Cyclical Fluctuations, Financial Frictions, and Productivity Differences across Firms

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Seminar presentation, 2026

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01

Motivation & evidence

∼30% of cyclical TFP variation is endogenous

- Productivity dispersion within industries is large and rises in weak-growth periods.
- This dispersion matters for aggregate TFP because it affects how efficiently capital and labor are allocated.
- Capturing this link fully requires firm-level distributions, which complicate standard representative-agent frameworks

This paper builds a heterogeneous-firm model that generates endogenous productivity dispersion from financial frictions, summarized by a small set of equilibrium conditions and *tractable* for estimation and business-cycle analysis.

Headline

Credit rationing makes part of *measured* TFP endogenous — about **30%** of the variance of TFP at business-cycle frequencies — and **strategic default** accounts for roughly **one third** of that endogenous component.

Adverse selection and strategic default ration credit

Each firm's production efficiency ω is *private information*. Two informational frictions interact to misallocate capital:

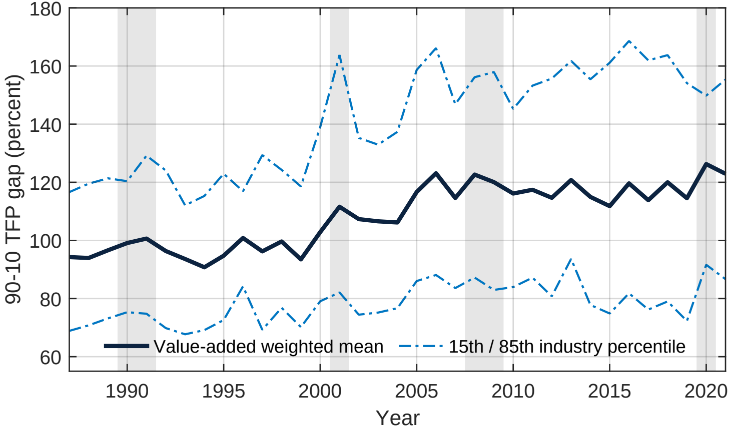
- **Adverse selection.** Outside investors cannot screen ω; a firm offering to pay more to borrow is not credible (any firm could claim high ω). Credit is *rationed* — as in **Stiglitz–Weiss (1981)**.
- **Moral hazard.** Low-ω firms find it profitable to divert borrowed funds to an outside option rather than produce — *strategic default*. This caps the borrowing capacity of even the most efficient firm.

The twist relative to Stiglitz–Weiss. There, default is *involuntary* (project returns fall short). Here it is a *strategic equilibrium choice*. SW is static and partial-equilibrium; ours is general equilibrium with capital, a goods market, and business cycles.

$\Rightarrow$ firms sort into three groups, and the resulting reallocation gives measured TFP an *endogenous* component.

Related literature

Dispersion is large and strongly countercyclical — the moments the model must hit



U.S. manufacturing, 86 four-digit NAICS industries (Census/BLS DiSP), 1987–2021.

- 90–10 gap ≈ 100%: the 90th-percentile plant is ≈ 2.1× as productive as the 10th.
- Pervasive — not driven by outliers.
- Widens when GDP growth is below average and in NBER recessions.

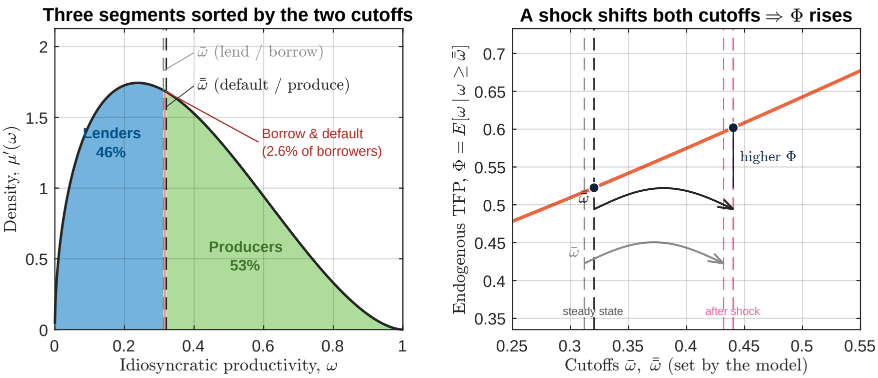
These are the moments we ask the model to reproduce.

02

A tractable model

Two endogenous cutoffs sort firms into three groups

Each firm draws $\omega \sim \text{Beta}(\eta_1, \eta_2)$ on $[0, 1]$ (*private*) and owns two technologies. Two *equilibrium* cutoffs — pinned down by the full model — sort firms into lenders / borrow-and-default / producers. We adopt the Beta form because its parameters let us match the steady-state targets that set the *masses* of each group (dispersion, default rate, bank-credit share, spread).



Full model setup

Cutoff conditions

Measured TFP = endogenous Φ × exogenous Z

Integrating output across producing firms (credit rationing ⇒ equal k, h):

$$Y_t = \Phi_t Z_t K_{t-1}^{\alpha} H_t^{1-\alpha}, \qquad \text{TFP}_t = \Phi_t \cdot Z_t, \qquad \Phi_t = c_\alpha \frac{1-\mu_{\eta_1+1,\eta_2}(\overline{\overline{\omega}}_{t-1})}{1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_{t-1})}.$$

$\Phi_t = E[\omega \mid \omega \geq \overline{\overline{\omega}}_{t-1}]$: endogenous · Z_t: exogenous

- The production cutoff $\overline{\overline{\omega}}$ responds to financial conditions ⇒ Φ_t moves even when Z_t is fixed; a tightening concentrates production in the best firms ⇒ $\Phi_t \uparrow$.
- Frictionless RBC limit: $\overline{\overline{\omega}} \to 1 \Rightarrow \Phi_t \to 1$, and measured TFP moves one-for-one with Z_t.

