\mathrm{d}\omega= \quad \text{(B.38)}$$

B.8 Beta Distribution: Diversion Function

Expressing F_t from equation (40):

$$\theta = \frac{1}{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t) - \mu_{\eta_1,\eta_2}(\overline{\omega}_t)} \int_{\overline{\omega}_t}^{\overline{\overline{\omega}}_t} \omega^{\psi} F_t \frac{\omega^{\eta_1-1}(1-\omega)^{\eta_2-1}}{B(\eta_1,\eta_2)} \mathrm{d}\omega =$$

$$\frac{1}{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t) - \mu_{\eta_1,\eta_2}(\overline{\omega}_t)} \int_{\overline{\omega}_t}^{\overline{\overline{\omega}}_t} F_t \frac{\omega^{\eta_1+\psi-1}(1-\omega)^{\eta_2-1}}{\Gamma(\eta_1)\Gamma(\eta_2)} \Gamma(\eta_1+\eta_2) \mathrm{d}\omega =$$

$$\frac{1}{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t) - \mu_{\eta_1,\eta_2}(\overline{\omega}_t)} \int_{\overline{\omega}_t}^{\overline{\overline{\omega}}_t} F_t \frac{\omega^{\eta_1+\psi-1}(1-\omega)^{\eta_2-1}}{\Gamma(\eta_1+\psi)\Gamma(\eta_2)} \Gamma(\eta_1+\psi+\eta_2) \frac{\Gamma(\eta_1+\eta_2)\Gamma(\eta_1+\psi)}{\Gamma(\eta_1)\Gamma(\eta_1+\psi+\eta_2)} \mathrm{d}\omega =$$

$$\frac{F_t}{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t) - \mu_{\eta_1,\eta_2}(\overline{\omega}_t)} \frac{\Gamma(\eta_1+\eta_2)\Gamma(\eta_1+\psi)}{\Gamma(\eta_1)\Gamma(\eta_1+\psi+\eta_2)} \left(\mu_{\eta_1+\psi,\eta_2}(\overline{\overline{\omega}}_t) - \mu_{\eta_1+\psi,\eta_2}(\overline{\omega}_t)\right) \quad \text{(B.39)}$$

Therefore,

$$F_t = \frac{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t) - \mu_{\eta_1,\eta_2}(\overline{\omega}_t)}{\mu_{\eta_1+\psi,\eta_2}(\overline{\overline{\omega}}_t) - \mu_{\eta_1+\psi,\eta_2}(\overline{\omega}_t)} \frac{\Gamma(\eta_1)\Gamma(\eta_1+\psi+\eta_2)}{\Gamma(\eta_1+\eta_2)\Gamma(\eta_1+\psi)} \theta \quad \text{(B.40)}$$

B.9 Equilibrium Conditions

This subsection lists the 29 equilibrium conditions that constitute a competitive equilibrium and identifies the derivation or earlier statement from which each is obtained.

Asset Euler equation. Equation (B.1) from Appendix B.1, rearranged to place λ_{ct} on the left-hand side:

$$\lambda_{ct} = \beta E_t \left\{ \lambda_{ct+1} R^A_{t+1} \right\}. \quad \text{(B.41)}$$

Consumption Euler equation. Equation (B.2) from Appendix B.1:

$$\beta E_t \left\{ \lambda_{ct+1} R^B_t \right\} = \frac{1}{C_t - \nu_{ct}}. \quad \text{(B.42)}$$

Labor supply. Compact form obtained by combining equations (B.3), (B.4), and (B.2) from Appendix B.1. Substituting $\beta E_t\{\lambda_{ct+1} R^B_t\} = \lambda_{ct}$ from (B.4) into (B.3) gives $-\vartheta/(C_t -$

$\nu_{ct})H_t^{\nu} + \lambda_{ct}W_t = 0$. Applying $\lambda_{ct} = 1/(C_t - \nu_{ct})$ from (B.2) then yields:

$$W_t = \vartheta(C_t - \nu_{ct})H_t^{\nu}. \tag{B.43}$$

Bond Euler equation. Equation (B.4) from Appendix B.1, rearranged to place λ_{ct} on the left-hand side:

$$\lambda_{ct} = \beta E_t \left\{ \lambda_{ct+1} R_t^B \right\}. \tag{B.44}$$

Aggregate output. Beta-distribution specialization of equation (B.23) from Appendix B.3, with time indices shifted by one period (replacing $t+1$ by t and t by $t-1$). The integral $\int_{\overline{\overline{\omega}}_{t-1}}^{1} \omega\, \mu'_{\eta_1,\eta_2}(\omega)\, \mathrm{d}\omega$ is evaluated in Appendix B.5 by rewriting $\omega \cdot \mu'_{\eta_1,\eta_2}(\omega)$ as a scalar multiple of the Beta(η_1+1, η_2) density, using the recurrence $\Gamma(z+1) = z\,\Gamma(z)$:

$$Y_t = \frac{\eta_1}{\eta_1+\eta_2} \frac{1 - \mu_{\eta_1+1,\eta_2}(\overline{\overline{\omega}}_{t-1})}{1 - \mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_{t-1})} Z_t K_{t-1}^{\alpha} H_t^{1-\alpha}. \tag{B.45}$$

Capital financing. Aggregate counterpart of the individual firm budget constraint (9) from Section 4.2.1. Derived in Appendix B.3 as equation (B.24) by integrating $b_t^{tot}(\omega) = Q_t k_t(\omega)$ over the mass of producing firms:

$$B_t^{tot} = Q_t K_t. \tag{B.46}$$

Labor demand. Equation (12) from Section 4.2.1, the first-order condition for labor of an individual producing firm. Under constant returns to scale, the labor–capital ratio is equalized across all producing firms and coincides with the aggregate ratio:

$$W_t = (1-\alpha)\,\frac{Y_t}{H_t}. \tag{B.47}$$

Average return on capital. Beta-distribution specialization of equation (B.26) from Appendix B.3. The integral is evaluated in Appendix B.6 by the same method as Appendix B.5, rewriting $\omega \cdot \mu'_{\eta_1,\eta_2}(\omega)$ as a scalar multiple of the Beta(η_1+1, η_2) density:

$$R_t = \frac{\eta_1}{\eta_1+\eta_2} \frac{1 - \mu_{\eta_1+1,\eta_2}(\overline{\overline{\omega}}_{t})}{1 - \mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_{t})} \frac{1}{Q_{t-1}} \alpha\; Z_t \left(\frac{H_t}{K_{t-1}}\right)^{1-\alpha} + \frac{(1-\delta)}{Q_{t-1}} Q_t. \tag{B.48}$$

Investment supply. Equation (22) from Section 4.3:

$$I_t^n = Z_{It}\left[1 - \frac{\phi}{2}\left(\frac{I_t^g}{I_{t-1}^g} - 1\right)^2\right] I_t^g. \tag{B.49}$$

Capital accumulation. Equation (24) from Section 4.3:

$$K_t = I_t^n + (1-\delta)K_{t-1}. \tag{B.50}$$

Capital-firm profits. Equation (26) from Section 4.3, the zero-profit condition for competitive capital-producing firms:

$$\Pi_t = Q_t Z_{It}\left[1 - \frac{\phi}{2}\left(\frac{I_t^g}{I_{t-1}^g} - 1\right)^2\right] I_t^g - I_t^g. \tag{B.51}$$

Government budget constraint. Equation (27) from Section 4.4:

$$T_t = B_t^G - R_{t-1}^B B_{t-1}^G. \tag{B.52}$$

Bond market identity. Equation (28) from Section 4.4:

$$B_t^G = B_t^H + D_t. \tag{B.53}$$

Household bond holdings. Equation (29) from Section 4.4. By assumption, government bonds are sold only to firms, so households hold none:

$$B_t^H = 0. \tag{B.54}$$

Haircut rebate to households. Equation (31) from Section 4.4:

$$\Xi_t = \xi D_{t-1}. \tag{B.55}$$

Lower cutoff condition. Equation (32) from Section 5.1, unchanged. The expected return from producing at the marginal lending firm (productivity $\overline{\omega}_t$) equals the expected return from the outside option:

$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(\frac{1}{Q_t}\alpha\ Z_{t+1}\left(\frac{H_{t+1}}{K_t}\right)^{1-\alpha}\overline{\omega}_t+\frac{(1-\delta)}{Q_t}Q_{t+1}\right)\right]=E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_t^B-\xi\right)\right]. \quad \text{(B.56)}$$

Aggregate lending. Equation (B.13) from Appendix B.3, with the generic cumulative distribution function μ replaced by its Beta-distribution counterpart μ_{η_1,η_2}:

$$L_t=\mu_{\eta_1,\eta_2}(\overline{\omega}_t)A_t. \quad \text{(B.57)}$$

Loan market clearing. Equation (B.9) from Appendix B.3:

$$B_t=L_t. \quad \text{(B.58)}$$

Aggregate diversion. Equation (B.16) from Appendix B.3, with μ replaced by μ_{η_1,η_2} throughout:

$$D_t=\left(\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t)-\mu_{\eta_1,\eta_2}(\overline{\omega}_t)\right)A_t+\frac{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t)-\mu_{\eta_1,\eta_2}(\overline{\omega}_t)}{1-\mu_{\eta_1,\eta_2}(\overline{\omega}_t)}\theta B_t. \quad \text{(B.59)}$$

Aggregate total borrowing. Equation (B.19) from Appendix B.3, with μ replaced by μ_{η_1,η_2} throughout:

$$B_t^{tot}=\left(1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t)\right)A_t+\frac{1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t)}{1-\mu_{\eta_1,\eta_2}(\overline{\omega}_t)}B_t. \quad \text{(B.60)}$$

Equity return market clearing. Equation (B.28) from Appendix B.3, with μ replaced by μ_{η_1,η_2} throughout. The general form is derived in Appendix B.4 by aggregating the per-firm equity returns (21), (17), and (15) across the three firm segments; Appendix B.7 verifies the Beta-specific version:

$$R_t^A A_{t-1}=B_{t-1}^{tot}R_t+(R_{t-1}^B-\xi)D_{t-1}+(1-\theta)\frac{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_{t-1})-\mu_{\eta_1,\eta_2}(\overline{\omega}_{t-1})}{1-\mu_{\eta_1,\eta_2}(\overline{\omega}_{t-1})}B_{t-1}. \quad \text{(B.61)}$$

Goods market clearing. Equation (B.29) from Appendix B.3:

$$Y_t=C_t+I_t^g. \quad \text{(B.62)}$$

Loan rate equation. Aggregate Beta-distribution form of equation (33) from Section 5.1.

