Online Appendix

Cyclical Fluctuations, Financial Frictions, and Productivity Differences across Firms

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The main body of the paper can be found here:
https://www.lguerrieri.com/downloads/GKM_Productivity_Differences.pdf

This appendix is organized as follows.

Appendix A provides formal proofs of the three propositions that establish the existence and properties of the two firm-segment cutoff points and the inter-firm loan rate in equilibrium.

Appendix B provides a structural interpretation for our diversion function.

Appendix C derives the first-order conditions for the household and capital-producing-firm optimization problems; provides a formal definition of competitive equilibrium; derives key model aggregates—including the aggregate equity return and the aggregate resource constraint—by integrating firm-level outcomes across the three firm segments using the Beta distribution; and lists all equilibrium conditions in compact notation.

Appendix D solves for the non-stochastic steady state, describing an iterative algorithm that guesses the cutoff points and bond rate and verifies consistency across all equilibrium conditions.

Appendix E explains how to compute model-implied counterparts of the empirical TFP dispersion measures by rescaling the productivity distribution to the producing-firm segment.

Appendix F develops a canonical representative-firm RBC model that removes the informational friction by eliminating idiosyncratic productivity and decentralising the model via the standard household-owns-capital structure.

Appendix G develops a variant of the baseline model in which the outside option is unavailable, so that strategic default is eliminated and firms sort into only two segments (lenders and producers) rather than three; derives the single-cutoff equilibrium conditions, steady state, and connections to the baseline and canonical RBC model.

Appendix H compares alternative solution methods.

A Proofs of Propositions

We prove here the three propositions stated in Section 5.1.

A.1 Proof of Proposition 1

Equation (32) can be described as

$$
\begin{aligned}
&E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(\rho_t\frac{1-\mu(\overline{\overline{\omega}}_t)}{1-\mu(\overline{\omega}_t)}+(1-\theta)\frac{\mu(\overline{\overline{\omega}}_t)-\mu(\overline{\omega}_t)}{1-\mu(\overline{\omega}_t)}\right)a_t\right]= \\
&E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_t^B-\xi\right)(a_t+\Theta_t(\omega)b_t)\right]
\end{aligned}
\tag{A.1}
$$

when $\omega=\overline{\omega}_t$. The left-hand side of equation (A.1) does not depend on ω, while the right-hand side of equation (A.1) increases in ω due to our assumption that $\Theta_t(\omega)$ is increasing in ω. Therefore, firms with $\omega<\overline{\omega}_t$ will have no incentive to deviate to the outside option because they get higher expected profits from lending than from diverting funds.

It is left to show that firms with $\omega<\overline{\omega}_t$ will have no incentive to deviate to production. We need to establish that the expected profits from lending are higher than the expected profits from producing, i.e.,

$$
E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(\rho_t\frac{1-\mu(\overline{\overline{\omega}}_t)}{1-\mu(\overline{\omega}_t)}+(1-\theta)\frac{\mu(\overline{\overline{\omega}}_t)-\mu(\overline{\omega}_t)}{1-\mu(\overline{\omega}_t)}\right)a_t\right]>E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_{t+1}(\omega)a_t\right)\right], \tag{A.2}
$$

for all firms with $\omega<\overline{\omega}_t$. Notice that these firms cannot borrow in the inter-firm market because of the screening technology.

Evaluating equation (13) at $\omega=\overline{\omega}_t$ and combining with the cutoff condition (31), we get

$$
E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}R_{t+1}(\overline{\omega}_t)\right]=E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_t^B-\xi\right)\right]. \tag{A.3}
$$

Plug this result into equation (32):

$$
\begin{aligned}
&E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(\rho_t\frac{1-\mu(\overline{\overline{\omega}}_t)}{1-\mu(\overline{\omega}_t)}+(1-\theta)\frac{\mu(\overline{\overline{\omega}}_t)-\mu(\overline{\omega}_t)}{1-\mu(\overline{\omega}_t)}\right)a_t\right]= \\
&\qquad\qquad E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}R_{t+1}(\overline{\omega})\left(a_t+\Theta_t(\overline{\omega}_t)b_t\right)\right].
\end{aligned}
\tag{A.4}
$$

Notice that

$$
E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}R_{t+1}(\overline{\omega})a_t\right]>E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_{t+1}(\omega)a_t\right)\right] \tag{A.5}
$$

for all firms with $\omega<\overline{\omega}_t$ since $R_{t+1}(\omega)$ is increasing in ω. Combining this result with

$\Theta_t(\overline{\omega}_t)b_t \geq 0$ and $\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}R_{t+1}(\overline{\omega}) \geq 0$, we get

$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(\rho_t\frac{1-\mu(\overline{\overline{\omega}}_t)}{1-\mu(\overline{\omega}_t)}+(1-\theta)\frac{\mu(\overline{\overline{\omega}}_t)-\mu(\overline{\omega}_t)}{1-\mu(\overline{\omega}_t)}\right)a_t\right]=$$
$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}R_{t+1}(\overline{\omega})\left(a_t+\Theta_t(\overline{\omega}_t)b_t\right)\right]\geq$$
$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}R_{t+1}(\overline{\omega})a_t+\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}R_{t+1}(\overline{\omega})\Theta_t(\overline{\omega}_t)b_t\right]>E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_{t+1}(\omega)a_t\right)\right], \quad \text{(A.6)}$$

for all firms with $\omega < \overline{\omega}_t$. Thus, equation (A.2) is verified. □