Closed-form derivation

Few targets, yet the model fits the cyclical moments

Steady-state targets pin $\eta_1, \eta_2, \xi, \theta$ jointly:

- 90–10 within-industry dispersion ($2.08\times$)
- prime—Treasury spread
- bank-credit share ($\approx 47\%$)
- delinquency / default rate (2.6%)

SMM sizes the shocks (Z, Z_I, ν_c) and the adjustment cost ϕ to match second moments – variances, correlations, and autocorrelations of real GDP, consumption, the relative investment price, and the delinquency rate (HP-filtered, $\lambda = 1600$).

Fit (targeted moments)

- Matches the *sign* of all 14 moments.
- Data and model 90% confidence intervals overlap for 10 of 14.
- Solved by second-order perturbation (pruned).

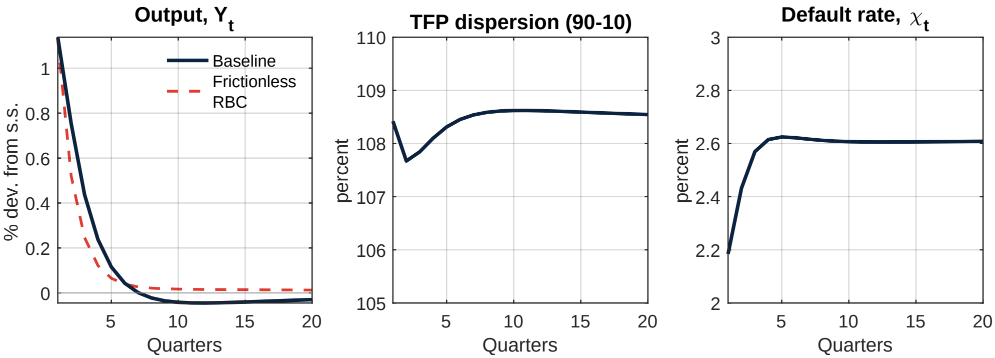
SMM moments

Parameter table

03

Results & assessment

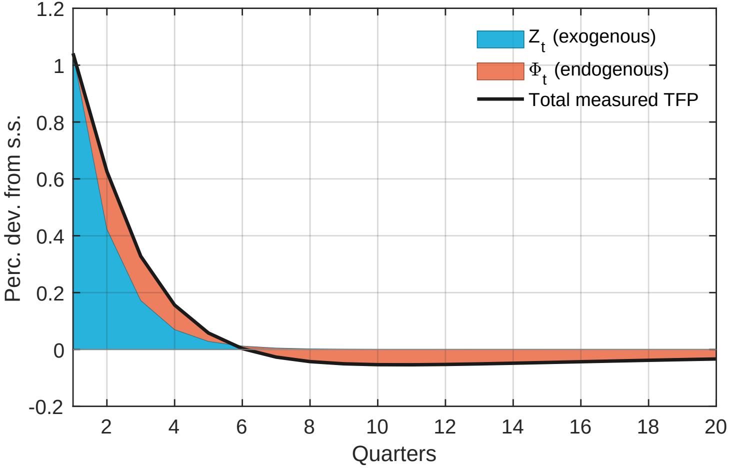
Technology shock amplified by endogenous dispersion



From period 2 the decaying shock lowers the expected return on capital $\Rightarrow \overline{\overline{\omega}} \uparrow$, dispersion $\downarrow$, production concentrates: output and investment rise *more* than RBC; default is countercyclical.

All 8 panels

Endogenous Φ adds amplification and persistence

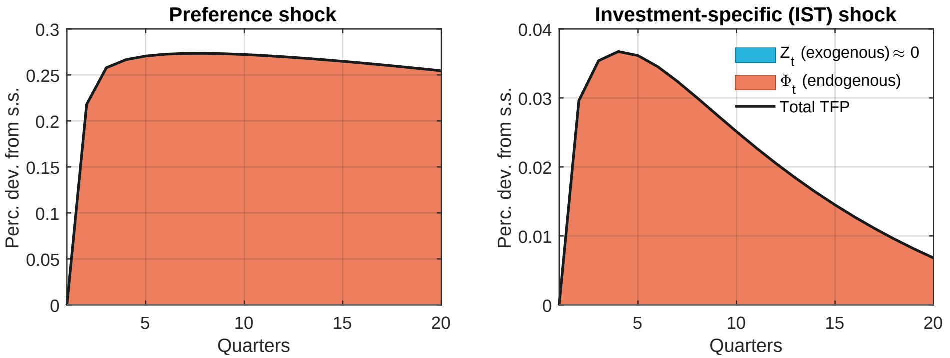


Measured TFP $= \log Z_t + \log \Phi_t$.

- Blue: exogenous Z_t.
- Orange: endogenous Φ_t, switching on from period 2.
- The endogenous component stacks *on top* of the shock — amplification and added persistence.

Next: does this require a Z shock at all?