Individual-firm variables are replaced using $a_t = A_t$ from (B.10) and $b_t = B_t/(1-\mu_{\eta_1,\eta_2}(\overline{\omega}_t))$ from (B.11); the diversion function $\Theta_t(\overline{\omega}_t)b_t$ is substituted using the functional form (39), giving $F_t\overline{\omega}_t^{\psi}B_t/(1-\mu_{\eta_1,\eta_2}(\overline{\omega}_t))$, where F_t is defined in equation (B.69) below:

$$\left(\rho_t \frac{1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t)}{1-\mu_{\eta_1,\eta_2}(\overline{\omega}_t)} + (1-\theta)\frac{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t)-\mu_{\eta_1,\eta_2}(\overline{\omega}_t)}{1-\mu_{\eta_1,\eta_2}(\overline{\omega}_t)}\right)A_t = \left(R_t^B-\xi\right)\left(A_t + F_t\overline{\omega}_t^{\psi}\frac{B_t}{1-\mu_{\eta_1,\eta_2}(\overline{\omega}_t)}\right). \tag{B.63}$$

Upper cutoff condition. Aggregate Beta-distribution form of equation (34) from Section 5.1. The same substitutions as for the loan rate equation above are applied: a_t, b_t, and $\Theta_t(\overline{\overline{\omega}}_t)$ are replaced using (B.10), (B.11), and (39) respectively:

$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_t^B-\xi\right)\left(A_t + F_t\overline{\overline{\omega}}_t^{\psi}\frac{B_t}{1-\mu_{\eta_1,\eta_2}(\overline{\omega}_t)}\right)\right] = E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(\left(\frac{1}{Q_t}\alpha\ Z_{t+1}\left(\frac{H_{t+1}}{K_t}\right)^{1-\alpha}\overline{\overline{\omega}}_t + \frac{(1-\delta)}{Q_t}Q_{t+1}\right)\left(A_t + \frac{B_t}{1-\mu_{\eta_1,\eta_2}(\overline{\omega}_t)}\right) - \rho_t\frac{B_t}{1-\mu_{\eta_1,\eta_2}(\overline{\omega}_t)}\right)\right]. \tag{B.64}$$

Capital firm first-order condition (Tobin's q). Equation (B.5) from Appendix B.2, derived from the capital-producing firm's optimization problem (25) stated in Section 4.3:

$$0 = E_t\left\{\begin{array}{c} Q_tZ_{It}\left[-\phi\left(\frac{I_t^g}{I_{t-1}^g}-1\right)\frac{1}{I_{t-1}^g}\right]I_t^g + Q_tZ_{It}\left[1-\frac{\phi}{2}\left(\frac{I_t^g}{I_{t-1}^g}-1\right)^2\right]-1 \\ +\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}Q_{t+1}Z_{It+1}\phi\left(\frac{I_{t+1}^g}{I_t^g}-1\right)\left(\frac{I_{t+1}^g}{I_t^g}\right)^2 \end{array}\right\}. \tag{B.65}$$

Aggregate TFP process. Equation (6) from Section 4.2:

$$\log Z_t = \rho_z \log Z_{t-1} + \varepsilon_t^z. \tag{B.66}$$

Investment-specific technology process. Equation (23) from Section 4.3:

$$\ln(Z_{It}) = \rho_I \ln(Z_{It-1}) + \varepsilon_{It}. \tag{B.67}$$

Consumption demand shock process. Equation (4) from Section 4.1:

$$\nu_{ct} = \rho_\nu \nu_{ct-1} + \varepsilon_{\nu t}. \tag{B.68}$$

Diversion scaling factor (F_t). Beta-distribution closed form of the integral definition (40) from Section 5.1. The derivation in Appendix B.8 rewrites $\omega^\psi \mu'_{\eta_1,\eta_2}(\omega)$ as a scalar multiple of the Beta$(\eta_1 + \psi, \eta_2)$ density kernel, using $\Gamma(\eta_1 + \psi)/\Gamma(\eta_1)$ to convert the normalizing constants:

$$F_t = \frac{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t) - \mu_{\eta_1,\eta_2}(\overline{\omega}_t)}{\mu_{\eta_1+\psi,\eta_2}(\overline{\overline{\omega}}_t) - \mu_{\eta_1+\psi,\eta_2}(\overline{\omega}_t)} \frac{\Gamma(\eta_1)\Gamma(\eta_1 + \psi + \eta_2)}{\Gamma(\eta_1 + \eta_2)\Gamma(\eta_1 + \psi)}\theta \tag{B.69}$$

and ω follows the Beta distribution with parameters η_1 and η_2, i.e.,

$$\mu'_{\eta_1,\eta_2}(\omega) = \frac{\omega^{\eta_1 - 1}(1-\omega)^{\eta_2 - 1}}{B(\eta_1, \eta_2)},$$

where $B(\eta_1, \eta_2) = \frac{\Gamma(\eta_1)\Gamma(\eta_2)}{\Gamma(\eta_1+\eta_2)}$ and Γ is the Gamma function.

C Steady-State Conditions

We derived 29 equilibrium conditions for 29 endogenous variables:

$\lambda_{ct}, R^A_t, C_t, H_t, W_t, R^B_t, Y_t, \overline{\omega}_t, \overline{\overline{\omega}}_t, Z_t, Z_{It}, \nu_{ct}, K_t, B^{tot}_t, Q_t, R_t, I^n_t, I^g_t, \Pi_t, T_t, B^G_t, B^H_t, D_t, \Xi_t, L_t,$
$A_t, B_t, \rho_t,$ and F_t.

Next, we consider a strategy for finding the non-stochastic steady state. The strategy will involve guessing the values of $\overline{\omega}$, $\overline{\overline{\omega}}$, and R and iterating to a fixed point.

Given our guesses, we can find

$$F = \frac{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}) - \mu_{\eta_1,\eta_2}(\overline{\omega})}{\mu_{\eta_1+\psi,\eta_2}(\overline{\overline{\omega}}) - \mu_{\eta_1+\psi,\eta_2}(\overline{\omega})} \frac{\Gamma(\eta_1)\Gamma(\eta_1 + \psi + \eta_2)}{\Gamma(\eta_1 + \eta_2)\Gamma(\eta_1 + \psi)}\theta \tag{C.1}$$

Continue by fixing

$$H = H^{ss} = 1. \tag{C.2}$$

We can support any choice of H by choosing ϑ appropriately. For example, from the intratemporal condition (B.43) evaluated in steady state, we can find that $\vartheta = \frac{W}{C\,H^{\nu}}$ supports $H = 1$ in the steady state.

From equation (B.44), we can see that in the steady state

$$R^B = \frac{1}{\beta}. \tag{C.3}$$

Similarly, from equation (B.41)

$$R^A = \frac{1}{\beta}. \tag{C.4}$$

From equation (B.66)

$$Z = 1. \tag{C.5}$$

From equation (B.67)

$$Z_I = 1. \tag{C.6}$$

From equation (B.68)

$$\nu_c = 0. \tag{C.7}$$

From equation (B.65)

$$Q = 1. \tag{C.8}$$

From equation (B.48) in the steady state

$$R = \frac{\eta_1}{\eta_1 + \eta_2} \frac{1 - \mu_{\eta_1+1,\eta_2}(\overline{\overline{\omega}})}{1 - \mu_{\eta_1,\eta_2}(\overline{\overline{\omega}})} \alpha \left(\frac{H}{K}\right)^{1-\alpha} + 1 - \delta,$$

we can solve for K

$$K = H \left[(R - (1-\delta)) \frac{\eta_1 + \eta_2}{\alpha \eta_1} \frac{1 - \mu_{\eta_1,\eta_2}(\overline{\overline{\omega}})}{1 - \mu_{\eta_1+1,\eta_2}(\overline{\overline{\omega}})} \right]^{\frac{1}{\alpha-1}}. \tag{C.9}$$

Combining equation (B.46) and our result (C.8), we get

$$B^{tot} = K. \tag{C.10}$$

From equation (B.45)

$$Y = \frac{\eta_1}{\eta_1 + \eta_2} \frac{1 - \mu_{\eta_1+1,\eta_2}(\overline{\overline{\omega}})}{1 - \mu_{\eta_1,\eta_2}(\overline{\overline{\omega}})} K^{\alpha} H^{1-\alpha} \tag{C.11}$$

From equation (B.47)

$$W = (1 - \alpha) \frac{Y}{H}. \tag{C.12}$$

From equation (B.50)

$$I^n = \delta K. \tag{C.13}$$

From equation (B.49)

$$I^g = I^n. \tag{C.14}$$

Therefore, from equation (B.62), we can find

$$C = Y - I^g. \tag{C.15}$$

Therefore, from equation (B.42), we can find

$$\lambda_c = \frac{1}{C}. \tag{C.16}$$

Combining equations (B.57) and (B.58) and plugging the result into equation (B.60),

$$B^{tot} = \left(1 - \mu_{\eta_1,\eta_2}(\overline{\overline{\omega}})\right) A + \frac{1 - \mu_{\eta_1,\eta_2}(\overline{\overline{\omega}})}{1 - \mu_{\eta_1,\eta_2}(\overline{\omega})} \mu_{\eta_1,\eta_2}(\overline{\omega}) A$$

we can find A

$$A = \frac{B^{tot}}{1 - \mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}) + \frac{1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}})}{1-\mu_{\eta_1,\eta_2}(\overline{\omega})} \mu_{\eta_1,\eta_2}(\overline{\omega})}. \tag{C.17}$$