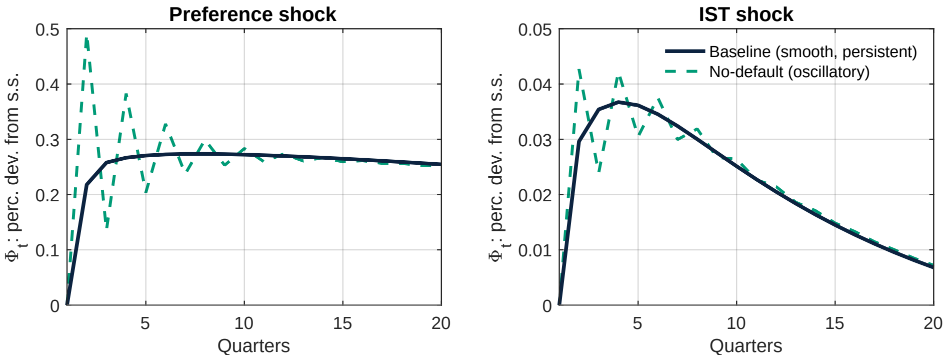
Non-technology shocks move endogenous TFP too



Preference and IST shocks leave Z_t *exactly* unchanged, yet TFP moves: a higher real rate raises borrowing costs ⇒ production concentrates in high-ω firms ⇒ dispersion ↓, Φ_t ↑ — a purely endogenous wealth effect.

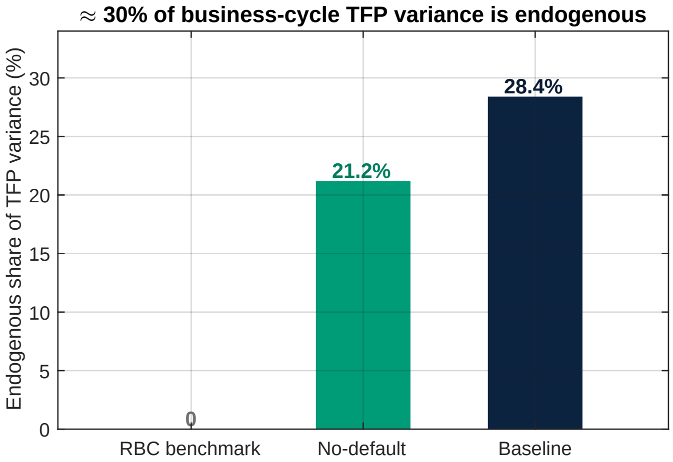
Preference IRFs IST IRFs

Strategic default: amplifier and stabilizer



With one cutoff doing double duty, its forward-looking jump and the lagged selection effect have *opposite* signs $\Rightarrow$ oscillation. The default margin absorbs shocks first, restoring a smooth, persistent Φ_t.

About 28% of TFP variance is endogenous



Gap vs. RBC $= 1 - \dfrac{\text{Var}(\log Z_t)}{\text{Var}(\log \text{TFP}_t)}$, all three shocks active.

- $\Rightarrow$ strategic default accounts for about one third of the endogenous component.

Decomposition table

Untargeted moments: the model matches the signs in the data

	90–10 dispersion		Delinquency rate	
	Data	Model	Data	Model
GDP-per-capita growth	−2.86	−0.70	−0.22	−0.11
Delinquency rate	−4.22	−4.28		
Lagged delinquency			0.84	0.41
Real interest rate			0.14	0.26

- Same OLS run on data and on 1,000 simulated samples; *not* targeted in calibration.
- Dispersion narrows when growth is high; delinquencies rise when growth is depressed and real rates are high. The model reproduces every sign.

Full regression tables

Takeaways

- A *tractable* representative-agent model with endogenous productivity dispersion and **strategic default** as an equilibrium outcome.
- Credit rationing makes $\approx 30\%$ of business-cycle TFP variance endogenous; strategic default delivers about a third of it and stabilizes its dynamics.
- The model reproduces, in sign, untargeted correlations of dispersion, growth, and delinquencies.
- Intended as a reusable *building block* for policy analysis.
  - Beyond this paper: expansionary fiscal policy could have additional desirable effects, as the associated increase in real interest rates would force some unproductive firms to quit.
  - Monetary policy faces additional challenges: the typical New Keynesian rationale to lower policy rates in a downturn could inefficiently keep low-productivity firms afloat.

Thank you

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Endogenous TFP & Financial Frictions

Backup slides follow.

Backup — related literature (1/2)

Credit frictions & amplification (costly state verification).

- Carlstrom–Fuerst (1997); Bernanke–Gertler–Gilchrist (1999); Christiano–Motto–Rostagno (2014): agency costs of intermediation amplify and propagate shocks.
- But productivity dispersion is *static / exogenous*, and default is *not strategic* (inability to repay). Heterogeneity is project-return risk, not a producing-firm selection margin.

Credit rationing.

- Stiglitz–Weiss (1981): the interest rate screens applicants (adverse selection) and shifts risk-taking (incentives). We add a *strategic-diversion* margin inside general equilibrium.

More (2/3) ◂ Back

Backup — related literature (2/3)

Endogenous productivity dispersion.

- Khan–Thomas (2013): capital reallocation across heterogeneous firms drives endogenous TFP — via collateral constraints + irreversibility, with default ruled out. We obtain the same channel more parsimoniously, with default as an equilibrium outcome.
- Buera–Moll (2015); Zheng (2014); Dong (2025): collateral constraints, cutoffs set by parameters, no equilibrium default. Ours: equilibrium sorting and strategic default.
- Moll (2014): persistent shocks let firms self-finance out of misallocation; with i.i.d. shocks TFP is exogenous. We deliver endogenous TFP *even with i.i.d.* productivity.

More (3/3)   ◂ Back

Backup — related literature (3/3)

Misallocation: size and aggregate cost.

- Restuccia–Rogerson (2008); Hsieh–Klenow (2009): policy distortions and marginal-product wedges imply large aggregate TFP losses.
- Midrigan–Xu (2014); Gopinath et al. (2017); David–Venkateswaran (2019): financial frictions and capital misallocation in the U.S. and Southern Europe.