Therefore, from the combination of equations (B.57) and (B.58), we can find

$$B = \mu_{\eta_1,\eta_2}(\overline{\omega}) A. \tag{C.18}$$

From equation (B.58)

$$L = B. \tag{C.19}$$

From equation (B.59)

$$D = \left(\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}) - \mu_{\eta_1,\eta_2}(\overline{\omega})\right) A + \frac{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}) - \mu_{\eta_1,\eta_2}(\overline{\omega})}{1-\mu_{\eta_1,\eta_2}(\overline{\omega})}\theta B. \tag{C.20}$$

From equation (B.54)

$$B^H = 0. \tag{C.21}$$

Therefore, from equation (B.53)

$$B^G = D. \tag{C.22}$$

From equation (B.52)

$$T = B^G - R^B B^G, \tag{C.23}$$

From equation (B.55)

$$\Xi = \xi D. \tag{C.24}$$

From equation (B.51)

$$\Pi = 0. \tag{C.25}$$

From equation (B.63)

$$\left(\rho \frac{1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}})}{1-\mu_{\eta_1,\eta_2}(\overline{\omega})} + (1-\theta)\frac{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}) - \mu_{\eta_1,\eta_2}(\overline{\omega})}{1-\mu_{\eta_1,\eta_2}(\overline{\omega})}\right) A = \left(R^B - \xi\right)\left(A + \frac{F\overline{\omega}^{\psi} B}{1-\mu_{\eta_1,\eta_2}(\overline{\omega})}\right),$$

we can find

$$\rho = \frac{1-\mu_{\eta_1,\eta_2}(\overline{\omega})}{1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}})}\left(\left(R^B - \xi\right)\left(1 + \frac{F\overline{\omega}^{\psi}\mu_{\eta_1,\eta_2}(\overline{\omega})}{(1-\mu_{\eta_1,\eta_2}(\overline{\omega}))}\right) - (1-\theta)\frac{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}) - \mu_{\eta_1,\eta_2}(\overline{\omega})}{1-\mu_{\eta_1,\eta_2}(\overline{\omega})}\right), \tag{C.26}$$

where we use equation (C.18) to substitute for $\frac{B}{A}$.

We have 3 equations to verify our 3 guesses

1. From equation (B.56), verify the guess for $\overline{\omega}$

$$\alpha \left(\frac{H}{K}\right)^{1-\alpha} \overline{\omega} + 1 - \delta = R^B - \xi,$$

which can be re-written as

$$0 = -1 + \frac{R^B - \xi - (1-\delta)}{\alpha \left(\frac{H}{K}\right)^{1-\alpha} \overline{\omega}}. \tag{C.27}$$

2. From equation (B.64), verify the guess for $\overline{\overline{\omega}}$

$$\left(R^B - \xi\right)\left(A + F\overline{\overline{\omega}}^{\psi} \frac{B}{1-\mu_{\eta_1,\eta_2}(\overline{\omega})}\right) = \\ \left(\alpha \left(\frac{H}{K}\right)^{1-\alpha} \overline{\overline{\omega}} + 1 - \delta\right)\left(A + \frac{B}{1-\mu_{\eta_1,\eta_2}(\overline{\omega})}\right) - \rho \frac{B}{1-\mu_{\eta_1,\eta_2}(\overline{\omega})},$$

which can be re-written as

$$0 = -1 + \left(\alpha \left(\frac{H}{K}\right)^{1-\alpha} \overline{\overline{\omega}} + 1 - \delta\right) \frac{1}{\rho}\left(\frac{1-\mu_{\eta_1,\eta_2}(\overline{\omega})}{\mu_{\eta_1,\eta_2}(\overline{\omega})} + 1\right) - \\ \left(R^B - \xi\right)\frac{1}{\rho}\left(\frac{1-\mu_{\eta_1,\eta_2}(\overline{\omega})}{\mu_{\eta_1,\eta_2}(\overline{\omega})} + F\overline{\overline{\omega}}^{\psi}\right), \tag{C.28}$$

where we use equation (C.18) to substitute for $\frac{A}{B}$

3. From equation (B.61), verify the guess for R

$$R^A A = B^{tot} R + (R^B - \xi)D + (1-\theta)\frac{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}) - \mu_{\eta_1,\eta_2}(\overline{\omega})}{1-\mu_{\eta_1,\eta_2}(\overline{\omega})} B,$$

which can be re-written as

$$0 = -1 + \frac{B^{tot}R}{R^A A} + \frac{(R^B - \xi)D}{R^A A} + (1-\theta)\frac{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}) - \mu_{\eta_1,\eta_2}(\overline{\omega})}{(1-\mu_{\eta_1,\eta_2}(\overline{\omega}))R^A A} B. \tag{C.29}$$

D Data and Model Counterparts

We are interested in the segment for the producing firms. To calculate the 90^{th} percentile for that segment, we need to rescale the density so that it integrates to 1 over the segment.

Accordingly, for the 10^{th} percentile, we can solve for ω_{10} from

$$\int_{\overline{\overline{\omega}}}^{\omega_{10}} \frac{\mu'_{\eta_1,\eta_2}(\omega)}{1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}})} \mathrm{d}\omega = 0.10. \tag{D.30}$$

Working on the derivation:

$$\frac{1}{1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}})} \left(\mu_{\eta_1,\eta_2}(\omega_{10}) - \mu_{\eta_1,\eta_2}(\overline{\overline{\omega}})\right) = 0.10.$$

Hence,

$$\omega_{10} = \mu^{-1}_{\eta_1,\eta_2}\left[0.10\left(1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}})\right) + \mu_{\eta_1,\eta_2}(\overline{\overline{\omega}})\right], \tag{D.31}$$

where $\mu^{-1}_{\eta_1,\eta_2}$ is the inverse of the Beta distribution.

Analogously for the 90^{th} percentile, the expression will be

$$\omega_{90} = \mu^{-1}_{\eta_1,\eta_2}\left[0.90\left(1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}})\right) + \mu_{\eta_1,\eta_2}(\overline{\overline{\omega}})\right]. \tag{D.32}$$

Therefore, our target ratio is

$$\frac{\omega_{90}}{\omega_{10}} = \frac{\mu^{-1}_{\eta_1,\eta_2}\left[0.90\left(1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}})\right) + \mu_{\eta_1,\eta_2}(\overline{\overline{\omega}})\right]}{\mu^{-1}_{\eta_1,\eta_2}\left[0.10\left(1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}})\right) + \mu_{\eta_1,\eta_2}(\overline{\overline{\omega}})\right]}. \tag{D.33}$$

E Representative-Firm RBC Benchmark

In this appendix we show that equilibrium conditions of our baseline real model are equivalent to those of a canonical RBC model for the special case in which production is efficient, i.e. $\bar{\omega} = \bar{\bar{\omega}} = 1$.

We describe the canonical RBC model before tackling the equivalence proof.

E.1 The canonical RBC model

The model adopts the canonical RBC decentralization: the household directly owns the capital stock and rents it to the firm, rather than holding equity issued by firms as in the baseline. Capital-producing firms with investment adjustment costs are kept unchanged.

E.1.1 Firm's Problem

In each period t the representative goods-producing firm rents capital K_{t-1} from the household at the competitive rental rate r_t^K and hires labor H_t at the competitive wage W_t. It produces output

$$Y_t = Z_t K_{t-1}^{\alpha} H_t^{1-\alpha} \tag{E.1}$$

and maximizes static profit $\Pi_t^f = Y_t - r_t^K K_{t-1} - W_t H_t$. With constant returns to scale, equilibrium profits are zero and the first-order conditions give

$$W_t = (1-\alpha)\frac{Y_t}{H_t}, \tag{E.2}$$

$$r_t^K = \alpha \frac{Y_t}{K_{t-1}}. \tag{E.3}$$

E.1.2 Capital Producers' Problem

Competitive capital-producing firms solve exactly the same problem as in the baseline model (Section 4.3). They purchase I_t^g units of the consumption good, install

$$I_t^n = Z_{It}\left[1 - \frac{\phi}{2}\left(\frac{I_t^g}{I_{t-1}^g} - 1\right)^2\right] I_t^g \tag{E.4}$$

units of new capital, and sell it at the shadow price Q_t (Tobin's q), earning profits

$$\Pi_t = Q_t Z_{It}\left[1 - \frac{\phi}{2}\left(\frac{I_t^g}{I_{t-1}^g} - 1\right)^2\right] I_t^g - I_t^g. \tag{E.5}$$

Profit maximization subject to the investment technology yields

$$0 = E_t \left\{ \begin{array}{c} Q_t Z_{It}\left[-\phi\left(\frac{I_t^g}{I_{t-1}^g} - 1\right)\frac{1}{I_{t-1}^g}\right] I_t^g + Q_t Z_{It}\left[1 - \frac{\phi}{2}\left(\frac{I_t^g}{I_{t-1}^g} - 1\right)^2\right] - 1 \\ +\beta \frac{\lambda_{ct+1}}{\lambda_{ct}} Q_{t+1} Z_{It+1} \phi \left(\frac{I_{t+1}^g}{I_t^g} - 1\right)\left(\frac{I_{t+1}^g}{I_t^g}\right)^2 \end{array} \right\}. \tag{E.6}$$

E.1.3 Household's Problem

The representative household has the same KPR preferences as in the baseline model. It directly owns the capital stock K_{t-1}, rents it to the representative firm, receives the profits of capital-producing firms, and chooses consumption C_t, labor H_t, gross investment I_t^g, and the future capital stock K_t. The household solves

$$\max_{\{C_{t+\tau}, H_{t+\tau}, I^g_{t+\tau}, K_{t+\tau}\}_{\tau=0}^{\infty}} E_0 \sum_{\tau=0}^{\infty} \beta^\tau \left[\ln(C_{t+\tau} - \nu_{c,t+\tau}) - \frac{\vartheta}{1+\nu} (H_{t+\tau})^{1+\nu} \right], \tag{E.7}$$

subject to the flow budget constraint

$$C_t + I_t^g = W_t H_t + r_t^K K_{t-1} + \Pi_t \tag{E.8}$$

and the capital accumulation equation

$$K_t = I_t^n + (1-\delta) K_{t-1}, \tag{E.9}$$

where I_t^n is determined by the capital producers' technology (E.4) and Π_t denotes the profits of capital-producing firms rebated to the household.

Let λ_{ct} be the Lagrange multiplier on (E.8) and Q_t the multiplier (in units of the consumption good) on (E.9). With KPR log utility in consumption, the first-order conditions are:

$$\lambda_{ct} = \frac{1}{C_t - \nu_{ct}}, \tag{E.10}$$

$$W_t = \vartheta (C_t - \nu_{ct}) H_t^\nu, \tag{E.11}$$

$$\lambda_{ct} = \beta E_t \{ \lambda_{ct+1} R_{t+1} \}, \tag{E.12}$$

where the gross return on capital is

$$R_{t+1} \equiv \frac{r_{t+1}^K + (1-\delta) Q_{t+1}}{Q_t} = \frac{\alpha Z_{t+1} (H_{t+1}/K_t)^{1-\alpha} + (1-\delta) Q_{t+1}}{Q_t}. \tag{E.13}$$

The first-order condition for I_t^g coincides with the capital producers' optimality condition

(E.6).

E.1.4 Equilibrium Conditions

The equilibrium consists of sequences for $\{Y_t, C_t, H_t, W_t, K_t, I_t^n, I_t^g, R_t, Q_t, \Pi_t, \lambda_{ct}, Z_t, Z_{It}\}$ satisfying the following conditions:

$$\lambda_{ct} = \beta E_t\{\lambda_{ct+1} R_{t+1}\}, \tag{E.14}$$

$$\lambda_{ct} = \frac{1}{C_t - \nu_{ct}}, \tag{E.15}$$

$$W_t = \vartheta (C_t - \nu_{ct}) H_t^{\nu}, \tag{E.16}$$

$$Y_t = Z_t K_{t-1}^{\alpha} H_t^{1-\alpha}, \tag{E.17}$$

$$W_t = (1-\alpha)\frac{Y_t}{H_t}, \tag{E.18}$$

$$R_t = \frac{1}{Q_{t-1}} \alpha\, Z_t \left(\frac{H_t}{K_{t-1}}\right)^{1-\alpha} + \frac{(1-\delta)}{Q_{t-1}} Q_t, \tag{E.19}$$

$$I_t^n = Z_{It}\left[1 - \frac{\phi}{2}\left(\frac{I_t^g}{I_{t-1}^g} - 1\right)^2\right] I_t^g, \tag{E.20}$$

$$K_t = I_t^n + (1-\delta) K_{t-1}, \tag{E.21}$$

$$\Pi_t = Q_t Z_{It}\left[1 - \frac{\phi}{2}\left(\frac{I_t^g}{I_{t-1}^g} - 1\right)^2\right] I_t^g - I_t^g, \tag{E.22}$$

$$0 = E_t \left\{ \begin{array}{c} Q_t Z_{It}\left[-\phi\left(\frac{I_t^g}{I_{t-1}^g} - 1\right)\frac{1}{I_{t-1}^g}\right] I_t^g + Q_t Z_{It}\left[1 - \frac{\phi}{2}\left(\frac{I_t^g}{I_{t-1}^g} - 1\right)^2\right] - 1 \\ + \beta \frac{\lambda_{ct+1}}{\lambda_{ct}} Q_{t+1} Z_{It+1} \phi \left(\frac{I_{t+1}^g}{I_t^g} - 1\right)\left(\frac{I_{t+1}^g}{I_t^g}\right)^2 \end{array} \right\}, \tag{E.23}$$

$$Y_t = C_t + I_t^g, \tag{E.24}$$

$$\log Z_t = \rho_z \log Z_{t-1} + \varepsilon_t^z, \tag{E.25}$$

$$\ln(Z_{It}) = \rho_I \ln(Z_{It-1}) + \varepsilon_{It}, \tag{E.26}$$

$$\nu_{ct} = \rho_\nu \nu_{ct-1} + \varepsilon_t^{\nu}. \tag{E.27}$$

Equations (E.14)–(E.16) are the household's first-order conditions. Under KPR preferences, equation (E.15) gives $\lambda_{ct} = 1/(C_t - \nu_{ct})$ and equation (E.16) gives $W_t = \vartheta(C_t - \nu_{ct})H_t^{\nu}$, so a wealth effect on labor is present. Equation (E.27) is the consumption preference shock process. Equations (E.17)–(E.19) describe goods production and factor returns. Equations (E.20)–(E.22) govern investment and capital accumulation. Equation (E.23) is the capital producers' optimality condition (Tobin's q equation). Equations (E.24)–(E.25) are the goods market clearing condition and the TFP process, respectively. Equation (E.26) is the investment-efficiency shock process.

E.2 Equivalence with the Baseline Real Model

We show that the equilibrium conditions of the RBC model (Section E.1.4) are equivalent to those of the baseline real model (Appendix B.9) when $\bar{\omega}_t = \bar{\bar{\omega}}_t = 1$ for all t, in the sense that each RBC condition (E.14)–(E.27) coincides with the corresponding baseline condition once the financial variables absent from the RBC model are eliminated by substitution. We derive a key limit first and then proceed in three steps.

A key limit. Both $1 - \mu_{\eta_1+1,\eta_2}(\omega)$ and $1 - \mu_{\eta_1,\eta_2}(\omega)$ vanish as $\omega \to 1$. Using the local behaviour of the Beta c.d.f. near 1,

$$1 - \mu_{\eta_1,\eta_2}(\omega) \ \sim\ \frac{(1-\omega)^{\eta_2}}{B(\eta_1,\eta_2)\,\eta_2} \qquad \text{as } \omega \to 1,$$

one obtains

$$\lim_{\bar{\bar{\omega}}\to 1} \frac{\eta_1}{\eta_1+\eta_2}\frac{1-\mu_{\eta_1+1,\eta_2}(\bar{\bar{\omega}})}{1-\mu_{\eta_1,\eta_2}(\bar{\bar{\omega}})} \ =\ \frac{\eta_1}{\eta_1+\eta_2}\cdot\frac{B(\eta_1,\eta_2)}{B(\eta_1+1,\eta_2)} \ =\ \frac{\eta_1}{\eta_1+\eta_2}\cdot\frac{\eta_1+\eta_2}{\eta_1} \ =\ 1, \qquad \text{(E.28)}$$

where we use $B(\eta_1,\eta_2)/B(\eta_1+1,\eta_2) = \Gamma(\eta_1+\eta_2+1)/\Gamma(\eta_1+\eta_2)\cdot\Gamma(\eta_1)/\Gamma(\eta_1+1) = (\eta_1+\eta_2)/\eta_1$.

Step 1: Financial variables degenerate. At $\bar{\omega}_t = \bar{\bar{\omega}}_t = 1$ the CDF satisfies $\mu_{\eta_1,\eta_2}(1) = 1$, so the measures of the three firm segments collapse:

- Equation (B.54): $B_t^H = 0$.
- Equation (B.59): $D_t = \big(\mu(1) - \mu(1)\big)A_t + \frac{\mu(1)-\mu(1)}{1-\mu(1)}\theta B_t = 0$.
- Equations (B.53)–(B.52): $B_t^G = 0,\ T_t = 0$.

- Equation (B.55): $\Xi_t = \xi D_{t-1} = 0$.
- Equations (B.57)–(B.58): $L_t = B_t = \mu(1)A_t = A_t$.

For B_t^{tot}, equation (B.60) reads $B_t^{tot} = (1-\mu(\bar{\omega}_t))A_t + \frac{1-\mu(\bar{\bar{\omega}}_t)}{1-\mu(\bar{\omega}_t)}B_t$. Since $\bar{\omega}_t = \bar{\bar{\omega}}_t$, the second ratio equals unity for every $\bar{\omega}_t < 1$; taking the limit as $\bar{\omega}_t \to 1$ (and using $B_t = A_t$) gives

$$B_t^{tot} = A_t. \tag{E.29}$$

Combined with equation (B.46) this yields $A_t = Q_t K_t$.

Substituting $D_{t-1} = 0$, $B_{t-1}^{tot} = A_{t-1}$, and $\mu(\bar{\bar{\omega}}_{t-1}) - \mu(\bar{\omega}_{t-1}) = 0$ into equation (B.61):

$$R_t^A A_{t-1} = A_{t-1}R_t \implies R_t^A = R_t. \tag{E.30}$$

Step 2: Real conditions. Table A.1 lists each RBC equilibrium condition alongside the Appendix B condition(s) from which it follows at $\bar{\omega}_t = \bar{\bar{\omega}}_t = 1$.

Step 3: Remaining baseline conditions are vacuous or determine financial variables only.

- *Government block* ((B.52)–(B.55)): shown in Step 1 to give $T_t = \Xi_t = B_t^G = B_t^H = 0$, trivially consistent.
- *Aggregation* ((B.57)–(B.60) and (B.61)): used in Step 1 to derive $B_t^{tot} = A_t = Q_t K_t$ and $R_t^A = R_t$, both already absorbed into conditions (E.14) and (E.19).
- *Lower cutoff* (B.56): at $\bar{\omega}_t = 1$, the left-hand side equals $E_t[\beta \frac{\lambda_{ct+1}}{\lambda_{ct}} R_{t+1}] = 1$ (by (E.14)), so the condition becomes $\lambda_{ct} = \beta E_t\{\lambda_{ct+1}(R_t^B - \xi)\}$—a relation that pins the financial variable R_t^B given the real allocation, without further constraining real quantities.
- *Upper cutoff and loan rate* ((B.63)–(B.64)): at $\bar{\omega}_t = \bar{\bar{\omega}}_t = 1$ these conditions degenerate to equalities between ρ_t and $R_t^B - \xi$, jointly pinning the financial variables R_t^B and ρ_t without restricting real quantities.

Hence every condition determining a real quantity in the RBC model corresponds to a baseline condition, and the baseline conditions that do not appear in the RBC model either become trivially satisfied or serve only to determine financial variables. □

Table A.1: Correspondence between the RBC model's equilibrium conditions (Section E.1.4) and the baseline model's conditions (Appendix B.9) at $\bar{\omega}_t = \bar{\bar{\omega}}_t = 1$.