Heterogeneous firms & default.

- Gilchrist–Sim–Zakrajsek (2014); Arellano–Bai–Kehoe (2019); Gomes–Jermann–Schmid (2021): credit spreads and equilibrium default; we emphasize consistency with productivity dispersion.

◂ Back

Backup — full model setup

Households (KPR). Choose C, H, equity A, government bonds; supply the stochastic discount factor.

Goods firms. Two-period overlapping structure. Raise equity a_t (aliquot, independent of ω), draw ω, then sort:

- $\omega \geq \overline{\overline{\omega}}$: borrow & *produce*; $R_{t+1}(\omega) = \frac{\kappa_{t+1}}{Q_t}\omega + \frac{(1-\delta)}{Q_t}Q_{t+1}$, where $\kappa_{t+1} \equiv \alpha Z_{t+1}\left(\frac{H_{t+1}}{K_t}\right)^{1-\alpha}$ is a common factor across all producing firms.
- $\overline{\omega} < \omega < \overline{\overline{\omega}}$: borrow & *default*; divert fraction $\Theta_t(\omega) = \omega^{\psi} F_t$ to the outside option (haircut ξ).
- $\omega \leq \overline{\omega}$: *lend* at inter-firm rate ρ_t.

Capital producers. Build capital with quadratic adjustment costs (ϕ), Tobin's q price Q_t.

Government. Issues bonds bought only by firms; rebates the haircut $\Xi_t = \xi D_{t-1}$ lump-sum; balanced budget.

Frictions. (i) Adverse selection: ω private ⇒ credit rationing (Stiglitz–Weiss). (ii) Moral hazard: low-ω firms divert ⇒ caps borrowing of even the best firm.

Shocks. AR(1) in Z (TFP), Z_I (IST), ν_c (preference).

Back

Backup – households

Infinitely-lived households with KPR preferences over consumption C_t and labor H_t solve the following problem:

$$\max_{\left\{A_{t+\tau}, C_{t+\tau}, H_{t+\tau}, B^H_{t+\tau}\right\}_{\tau=0}^{\infty}} E_t \sum_{\tau=0}^{\infty} \beta^{\tau} \left[\ln(C_{t+\tau} - \nu_{ct+\tau}) - \frac{\vartheta}{1+\nu} (H_{t+\tau})^{1+\nu} \right],$$

subject to

$$C_{t+\tau} + A_{t+\tau} + B^H_{t+\tau} = R^A_{t+\tau} A_{t+\tau-1} + W_{t+\tau} H_{t+\tau} + R^B_{t+\tau-1} B^H_{t+\tau-1} + \Pi_{t+\tau} + T_{t+\tau} + \Xi_{t+\tau},$$

where $\nu_{ct+\tau}$ is an exogenous shock to consumption, which follows an autoregressive process of order one.

Households finance firms through a mutual fund A_t that earns a return R^A_t. B^H_t is a government bond held by households. Π_t is profits from ownership of goods-producing and capital-producing firms. T_t represents a lump-sum transfer from the government. Ξ_t refers to transfers that the household receives because firms that take the outside option are subject to a haircut on the returns from the outside option.

Back

Backup – firms, overview

	Period t	Period $t+1$
1	Raise equity a_t	Produce, **outside option matures**
2	**Productivity level** $\omega \in [0,1]$ **is drawn**	Repay loans to other firms
3	**Lend or borrow in inter-firm market**	Pay households
4	**Some borrowing firms take the outside option and default**	
5	Purchase physical capital	

- The outside option consists of purchasing government bonds.
- Firms that borrow decide whether to exercise the outside option or purchase capital goods to produce.
- If firm (ω) walks away with the outside option, it can retain a fraction $\Theta_t(\omega)$ of the funds borrowed from other firms $b_t^i(\omega)$. We assume $\Theta_t(\omega)$ increases in ω.
- Firms that take the outside option are subject to a haircut cost ξ on their investment.

Back

Backup – firms' production technology

- Firms have this production technology

$$y_{t+1}(\omega) = Z_{t+1}\omega k_t(\omega)^{\alpha} h_{t+1}(\omega)^{1-\alpha}.$$

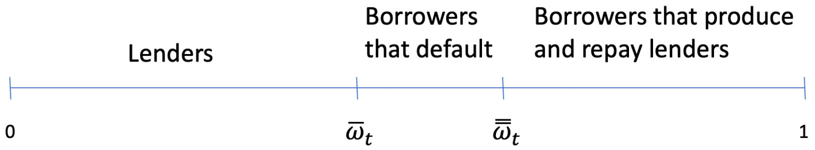
- There are two types of productivity shocks:
  a. An aggregate technology shock Z_{t+1} evolves according to

$$\log Z_{t+1} = \rho_z \log Z_t + \varepsilon^z_{t+1}.$$

  b. A firm-specific productivity shock ω follows the cumulative distribution function $\mu(\omega)$ on the interval $[0, 1]$, satisfying some regularity conditions – $\mu(0) = 0$, $\mu(1) = 1$, and $\mu'(\omega) > 0$.

Back

Backup – sorting firms into groups



- In equilibrium, firms with the lowest level of productivity become lenders – think of them as financial intermediaries.
- A group of firms with intermediate productivity borrows from financial intermediaries and defaults.
- Only firms with sufficiently high productivity produce.
- The cutoff points that determine the mass of these groups of firms will be influenced by economic conditions.
- We show these results in a series of 3 propositions. Our strategy is to posit a solution that satisfies the FOCs of firms and then verify that no firm can or has an incentive to switch to a different segment.

Backup – Firms that produce ($\omega \geq \overline{\overline{\omega}}_t$)

Producing firms maximize expected discounted profits

$$\max_{k_t(\omega),\, h_{t+1}(\omega),\, b_t^{tot}(\omega)} E_t \left\{ \beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \pi_{t+1}(\omega) \right\},$$

where

$$\pi_{t+1}(\omega) = z_{t+1}\omega k_t(\omega)^{\alpha} h_{t+1}(\omega)^{1-\alpha} + (1-\delta) Q_{t+1} k_t(\omega) - R_{t+1}(\omega) b_t^{tot}(\omega) - W_{t+1} h_{t+1}(\omega),$$

and subject to

$$b_t^{tot}(\omega) = a_t(\omega) + b_t(\omega),$$

and to

$$b_t^{tot}(\omega) = Q_t k_t(\omega).$$

The labor−capital ratio $h_{t+1}(\omega)/k_t(\omega) = \tilde{h}_{t+1}/\tilde{k}_t$ is equalized across producing firms because both capital and labor are committed before ω is observed, and all producing firms are ex-ante identical,

Backup – returns for producing firms

- Upon observing ω, the producing firm determines the demand for inter-firm loans by solving the following problem:

$$\max_{b_t(\omega)|\omega} E_t \left[\beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \left(R_{t+1}(\omega) \left(a_t(\omega) + b_t(\omega) \right) - R^A_{t+1}(\omega) a_t(\omega) - \rho_t b_t(\omega) \right) \right]$$

- Producing firms, given constant returns to scale technology, would choose not to borrow in the inter-firm market if the average returns to production were lower than the cost of inter-firm funding.
- So, the inter-firm lending rate is such that $\rho_t \leq E_t R_{t+1}$.
- As a corollary, firms that borrow could increase returns by borrowing even more — in other words, credit is rationed.
- The returns paid to households by firms in this segment are derived from the zero-profit condition:

$$R^A_{t+1}(\omega) = \frac{R_{t+1}(\omega) \left(a_t(\omega) + b_t(\omega) \right) - \rho_t b_t(\omega)}{a_t(\omega)}.$$

◂ Back

Backup – firms that borrow and default ($\overline{\omega}_t < \omega < \overline{\overline{\omega}}_t$)

The problem of the firm that diverts the borrowed funds by choosing the outside option can be described as follows:

$$\max_{b_t(\omega)|\omega} E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\Big(\left(R_t^B-\xi\right)\left(a_t(\omega)+\Theta_t(\omega)b_t(\omega)\right)-R_{t+1}^A(\omega)a_t(\omega)\Big)\right]$$

We show that the returns to households from firms in this group will be:

$$R_{t+1}^A(\omega)=\frac{\left(R_t^B-\xi\right)\left(a_t(\omega)+\Theta_t(\omega)b_t(\omega)\right)}{a_t(\omega)}.$$

Back

Backup – diversion function $\Theta_t(\omega)$

To align our functional choice of the diversion function $\Theta_t(\omega)$ with the structural justification for this function, this function must be non-negative, increasing, and concave in ω. Among the many functional forms consistent with these requirements, we choose the power specification

$$\Theta_t(\omega) = \omega^{\psi} F_t,$$

where ψ is a parameter and F_t is determined endogenously to satisfy

$$\theta = \int_{\underline{\omega}_t}^{\overline{\overline{\omega}}_t} \omega^{\psi} F_t \frac{\mu'(\omega)}{\mu(\overline{\overline{\omega}}_t) - \mu(\overline{\omega}_t)} \mathrm{d}\omega.$$

◂ Back

Backup – lending firms ($\omega < \overline{\omega}_t$)

The problem of firms that lend in the inter-firm market is

$$\max_{l_t(\omega)|\omega} E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(\left(\rho_t\frac{1-\mu(\overline{\overline{\omega}}_t)}{1-\mu(\overline{\omega}_t)}l_t(\omega)+\frac{\mu(\overline{\overline{\omega}}_t)-\mu(\overline{\omega}_t)}{1-\mu(\overline{\omega}_t)}(1-\theta)l_t(\omega)\right)-R^A_{t+1}(\omega)a_t(\omega)\right)\right],$$

subject to the constraint that $l_t(\omega) \leq a_t(\omega)$.

Returns

$$R^A_{t+1}(\omega)=\rho_t\frac{1-\mu(\overline{\overline{\omega}}_t)}{1-\mu(\overline{\omega}_t)}+\frac{\mu(\overline{\overline{\omega}}_t)-\mu(\overline{\omega}_t)}{1-\mu(\overline{\omega}_t)}(1-\theta),$$

where θ is the average recovery rate on defaulted loans.

Back

Backup – pinning down lending rate ρ_t

For the cutoff firm $\overline{\omega}_t$ the return from lending matches the returns from borrowing and defaulting:

$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(\rho_t\frac{1-\mu(\overline{\overline{\omega}}_t)}{1-\mu(\overline{\omega}_t)}+(1-\theta)\frac{\mu(\overline{\overline{\omega}}_t)-\mu(\overline{\omega}_t)}{1-\mu(\overline{\omega}_t)}\right)a_t\right]= E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_t^B-\xi\right)(a_t+\Theta_t(\overline{\omega}_t)b_t)\right],$$

where $1-\theta$ is the average recovery rate on defaulted loans, so

$$\theta=\int_{\overline{\omega}_t}^{\overline{\overline{\omega}}_t}\Theta_t(\omega)\frac{\mu'(\omega)}{\mu(\overline{\overline{\omega}}_t)-\mu(\overline{\omega}_t)}\mathrm{d}\omega.$$

- On the LHS, the return for lending is not firm specific, and conversely, the RHS increases in ω since $\Theta_t(\omega)$ is increasing in ω.
- There is no dependence of b_t on ω because $b_t=b_t(\omega)$ for all ω due to asymmetric information.