RBC eq.	Condition	Baseline source
(E.14)	$\lambda_{ct} = \beta E_t\{\lambda_{ct+1} R_{t+1}\}$	Substitute $R_t^A = R_t$ from (E.30) into (B.41).
(E.15)	$\lambda_{ct} = 1/(C_t - \nu_{ct})$	Equations (B.42)–(B.44): both give $\lambda_{ct} = \beta E_t\{\lambda_{ct+1} R_t^B\} = 1/(C_t - \nu_{ct})$.
(E.16)	$W_t = \vartheta(C_t - \nu_{ct}) H_t^{\nu}$	Identical to (B.43).
(E.17)	$Y_t = Z_t K_{t-1}^{\alpha} H_t^{1-\alpha}$	Equation (B.45) with the limit (E.28) at $\bar{\bar{\omega}}_t = 1$.
(E.18)	$W_t = (1-\alpha) Y_t / H_t$	Identical to (B.47).
(E.19)	$R_t = \frac{1}{Q_{t-1}} \alpha Z_t \left(\frac{H_t}{K_{t-1}}\right)^{1-\alpha} + (1-\delta)\frac{Q_t}{Q_{t-1}}$	Equation (B.48) with the limit (E.28) at $\bar{\bar{\omega}}_{t-1} = 1$.
(E.20)	$I_t^n = Z_{It}[1 - \frac{\phi}{2}(\cdot)^2] I_t^g$	Identical to (B.49).
(E.21)	$K_t = I_t^n + (1-\delta) K_{t-1}$	Identical to (B.50).
(E.22)	$\Pi_t = Q_t Z_{It}[1 - \frac{\phi}{2}(\cdot)^2] I_t^g - I_t^g$	Identical to (B.51).
(E.23)	Tobin's q equation	Identical to (B.65).
(E.24)	$Y_t = C_t + I_t^g$	Identical to (B.62).
(E.25)	$\log Z_t = \rho_z \log Z_{t-1} + \varepsilon_t^z$	Identical to (B.66).
(E.26)	$\ln(Z_{It}) = \rho_I \ln(Z_{It-1}) + \varepsilon_{It}$	Identical to (B.67).
(E.27)	$\nu_{ct} = \rho_{\nu} \nu_{ct-1} + \varepsilon_t^{\nu}$	Identical to (B.68).

Note: "Identical" means the equation is structurally unchanged between the two models. Equations (B.52)–(B.55) (government block), (B.56) (lower cutoff), (B.59)–(B.60) (aggregation), (B.61) (equity return), (B.63)–(B.64) (cutoffs and loan rate), (B.68)–(B.69) (preference shock, diversion shock, and diversion scaling) are used only to determine financial variables absent from the RBC model or become vacuously satisfied at $\bar{\omega}_t = \bar{\bar{\omega}}_t = 1$; see Step 3 below.

E.3 Steady-State Conditions

We solve for the non-stochastic steady state sequentially. For every variable that also appears in the baseline model we verify that the steady-state value coincides with the expression in Appendix C evaluated at $\bar{\omega} = \bar{\bar{\omega}} = 1$; the equivalence of equilibrium conditions established in Section E.2 implies that any verification of a dynamic condition in steady state automatically checks the corresponding Appendix B condition.

Sequential derivation. From equation (E.25),

$$Z = 1. \tag{E.31}$$

From equation (E.26),

$$Z_I = 1. \tag{E.32}$$

From equation (E.23) with $Z_I = 1$,

$$Q = 1. \tag{E.33}$$

Matches Appendix C (C.5)–(C.8).

From equation (E.14),

$$R = \frac{1}{\beta}. \tag{E.34}$$

Matches Appendix C (C.4) (the baseline gives $R^A = 1/\beta$; here $R_t^A = R_t$ by (E.30)).

Fix $H = H^{ss} = 1$ and choose $\vartheta = W/(C\,H^{\nu})$ to support this normalisation. From equation (E.19) in steady state ($Z = Q = 1$),

$$R = \alpha\left(\frac{H}{K}\right)^{1-\alpha} + (1-\delta), \tag{E.35}$$

so

$$K = H\left[\frac{R-(1-\delta)}{\alpha}\right]^{1/(\alpha-1)}. \tag{E.36}$$

Matches Appendix C (C.9): the prefactor in (B.48) tends to 1 as $\bar{\bar{\omega}} \to 1$ by (E.28).

From equation (E.17),

$$Y = K^{\alpha}H^{1-\alpha}. \tag{E.37}$$

Matches Appendix C (C.11) at $\bar{\bar{\omega}} = 1$ by (E.28).

From equation (E.18),

$$W = (1-\alpha)\frac{Y}{H}. \tag{E.38}$$

Matches Appendix C (C.12).

From equations (E.20)–(E.21) in steady state,

$$I^n = \delta K, \qquad I^g = I^n. \tag{E.39}$$

Matches Appendix C (C.13)–(C.14).

From equation (E.24),

$$C = Y - I^g. \tag{E.40}$$

Matches Appendix C (C.15).

From equation (E.15),

$$\lambda_c = \frac{1}{C}. \tag{E.41}$$

Matches Appendix C (C.16).

From equation (E.22) with $Q = 1$ and $I^g = I^n$,

$$\Pi = 0. \tag{E.42}$$

Matches Appendix C (C.25).

F Model Without Strategic Default

This appendix develops a variant of the baseline model in which the outside option is unavailable. Without an outside option, no firm has an incentive to divert borrowed funds, so strategic default is impossible. The two-cutoff structure of the baseline model—which is needed to demarcate lenders, strategic defaulters, and producers—collapses to a single cutoff that separates lenders from firms that borrow and produce. The resulting model is structurally simpler than the baseline and provides a useful intermediate case between the full model and the frictionless RBC benchmark of Appendix E: financial intermediation and capital misallocation are still present, but the default margin is shut down.

The appendix proceeds as follows. Section F.1 describes which elements of the baseline model change and which are preserved. Section F.2 restates the firm problems. Section F.3 derives the single cutoff condition and the equilibrium loan rate. Section F.5 lists all equilibrium conditions. Section F.6 derives the non-stochastic steady state.

F.1 Changes Relative to the Baseline Model

The following building blocks of the baseline model are *unchanged*: the household problem (Section 4.1 and Appendix B.1); the capital-producing firm problem (Section 4.3 and Appendix B.2); the aggregate technology shock process (equation (6)); the investment-specific technology process (equation (23)); and the consumption preference shock process (equation (4)).

The following elements are *removed or simplified*:

No outside option. Firms cannot divert borrowed funds into the government bond. Parameters ξ (the haircut on the outside-option return) and θ (the average diversion share) play no role. The firm-specific diversion function $\Theta_t(\omega)$, its scaling factor F_t, and the aggregate diversion D_t are all undefined. Because no funds are ever diverted, the haircut rebate to households is $\Xi_t = 0$ for all t.

Two firm segments instead of three. Since strategic default is impossible, the inter-firm intermediation market sorts firms into only two groups depending on the realization of idiosyncratic productivity ω:

1. Firms with $\omega \leq \overline{\omega}_t$ that lend their equity in the inter-firm market.

2. Firms with $\omega > \overline{\omega}_t$ that borrow and produce.

The defaulter segment $\overline{\omega}_t < \omega < \overline{\overline{\omega}}_t$ of the baseline model is absent. The upper cutoff $\overline{\overline{\omega}}_t$ does not arise.

Simplified government sector. Because no firm uses the government bond as an outside option, aggregate firm holdings of government bonds are $D_t = 0$. Retaining the assumption from the baseline that households do not hold government bonds, $B_t^H = 0$, the government budget constraint (equation (28)) implies $B_t^G = 0$ and hence $T_t = 0$ for all t. The government sector drops out of the model entirely. The risk-free rate R_t^B nonetheless remains well defined as the shadow rate implied by the household's bond Euler equation.

Loan rate. Because all inter-firm loans are repaid with certainty, the inter-firm loan market is free of default risk. We retain the intermediation-cost parameter $\xi > 0$ from the baseline model. In the baseline, ξ is the haircut on the outside option and the proximate source of strategic default; here, with no outside option, ξ captures a plain intermediation spread in the inter-firm lending market. Competition in the inter-firm market drives

$$\rho_t = R_t^B - \xi. \tag{F.1}$$

Keeping ξ at its baseline-calibrated value facilitates direct comparison across models; setting $\xi = 0$ recovers the frictionless RBC benchmark of Appendix E.

The household budget constraint simplifies to

$$C_t + A_t = R_t^A A_{t-1} + W_t H_t + \Pi_t, \tag{F.2}$$

dropping the terms B_t^H, $R_{t-1}^B B_{t-1}^H$, T_t, and Ξ_t that appear in equation (3) of the baseline.

F.2 Firm Problems

F.2.1 Firms Choosing to Produce ($\omega > \overline{\omega}_t$)

The problem of a producing firm is identical to that in Section 4.2.1. Each producing firm finances its capital purchase by combining household equity $a_t(\omega) = a_t$ with an inter-firm loan $b_t(\omega) = b_t$, so that total financing is $b_t^{tot}(\omega) = a_t + b_t = Q_t k_t(\omega)$. The first-order conditions for labor and capital are equations (12) and (13) of the baseline, reproduced here for convenience:

$$W_t = (1-\alpha)\frac{Y_t}{H_t}, \tag{F.3}$$

$$R_{t+1}(\omega) = \frac{1}{Q_t}\,\alpha\, Z_{t+1}\Big(\frac{H_{t+1}}{K_t}\Big)^{1-\alpha}\omega + \frac{(1-\delta)}{Q_t}Q_{t+1}. \tag{F.4}$$

The return paid to households by a producing firm is

$$R^A_{t+1}(\omega) = \frac{R_{t+1}(\omega)(a_t+b_t) - \rho_t\, b_t}{a_t}, \tag{F.5}$$

identical to equation (15) of the baseline.