Backup – pinning down $\overline{\omega}_t$

- Assume that a screening technology allows lenders to tell whether borrowers can be expected to make more than the outside option.
- We verify that if the firms that have a firm-specific productivity $\omega < \overline{\omega}_t$ will choose to lend, then for a firm with $\omega = \overline{\omega}_t$ the expected return from producing equals the return of the outside option:

$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(\frac{1}{Q_t}\alpha\, Z_{t+1}\left(\frac{H_{t+1}}{K_t}\right)^{1-\alpha}\overline{\omega}_t+\frac{(1-\delta)}{Q_t}Q_{t+1}\right)\right]=$$
$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R^B_t-\xi\right)\right].$$

◂ Back

Backup – squaring the claim that firms with $\omega \leq \overline{\omega}_t$ will lend

- For ω_t to be a cutoff point between firms that lend and firms that borrow and default, we still need to check that firm with $\omega < \overline{\omega}_t$ make higher expected profits from lending than from producing, i.e.,

$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(\rho_t\frac{1-\mu(\overline{\overline{\omega}}_t)}{1-\mu(\overline{\omega}_t)}+(1-\theta)\frac{\mu(\overline{\overline{\omega}}_t)-\mu(\overline{\omega}_t)}{1-\mu(\overline{\omega}_t)}\right)a_t\right]>$$
$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_{t+1}(\omega)a_t\right)\right],$$

  for all firms with $\omega < \overline{\omega}_t$.
- We've done the work on this one, but the proof is not pretty. Let's move on for the sake of this presentation.

◂ Back

Backup – pinning down $\overline{\overline{\omega}}_t$

- The firm with $\omega = \overline{\overline{\omega}}_t$ will be indifferent between diverting funds and producing, thus

$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_t^B - \xi\right)\left(a_t + \Theta(\overline{\overline{\omega}}_t)b_t\right)\right] =$$
$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_{t+1}(\overline{\overline{\omega}}_t)(a_t + b_t) - \rho_t b_t\right)\right].$$

- Notice that both the LHS and the RHS increase with ω. But we can ensure that the increase is slower for the LHS by the choice of the slope of the function $\Theta(\omega)$.
- For a concave $\Theta(\omega)$ a sufficient global condition is

$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_t^B - \xi\right)\Theta_t'(\overline{\omega}_t)b_t\right] < E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\frac{\kappa_{t+1}}{Q_t}(a_t + b_t)\right].$$

- We verify this condition computationally.

◂ Back

Backup – capital-producing firms

The aggregate capital stock evolves according to:

$$K_t = I_t^n + (1-\delta)K_{t-1},$$

where K_t is the amount of capital allocated to the goods-producing firms.

Investment is subject to quadratic adjustment costs

$$I_t^n = Z_{It}\left[1 - \frac{\phi}{2}\left(\frac{I_t^g}{I_{t-1}^g} - 1\right)^2\right] I_t^g,$$

where Z_{It} is an exogenous investment-specific technology shock, which follows an autoregressive process of order one.

The capital producing firms are owned by households, and solve the problem

$$\max_{I_{t+i}^g} E_t \sum_{i=0}^{\infty} \beta^i \frac{\lambda_{ct+i}}{\lambda_{ct}} \left\{ Q_{t+i} Z_{It+i}\left[1 - \frac{\phi}{2}\left(\frac{I_{t+i}^g}{I_{t+i-1}^g} - 1\right)^2\right] I_{t+i}^g - I_{t+i}^g \right\}.$$

Backup – the government and lump-sum transfers to households

- The only role of the government in the model is to provide an alternative source of returns for firms that borrow and default.

$$T_t = B_t^G - R_{t-1}^B B_{t-1}^G.$$

- Only firms can buy government bonds, so $B_t^G = D_t$.
- Since firms that take the outside option are subject to a haircut cost ξ for each unit invested in government bonds in the previous period, the amount of transfers rebated to the household to ensure that there are no deadweight losses in the economy is equal to

$$\Xi_t = \xi D_{t-1}.$$

- Households also receive profits from capital-producing firms, Π_t.

Back

Backup — closed-form TFP decomposition

Integrating individual output ($y_{t+1}(\omega) = Z_{t+1}\,\omega\,\tilde{k}_t^{\alpha}\tilde{h}_{t+1}^{1-\alpha}$, linear in ω) over producers:

$$Y_t = Z_t K_{t-1}^{\alpha} H_t^{1-\alpha}\,\frac{\int_{\overline{\overline{\omega}}_{t-1}}^{1} \omega\,\mu'(\omega)\,d\omega}{1-\mu(\overline{\overline{\omega}}_{t-1})}.$$

With $\omega \sim \text{Beta}(\eta_1,\eta_2)$, use $\omega\,\mu'_{\eta_1,\eta_2}(\omega) \propto$ the $\text{Beta}(\eta_1+1,\eta_2)$ kernel and $B(\eta_1+1,\eta_2)/B(\eta_1,\eta_2) = \eta_1/(\eta_1+\eta_2) \equiv c_\alpha$:

$$\Phi_t = c_\alpha\,\frac{1-\mu_{\eta_1+1,\eta_2}(\overline{\overline{\omega}}_{t-1})}{1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_{t-1})} = E[\omega \mid \omega \geq \overline{\overline{\omega}}_{t-1}].$$

As $\overline{\overline{\omega}} \to 1$, l'Hôpital gives $\Phi_t \to 1$ (RBC). $Z = 1$ at steady state $\Rightarrow \log \text{TFP}_t = \log \Phi_t + \log Z_t$.

◂ Back

Backup — SMM targeted moments

Quarterly, 1987:Q1–2021:Q4; HP-filtered ($\lambda = 1600$); modified optimal weighting matrix.

- Variables: real GDP, real consumption, relative price of investment, business-loan delinquency rate.
- Targeted: 4 variances, correlations, and first-order autocorrelations (14 moments total).
- Result: signs matched on all 14; data/model 90% CIs overlap on 10 (the four variances; correlations with GDP, consumption, default; GDP and consumption autocorrelations).
- Non-overlapping: corr(GDP, inv. price), corr(inv. price, default), and the two investment-price/default autocorrelations.
- Variance decomposition: TFP shocks drive GDP; *consumption* shocks drive TFP dispersion; IST least important.

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Backup – parameters

Conventional		
β	discount factor	0.9925
α	capital share	0.30
δ	depreciation	0.01
ν	inverse Frisch	2
Steady-state targets (90–10, spread, bank share, default)		
η_1, η_2	Beta shape	1.543, 2.735
ξ	outside-option haircut	0.007
θ	avg. divertible fraction	0.0003
SMM		
ϕ	investment adj. cost	0.239
ρ_z, σ_z	TFP shock	0.407, 0.010
ρ_I, σ_I	IST shock	0.990, 0.001
ρ_ν, σ_ν	preference shock	0.986, 0.021

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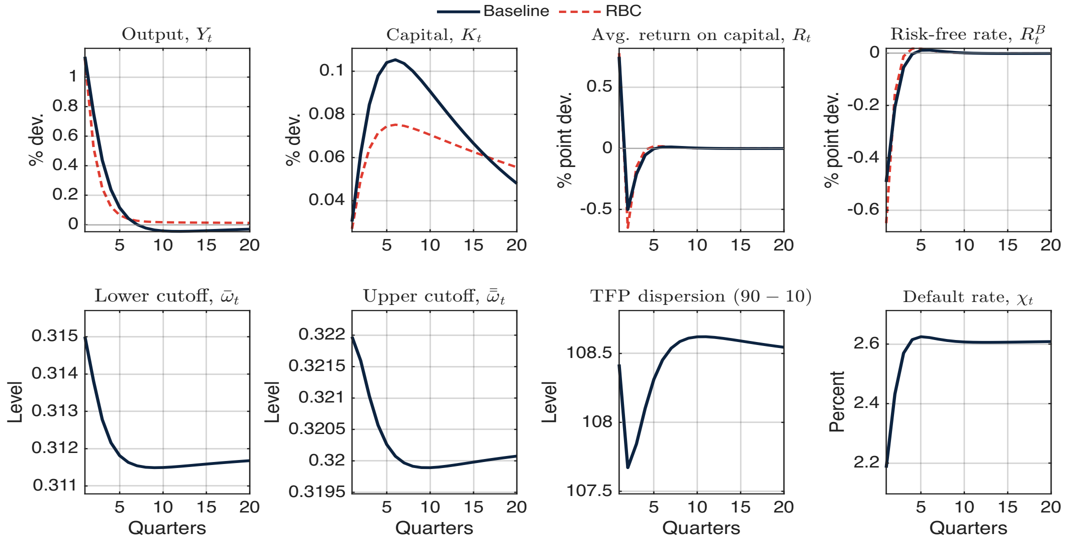
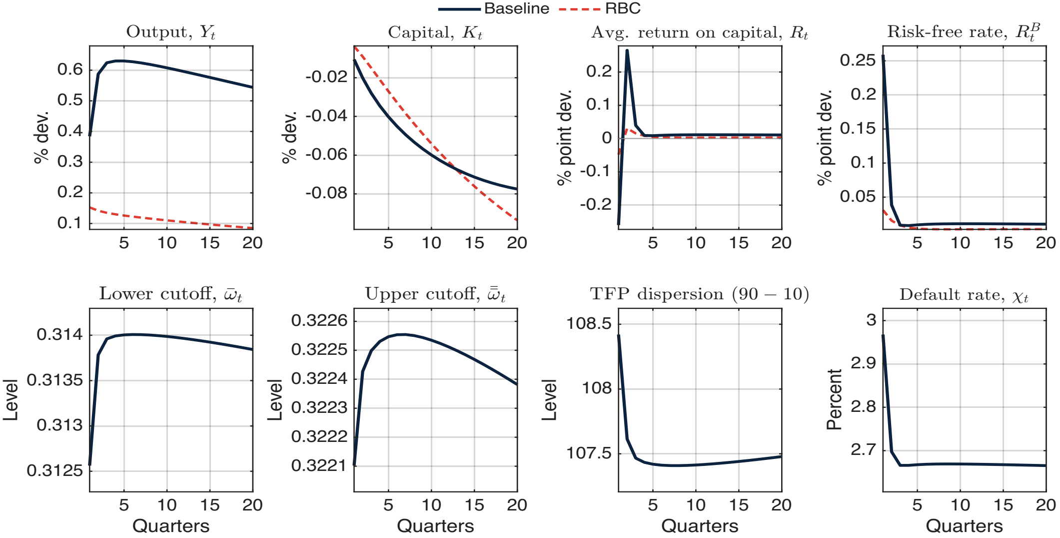
Backup – TFP shock, all variables