F.2.2 Firms Choosing to Lend ($\omega \leq \overline{\omega}_t$)

The problem of a lending firm simplifies substantially relative to Section 4.2.3. Because all borrowers repay at rate ρ_t, the lending firm faces no default loss. Its problem is

$$\max_{l_t(\omega)|\omega} E_t\Bigg[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\Big(\rho_t\, l_t(\omega) - R^A_{t+1}(\omega)\, a_t(\omega)\Big)\Bigg], \tag{F.6}$$

subject to $l_t(\omega) \leq a_t(\omega)$. Since revenues are increasing in $l_t(\omega)$, the constraint holds with equality: $l_t(\omega) = a_t(\omega)$. The zero-profit condition that holds under every state of nature is

$$R^A_{t+1}(\omega) = \rho_t \qquad \text{for all } \omega \leq \overline{\omega}_t. \tag{F.7}$$

Compare this to the baseline equation (21): the two terms involving the fraction of defaulting borrowers vanish, leaving a simple pass-through of the loan rate to households.

F.3 The Single Cutoff Point and the Loan Rate

With only lenders and producers, a single cutoff $\overline{\omega}_t$ characterizes the equilibrium sorting of firms. A firm with productivity exactly at $\overline{\omega}_t$ is indifferent between lending—earning ρ_t on its equity—and borrowing to produce—earning $R_{t+1}(\overline{\omega}_t)$ on total funds net of loan repayments.

Formally,

$$E_t\left[\beta \frac{\lambda_{ct+1}}{\lambda_{ct}}\, \rho_t\right] = E_t\left[\beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \left(R_{t+1}(\overline{\omega}_t)\frac{a_t+b_t}{a_t} - \rho_t \frac{b_t}{a_t}\right)\right].$$

Rearranging and collecting terms in ρ_t:

$$E_t\left[\beta \frac{\lambda_{ct+1}}{\lambda_{ct}}\, \rho_t\right] = E_t\left[\beta \frac{\lambda_{ct+1}}{\lambda_{ct}}\, R_{t+1}(\overline{\omega}_t)\right]. \tag{F.8}$$

Substituting $\rho_t = R_t^B - \xi$ from equation (F.1) and expanding $R_{t+1}(\overline{\omega}_t)$ using equation (13), the *single cutoff condition* is

$$\boxed{E_t\left[\beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \left(\frac{1}{Q_t}\, \alpha\, Z_{t+1}\left(\frac{H_{t+1}}{K_t}\right)^{1-\alpha} \overline{\omega}_t + \frac{(1-\delta)}{Q_t} Q_{t+1}\right)\right] = E_t\left[\beta \frac{\lambda_{ct+1}}{\lambda_{ct}}\, (R_t^B - \xi)\right].} \tag{F.9}$$

This condition is *identical* to the baseline lower-cutoff condition (32): ξ enters in exactly the same way in both models, which preserves a clean mapping between the two calibrations.

The following proposition establishes that equation (F.9) together with (F.1) is sufficient to support the two-segment equilibrium.

Proposition 4. *Suppose $R_{t+1}(\omega)$ is strictly increasing in ω and $\rho_t = R_t^B - \xi$ with $\xi \geq 0$. Then equation (F.9) defines a unique cutoff $\overline{\omega}_t$ such that*

1. *firms with $\omega \leq \overline{\omega}_t$ weakly prefer to lend;*
2. *firms with $\omega > \overline{\omega}_t$ strictly prefer to borrow and produce.*

Proof. Equation (F.8) equates the expected discounted return from lending, ρ_t (independent of ω), with the expected discounted return from producing at $\overline{\omega}_t$. Since $R_{t+1}(\omega)$ is strictly increasing in ω, the return from producing is strictly higher than the return from lending for all $\omega > \overline{\omega}_t$ and strictly lower for all $\omega < \overline{\omega}_t$. Because there is no outside option, no firm defaults strategically; the only alternatives are lending and producing, and the sorting follows directly from the monotonicity of $R_{t+1}(\omega)$. □

Remark. The baseline model requires two conditions (equations (32), (33)) and a verification that no firm deviates to the outside option (Proposition 1). Here a single condition (F.9) suffices because the outside option is unavailable and the only relevant comparison is lending versus producing.

F.4 Aggregation and Equilibrium Definition

With two firm segments instead of three, the aggregation of individual decisions into aggregate variables simplifies relative to Appendix B.3. The steps below follow the same procedure; we highlight the equations that change.

Inter-firm loan market clearing. A mass $\mu_{\eta_1,\eta_2}(\overline{\omega}_t)$ of firms lend and a mass $1-\mu_{\eta_1,\eta_2}(\overline{\omega}_t)$ borrow. The aggregate supply and demand of inter-firm loans are

$$L_t = \int_0^{\overline{\omega}_t} l_t(\omega)\,\mu'(\omega)\,\mathrm{d}\omega = \mu_{\eta_1,\eta_2}(\overline{\omega}_t)\,A_t, \tag{F.10}$$

$$B_t = \int_{\overline{\omega}_t}^1 b_t(\omega)\,\mu'(\omega)\,\mathrm{d}\omega, \tag{F.11}$$

and market clearing requires $L_t = B_t$. Individual borrowing is therefore $b_t = B_t/\Big(1 - \mu_{\eta_1,\eta_2}(\overline{\omega}_t)\Big)$.

Aggregate total borrowing. Every producing firm uses total funds $b_t^{tot}(\omega) = a_t + b_t$. Integrating over the producing segment:

$$B_t^{tot} = \Big(1 - \mu_{\eta_1,\eta_2}(\overline{\omega}_t)\Big)A_t + B_t. \tag{F.12}$$

Compare this with the baseline equation (B.19): when $\overline{\overline{\omega}}_t = \overline{\omega}_t$ the two expressions coincide, confirming that the no-default model is the special case in which the defaulter segment has zero mass.

Aggregate equity return. Using equations (F.7) and (F.5) and integrating over both segments:

$$\begin{aligned} R_t^A A_{t-1} &= \int_0^{\overline{\omega}_{t-1}} \rho_{t-1}\, \mu'(\omega)\, \mathrm{d}\omega\, A_{t-1} + \int_{\overline{\omega}_{t-1}}^1 \Big[R_t(\omega)(A_{t-1}+b_{t-1}) - \rho_{t-1} b_{t-1}\Big] \mu'(\omega)\, \mathrm{d}\omega \\ &= \rho_{t-1} L_{t-1} + R_t B_{t-1}^{tot} - \rho_{t-1} B_{t-1} \\ &= R_t\, B_{t-1}^{tot}, \end{aligned} \tag{F.13}$$

where the last equality uses loan market clearing $L_{t-1} = B_{t-1}$. This is the simplification of the baseline equation (B.28) obtained by setting $D_{t-1} = 0$ and $\theta = 0$: without diversion losses, all capital returns flow through to households intact.

Aggregate output and average return on capital. With $\omega_i \geq \overline{\omega}_t$ for every producing firm, output and capital returns aggregate to the same Beta-distribution expressions as in Appendix B, replacing the baseline $\overline{\overline{\omega}}_t$ with $\overline{\omega}_t$ throughout:

$$Y_t = \frac{\eta_1}{\eta_1+\eta_2} \frac{1-\mu_{\eta_1+1,\eta_2}(\overline{\omega}_{t-1})}{1-\mu_{\eta_1,\eta_2}(\overline{\omega}_{t-1})} Z_t K_{t-1}^{\alpha} H_t^{1-\alpha}, \tag{F.14}$$

$$R_t = \frac{\eta_1}{\eta_1+\eta_2} \frac{1-\mu_{\eta_1+1,\eta_2}(\overline{\omega}_t)}{1-\mu_{\eta_1,\eta_2}(\overline{\omega}_t)} \frac{\alpha\, Z_t}{Q_{t-1}} \left(\frac{H_t}{K_{t-1}}\right)^{1-\alpha} + \frac{(1-\delta)}{Q_{t-1}} Q_t. \tag{F.15}$$

A competitive equilibrium of the no-default model is an allocation

$$\Big\{C_t,\, H_t,\, Y_t,\, K_t,\, B_t^{tot},\, I_t^n,\, I_t^g,\, \Pi_t,\, \overline{\omega}_t,\, Z_t,\, Z_{It},\, \nu_{ct},\, L_t,\, A_t,\, B_t\Big\}_{t=0}^{\infty}$$

together with a sequence of prices $\{\lambda_{ct},\, R_t^A,\, W_t,\, R_t^B,\, Q_t,\, R_t,\, \rho_t\}_{t=0}^{\infty}$ satisfying the 22 equations listed in the next subsection, given initial conditions K_0, I_0^g, B_0^{tot}, $\overline{\omega}_0$, A_0, B_0, Z_0, $Z_{I,0}$, ν_{c0}, R_0^B, Q_0, and exogenous processes $\{\varepsilon_t^z,\, \varepsilon_{It},\, \varepsilon_{\nu t}\}$.

F.5 Equilibrium Conditions

This subsection lists the 22 equilibrium conditions of the no-default model, identifying the source of each. Equations that are *identical* to their baseline counterparts in Appendix B.9

are noted as such; the remaining equations are new or simplified.

Asset Euler equation. Identical to baseline equation (B.41):

$$\lambda_{ct} = \beta \, E_t\Big\{\lambda_{ct+1} \, R^A_{t+1}\Big\}\,. \tag{F.16}$$

Consumption Euler equation. Identical to baseline equation (B.42):

$$\beta \, E_t\Big\{\lambda_{ct+1} \, R^B_t\Big\} = \frac{1}{C_t - \nu_{ct}}. \tag{F.17}$$

Labor supply. Identical to baseline equation (B.43):

$$W_t = \vartheta \, (C_t - \nu_{ct}) \, H_t^{\nu}. \tag{F.18}$$

Bond Euler equation. Identical to baseline equation (B.44):

$$\lambda_{ct} = \beta \, E_t\Big\{\lambda_{ct+1} \, R^B_t\Big\}\,. \tag{F.19}$$

Aggregate output. Beta-distribution form of equation (F.14). The single cutoff $\overline{\omega}_{t-1}$ replaces the baseline $\overline{\overline{\omega}}_{t-1}$:

$$Y_t = \frac{\eta_1}{\eta_1 + \eta_2} \frac{1 - \mu_{\eta_1+1,\eta_2}(\overline{\omega}_{t-1})}{1 - \mu_{\eta_1,\eta_2}(\overline{\omega}_{t-1})} Z_t \, K^{\alpha}_{t-1} \, H_t^{1-\alpha}. \tag{F.20}$$

Capital financing. Identical to baseline equation (B.46):

$$B^{tot}_t = Q_t \, K_t. \tag{F.21}$$

Labor demand. Identical to baseline equation (B.47):

$$W_t = (1 - \alpha) \, \frac{Y_t}{H_t}. \tag{F.22}$$

Average return on capital. Beta-distribution form of equation (F.15). The single cutoff $\overline{\omega}_t$

replaces the baseline $\overline{\overline{\omega}}_t$:

$$R_t = \frac{\eta_1}{\eta_1+\eta_2}\frac{1-\mu_{\eta_1+1,\eta_2}(\overline{\omega}_t)}{1-\mu_{\eta_1,\eta_2}(\overline{\omega}_t)}\frac{\alpha\, Z_t}{Q_{t-1}}\left(\frac{H_t}{K_{t-1}}\right)^{1-\alpha} + \frac{(1-\delta)}{Q_{t-1}}\,Q_t. \tag{F.23}$$

Investment supply. Identical to baseline equation (B.49):

$$I_t^n = Z_{It}\left[1-\frac{\phi}{2}\left(\frac{I_t^g}{I_{t-1}^g}-1\right)^2\right] I_t^g. \tag{F.24}$$

Capital accumulation. Identical to baseline equation (B.50):

$$K_t = I_t^n + (1-\delta)\,K_{t-1}. \tag{F.25}$$

Capital-firm profits. Identical to baseline equation (B.51):

$$\Pi_t = Q_t\,Z_{It}\left[1-\frac{\phi}{2}\left(\frac{I_t^g}{I_{t-1}^g}-1\right)^2\right] I_t^g - I_t^g. \tag{F.26}$$

Capital-firm first-order condition (Tobin's q). Identical to baseline equation (B.65):

$$0 = E_t\left\{\begin{array}{c} Q_t Z_{It}\left[-\phi\left(\frac{I_t^g}{I_{t-1}^g}-1\right)\frac{1}{I_{t-1}^g}\right] I_t^g + Q_t Z_{It}\left[1-\frac{\phi}{2}\left(\frac{I_t^g}{I_{t-1}^g}-1\right)^2\right]-1 \\ +\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}Q_{t+1}Z_{It+1}\,\phi\left(\frac{I_{t+1}^g}{I_t^g}-1\right)\left(\frac{I_{t+1}^g}{I_t^g}\right)^2 \end{array}\right\}. \tag{F.27}$$

Single cutoff condition. Equation (F.9) from Section F.3, reproduced here. The expected discounted return from producing at the marginal lending firm equals the inter-firm lending rate $\rho_t = R_t^B - \xi$. This replaces the two baseline conditions (B.56) and (B.63)–(B.64):

$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(\frac{\alpha\,Z_{t+1}}{Q_t}\left(\frac{H_{t+1}}{K_t}\right)^{1-\alpha}\overline{\omega}_t + \frac{(1-\delta)\,Q_{t+1}}{Q_t}\right)\right] = E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_t^B-\xi\right)\right]. \tag{F.28}$$

Aggregate lending. Equation (F.10), with $\overline{\omega}_t$ in place of the baseline $\overline{\omega}_t$ (same label, but here it is the *only* cutoff):

$$L_t = \mu_{\eta_1,\eta_2}(\overline{\omega}_t)\,A_t. \tag{F.29}$$

Loan market clearing. Identical to baseline equation (B.58):

$$B_t = L_t. \tag{F.30}$$

Aggregate total borrowing. Equation (F.12), the two-segment simplification of the baseline equation (B.60):

$$B_t^{tot} = \left(1 - \mu_{\eta_1,\eta_2}(\overline{\omega}_t)\right) A_t + B_t. \tag{F.31}$$

Equity return market clearing. Equation (F.13), the simplification of the baseline equation (B.61) with $D_{t-1} = 0$ and $\theta = 0$. All capital returns flow to households without any diversion losses:

$$R_t^A\, A_{t-1} = R_t\, B_{t-1}^{tot}. \tag{F.32}$$

Goods market clearing. Identical to baseline equation (B.62):

$$Y_t = C_t + I_t^g. \tag{F.33}$$

Loan rate. Equation (F.1):

$$\rho_t = R_t^B - \xi. \tag{F.34}$$

Aggregate TFP process. Identical to baseline equation (B.66):

$$\log Z_t = \rho_z\, \log Z_{t-1} + \varepsilon_t^z. \tag{F.35}$$

Investment-specific technology process. Identical to baseline equation (B.67):

$$\ln(Z_{It}) = \rho_I\, \ln(Z_{It-1}) + \varepsilon_{It}. \tag{F.36}$$

Consumption demand shock process. Identical to baseline equation (B.68):

$$\nu_{ct} = \rho_\nu\, \nu_{ct-1} + \varepsilon_{\nu t}. \tag{F.37}$$

Table A.2 summarises the 22 equations and their relationship to the baseline model.

Table A.2: Equilibrium conditions of the no-default model and their relation to the baseline (Appendix B.9)

#	Condition	No-default eq.	Relation to baseline
1	Asset Euler	(F.16)	Identical to (B.41)
2	Consumption Euler	(F.17)	Identical to (B.42)
3	Labor supply	(F.18)	Identical to (B.43)
4	Bond Euler	(F.19)	Identical to (B.44)
5	Aggregate output	(F.20)	Baseline (B.45) with $\overline{\overline{\omega}}_{t-1} \to \overline{\omega}_{t-1}$
6	Capital financing	(F.21)	Identical to (B.46)
7	Labor demand	(F.22)	Identical to (B.47)
8	Average return on capital	(F.23)	Baseline (B.48) with $\overline{\overline{\omega}}_t \to \overline{\omega}_t$
9	Investment supply	(F.24)	Identical to (B.49)
10	Capital accumulation	(F.25)	Identical to (B.50)
11	Capital-firm profits	(F.26)	Identical to (B.51)
12	Tobin's q	(F.27)	Identical to (B.65)
13	Single cutoff	(F.28)	Replaces (B.56), (B.63), (B.64); identical to (B.56)
14	Aggregate lending	(F.29)	Baseline (B.57), same form
15	Loan market clearing	(F.30)	Identical to (B.58)
16	Aggregate total borrowing	(F.31)	Baseline (B.60) with $\overline{\overline{\omega}}_t \to \overline{\omega}_t$
17	Equity return MC	(F.32)	Baseline (B.61) with $D_{t-1} = 0$, $\theta = 0$
18	Goods market clearing	(F.33)	Identical to (B.62)
19	Loan rate	(F.34)	Simplified from (B.63); same ξ, no default term
20	TFP process	(F.35)	Identical to (B.66)
21	IST process	(F.36)	Identical to (B.67)
22	Consumption shock	(F.37)	Identical to (B.68)

Note: The baseline model has 29 endogenous variables and 29 equations (Appendix B.9). Removing the outside option eliminates seven variables ($\overline{\overline{\omega}}_t$, F_t, D_t, Ξ_t, T_t, B_t^G, B_t^H) and replaces three cutoff/loan-rate conditions by two simpler ones (the single cutoff (F.28) and the no-default loan rate (F.34)), yielding 22 equations in 22 unknowns.

F.6 Steady-State Conditions

We solve for the non-stochastic steady state sequentially. The strategy mirrors Appendix E.3 for the RBC benchmark and Appendix C for the baseline, and it is considerably simpler than the baseline because the government sector is absent and the loan rate is pinned directly by the risk-free rate.

The 22 endogenous variables in steady state are

$$\lambda_c,\ R^A,\ C,\ H,\ W,\ R^B,\ Y,\ \overline{\omega},\ Z,\ Z_I,\ \nu_c,\ K,\ B^{tot},\ Q,\ R,\ I^n,\ I^g,\ \Pi,\ L,\ A,\ B,\ \rho.$$

The solution uses one free initial guess: the single cutoff $\overline{\omega}$.

Step 1: Exogenous variables and prices that do not depend on $\overline{\omega}$.

From equations (F.35), (F.36), (F.37):

$$Z = 1, \qquad Z_I = 1, \qquad \nu_c = 0. \tag{F.38}$$

From equation (F.27) with $Z_I = 1$ and no investment growth:

$$Q = 1. \tag{F.39}$$

From the bond Euler equation (F.19) in steady state:

$$R^B = \frac{1}{\beta}. \tag{F.40}$$

From equation (F.34):

$$\rho = R^B - \xi = \frac{1}{\beta} - \xi. \tag{F.41}$$

Step 2: Real allocation given $\overline{\omega}$.

Because no default occurs, the equity return market clearing condition (F.32) in steady state gives $R^A A = RB^{tot}$. Since the asset Euler equation (F.16) implies $R^A = 1/\beta$ and the financial aggregation (shown in Step 3 below) implies $A = B^{tot}$, we obtain

$$R = R^A = \frac{1}{\beta}. \tag{F.42}$$

This result matches the baseline (Appendix C, equation (C.4)) and the RBC benchmark (Appendix E.3, equation (E.34)).

With $R = 1/\beta$, $Q = 1$, and $Z = 1$, equation (F.23) in steady state is

$$\frac{1}{\beta} = \frac{\eta_1}{\eta_1 + \eta_2} \frac{1 - \mu_{\eta_1+1,\eta_2}(\overline{\omega})}{1 - \mu_{\eta_1,\eta_2}(\overline{\omega})} \alpha \left(\frac{H}{K}\right)^{1-\alpha} + (1 - \delta).$$

Setting $H = H^{ss} = 1$ (choosing $\vartheta = W/(C\, H^{\nu})$ to support this normalization) and solving for K:

$$K = \left[\frac{1/\beta - (1-\delta)}{\alpha} \cdot \frac{\eta_1 + \eta_2}{\eta_1} \cdot \frac{1 - \mu_{\eta_1,\eta_2}(\overline{\omega})}{1 - \mu_{\eta_1+1,\eta_2}(\overline{\omega})}\right]^{\frac{1}{\alpha-1}}. \tag{F.43}$$

Compare with Appendix C equation (C.9): the formula is identical, with $\overline{\overline{\omega}}$ replaced by $\overline{\omega}$.