Back

Backup – preference shock, all variables



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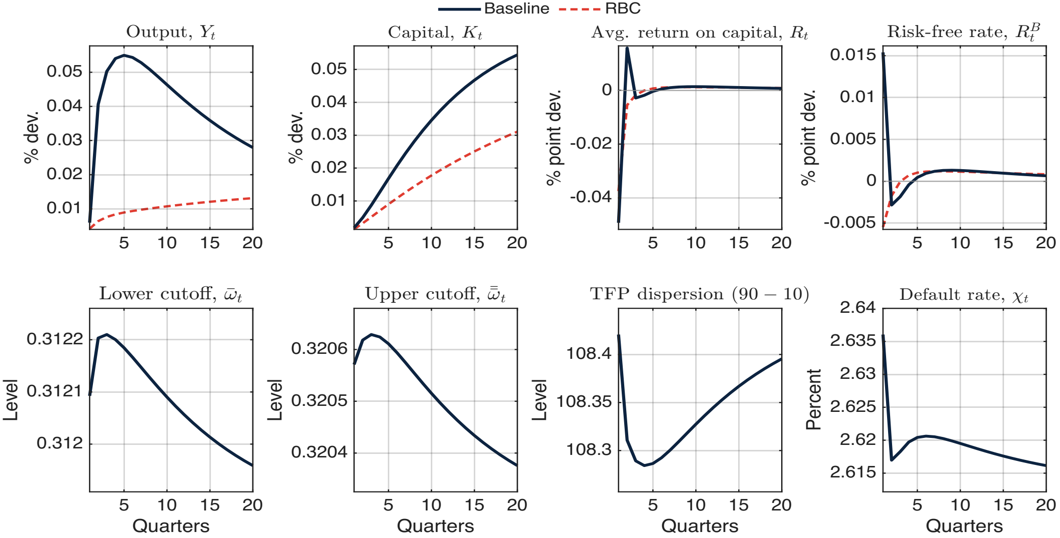
Backup – IST shock, all variables



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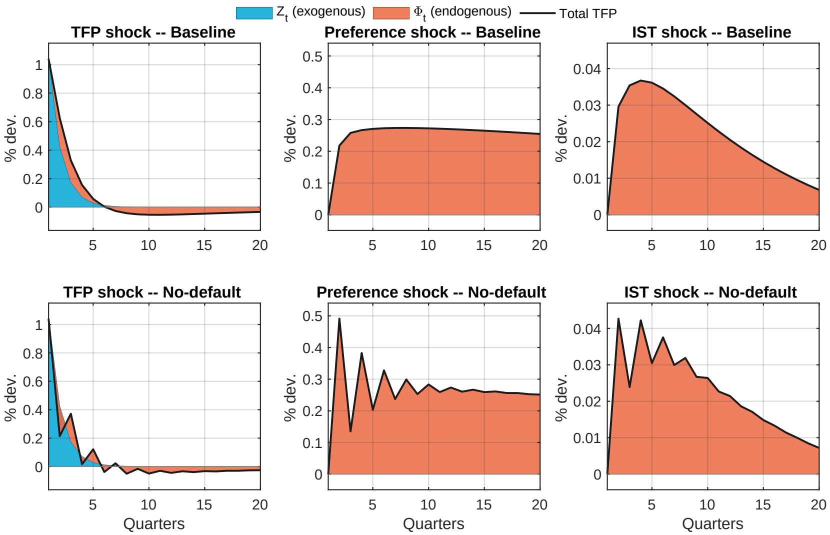
Backup — full TFP decomposition (3 shocks × 2 models)



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Backup — why the no-default model oscillates

One cutoff $\overline{\omega}_t$ must (i) govern production with a one-period lag and (ii) respond forward-looking to shocks.

- t: a positive shock raises the *expected* return to producing $\Rightarrow \overline{\omega}_t$ *falls*, admitting lower-ω producers.
- $t+1$: the lower cutoff lowers $E[\omega \mid \omega \geq \overline{\omega}_t] \Rightarrow$ effective TFP *falls*; R^B *rises*; $\overline{\omega}_{t+1}$ *up*.
- $t+2$: higher cutoff $\Rightarrow$ TFP *up*; R^B *down*; $\overline{\omega}_{t+2}$ *down* again . . . sign-alternating, decaying ≈ 0.69/period.

Baseline fix: the defaulting mass adjusts immediately and diversion-correction terms add positive feedback, so the dominant eigenvalue for $\overline{\overline{\omega}}_t$ stays positive — the oscillatory mode is suppressed.

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Backup — variance decomposition of the Solow residual

HP-filtered ($\lambda = 1600$), 50 draws of 500 quarters, all three shocks.

	$\text{Var}(\log Z_t)/\text{Var}(\log \text{TFP}_t)$	Gap vs. RBC (%)
RBC benchmark	1.000	0.0
Two-cutoff (baseline)	0.716	28.4
One-cutoff (no default)	0.788	21.2

The gap equals zero when Φ_t is constant (RBC); it measures the endogenous share, including its covariance with Z_t.

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Backup – regressions, data vs. model (90% intervals)

	(1) 90–10 disp.		(2) Delinquency	
	Data	Model	Data	Model
GDPpc growth	−2.86 [−4.16, −1.56]	−0.70 [−1.20, −0.34]	−0.22 [−0.31, −0.12]	−0.11 [−0.19, −0.04]
Delinquency rate	−4.22 [−5.56, −2.88]	−4.28 [−10.5, 0.04]		
Lagged delinquency			0.84 [0.74, 0.94]	0.41 [0.07, 0.72]
Real interest rate			0.14 [0.04, 0.24]	0.26 [0.19, 0.34]
N	35	34	35	34

Annual U.S. data 1987–2021; model = average slope over 1,000 simulated samples, 5th/95th percentiles in brackets.

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