The remaining real variables follow directly:

$$B^{tot} = Q\, K = K, \tag{F.44}$$

$$Y = \frac{\eta_1}{\eta_1 + \eta_2} \frac{1 - \mu_{\eta_1+1,\eta_2}(\overline{\omega})}{1 - \mu_{\eta_1,\eta_2}(\overline{\omega})} K^{\alpha} H^{1-\alpha}, \tag{F.45}$$

$$W = (1-\alpha)\, \frac{Y}{H}, \tag{F.46}$$

$$I^n = \delta\, K, \tag{F.47}$$

$$I^g = I^n = \delta\, K, \tag{F.48}$$

$$C = Y - I^g, \tag{F.49}$$

$$\lambda_c = \frac{1}{C}, \tag{F.50}$$

$$\Pi = 0. \tag{F.51}$$

Step 3: Financial variables given $\overline{\omega}$.

From the aggregate total borrowing equation (F.31) and the lending and market-clearing equations (F.29)–(F.30):

$$B^{tot} = \Big(1 - \mu_{\eta_1,\eta_2}(\overline{\omega})\Big) A + B,$$
$$B = L = \mu_{\eta_1,\eta_2}(\overline{\omega})\, A.$$

Substituting the second into the first:

$$B^{tot} = \Big(1 - \mu_{\eta_1,\eta_2}(\overline{\omega})\Big) A + \mu_{\eta_1,\eta_2}(\overline{\omega})\, A = A.$$

Therefore,

$$A = B^{tot} = K, \tag{F.52}$$

confirming that household equity equals total capital financing, and hence $R^A = R$ as used in Step 2. Individual lending and borrowing are then

$$B = \mu_{\eta_1,\eta_2}(\overline{\omega})\, A = \mu_{\eta_1,\eta_2}(\overline{\omega})\, K, \tag{F.53}$$
$$L = B. \tag{F.54}$$

Step 4: Verifying the cutoff $\overline{\omega}$.

The steady-state cutoff condition follows from equation (F.28) with $Q = 1$, $Z = 1$, and the stochastic discount factor evaluated in steady state:

$$\alpha\left(\frac{H}{K}\right)^{1-\alpha} \overline{\omega} + (1-\delta) = \rho = R^B - \xi = \frac{1}{\beta} - \xi. \tag{F.55}$$

This condition states that the return from producing at the marginal lender equals the inter-firm lending rate ρ. Rearranging:

$$\overline{\omega} = \frac{1/\beta - \xi - (1-\delta)}{\alpha\,(H/K)^{1-\alpha}}. \tag{F.56}$$

Equation (F.56) is one equation in the single unknown $\overline{\omega}$, given K from equation (F.43) (which itself depends on $\overline{\omega}$). Together they form a fixed-point problem in $\overline{\omega}$ alone, solvable by one-dimensional root finding.

Iterative algorithm.

1. Guess $\overline{\omega} \in (0, 1)$.

2. Compute K from equation (F.43) and Y from equation (F.45).

3. Check whether equation (F.55) holds. Specifically, evaluate the residual

$$\mathcal{R}(\overline{\omega}) \equiv \alpha\left(\frac{H}{K(\overline{\omega})}\right)^{1-\alpha}\overline{\omega} + (1-\delta) - \left(\frac{1}{\beta} - \xi\right), \tag{F.57}$$

   and update $\overline{\omega}$ until $\mathcal{R}(\overline{\omega}) = 0$.

4. Recover all remaining variables from Steps 2 and 3.

Unlike the baseline (Appendix C), which requires guessing *three* values ($\overline{\omega}$, $\overline{\overline{\omega}}$, R) and iterating on a three-dimensional fixed point, the no-default steady state requires guessing only $\overline{\omega}$, since $R = 1/\beta$ is determined analytically.

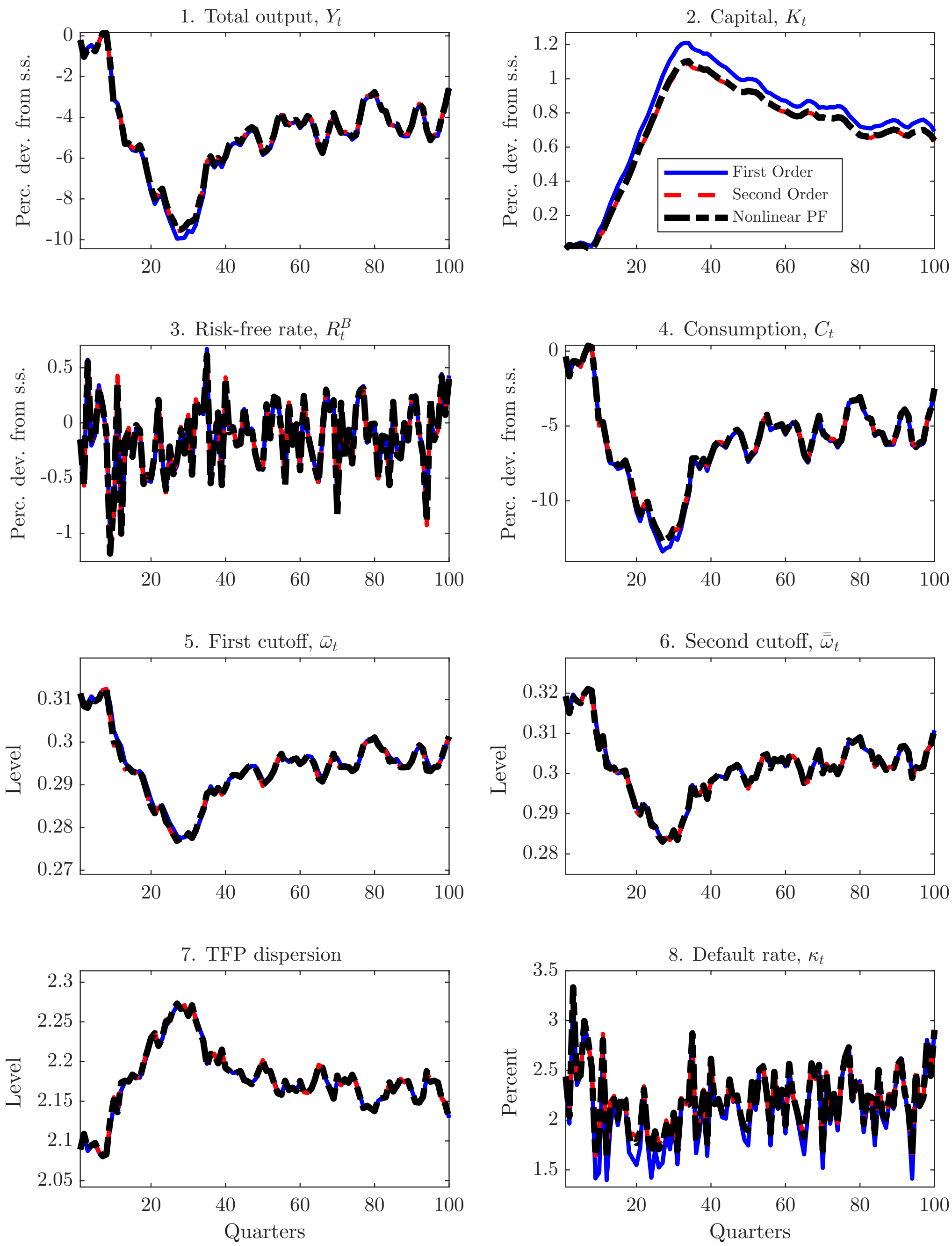
Comparison with the RBC benchmark. The frictionless RBC benchmark of Appendix E is recovered in the joint limit $\xi \to 0$ and $\overline{\omega} \to 1$. As $\xi \to 0$, the fixed-point condition (F.55) forces $\overline{\omega} \to 1$: the two equations become consistent only when all firms produce. In that limit the productivity-weighting ratio $\frac{\eta_1}{\eta_1+\eta_2}\frac{1-\mu_{\eta_1+1,\eta_2}(\overline{\omega})}{1-\mu_{\eta_1,\eta_2}(\overline{\omega})} \to 1$ by the limit established in Appendix E, and equations (F.43)–(F.51) coincide exactly with the RBC steady-state equations (E.36)–(E.42). For $\xi > 0$, the no-default model provides an intermediate case between the baseline (which adds strategic default on top of the same ξ) and the frictionless benchmark.

G Comparing Alternative Solution Methods

In the main body of the paper, all model calculations are based on an exact second-order perturbation solution. We show here a comparison between alternative solution methods, a first-order solution and a solution based on the fully nonlinear perfect-foresight shooting algorithm based on Fair and Taylor (1983) as implemented in Adjemian et al. (2026). We set the precision of the nonlinear solution close to machine precision. We have generalized the implementation of Adjemian et al. (2026) by taking the first guess of the solution paths from the first-order perturbation solution.

In a random sample, such as the ones that we simulate for our SMM estimation, the accumulation of deviations from the linearization point can render the first-order solution inaccurate. To compare the alternative solution methods, we draw a random sample including all the shocks with parameters set as described in Section 6. We find it reassuring that in Figure A.1, there is no daylight between the second-order perturbation solution and the fully-nonlinear solution. We prefer the second-order perturbation solution based on computational speed. By contrast, a noticeable gap opens up between the first-order perturbation solution and the perfect foresight solution. Note that, in the figure, the path from the first-order solution and from the perfect-foresight solution are shown in deviation from the non-stochastic steady state. For the second-order solution, the paths are shown in deviation from the stochastic fixed point.

Figure A.1: Comparing the evolution of a random sample across solutions methods



Note: "Omega bar" refers to $\overline{\omega}_t$ and "omega double bar" refers to $\overline{\overline{\omega}}_t$. For the first-order perturbation solution and the nonlinear perfect-foresight solution, the paths start from the non-stochastic steady state and are also in deviation from that point (where relevant). For the second-order solution the paths start from the stochastic steady state and are also in deviation from that point (where relevant).