A.2 Proof of Proposition 2

Let us prove it by contradiction. Assume that $\overline{\overline{\omega}}_t^* < \overline{\omega}_t$. Therefore, by our definition $\overline{\overline{\omega}}_t = \max(\overline{\omega}_t, \overline{\overline{\omega}}_t^*) = \overline{\omega}_t$. Plugging this result into equation (32):

$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\rho_t a_t\right]=E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_t^B-\xi\right)\left(a_t+\Theta_t(\overline{\omega}_t)b_t\right)\right]. \quad \text{(A.7)}$$

Consider the right-hand-side of equation (33) and substitute for $E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\rho_t\right]$ from equation (A.7).

$$\begin{aligned}&E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_{t+1}(\overline{\overline{\omega}}_t^*)(a_t+b_t)-\rho_t b_t\right)\right]=\\&E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_{t+1}(\overline{\overline{\omega}}_t^*)(a_t+b_t)-\left(R_t^B-\xi\right)\left(1+\Theta_t(\overline{\omega}_t)\frac{b_t}{a_t}\right)b_t\right)\right].\end{aligned} \quad \text{(A.8)}$$

Using that $R_{t+1}(\omega) > 0$ and strictly increasing in ω, together with equation (A.3) lead to the following inequality:

$$0<E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}R_{t+1}(\overline{\overline{\omega}}_t^*)\right]<E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}R_{t+1}(\overline{\omega}_t)\right]=E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_t^B-\xi\right)\right] \quad \text{(A.9)}$$

Plugging this inequality into equation (A.8) and collecting terms result in:

$$\begin{aligned}&E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_{t+1}(\overline{\overline{\omega}}_t^*)(a_t+b_t)-\rho_t b_t\right)\right]=\\&E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_{t+1}(\overline{\overline{\omega}}_t^*)(a_t+b_t)-\left(R_t^B-\xi\right)\left(1+\Theta_t(\overline{\omega}_t)\frac{b_t}{a_t}\right)b_t\right)\right]<\\&E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(\left(R_t^B-\xi\right)(a_t+b_t)-\left(R_t^B-\xi\right)\left(1+\Theta_t(\overline{\omega}_t)\frac{b_t}{a_t}\right)b_t\right)\right]=\\&E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(\left(R_t^B-\xi\right)\left(a_t-\Theta_t(\overline{\omega}_t)\frac{b_t^2}{a_t}\right)\right)\right]\leq E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(\left(R_t^B-\xi\right)a_t\right)\right]\leq\\&E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_t^B-\xi\right)\left(a_t+\Theta_t(\overline{\overline{\omega}}_t^*)b_t\right)\right],\end{aligned} \quad \text{(A.10)}$$

where we use that $\Theta_t(\overline{\omega}_t)\frac{b_t^2}{a_t} \geq 0$ and $\Theta_t(\overline{\overline{\omega}}_t^*)b_t \geq 0$. Therefore,

$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_t^B-\xi\right)\left(a_t+\Theta_t(\overline{\overline{\omega}}_t^*)b_t\right)\right] > E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_{t+1}(\overline{\overline{\omega}}_t^*)(a_t+b_t)-\rho_t b_t\right)\right]. \quad (A.11)$$

However, inequality (A.11) contradicts equation (33). Therefore, $\overline{\overline{\omega}}_t^* \geq \overline{\omega}_t$ $\square$

A.3 Proof of Proposition 3

Note that equations (32) and (33) are constructed such that a marginal firm with productivity level $\overline{\omega}_t$ receives the same expected profits from lending and diverting funds, while a marginal firm with productivity level $\overline{\overline{\omega}}_t$ receives the same expected profits from diverting funds and producing. For this proposition and in the main text, we resolve the tie by assuming these marginal firms choose to lend and produce, respectively. Since the probability distribution of ω is continuous, the probability that $\omega = \overline{\omega}_t$ or $\omega = \overline{\overline{\omega}}_t$ is zero.

Given the results of Propositions 1 and 2, we are left to show that

1. No firm with idiosyncratic productivity $\omega > \overline{\overline{\omega}}_t$ has an incentive to deviate from borrowing and producing.

2. No firm with idiosyncratic productivity $\overline{\omega}_t < \omega < \overline{\overline{\omega}}_t$ has an incentive to deviate from borrowing and defaulting.

We will show our proof in four smaller steps.

Step 1: Firms with $\omega > \overline{\omega}_t$ have higher expected profits from borrowing and defaulting than from lending.
Proof of step 1: Equation (32) can be described as

$$\begin{aligned}&E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(\rho_t\frac{1-\mu(\overline{\overline{\omega}}_t)}{1-\mu(\overline{\omega}_t)}+(1-\theta)\frac{\mu(\overline{\overline{\omega}}_t)-\mu(\overline{\omega}_t)}{1-\mu(\overline{\omega}_t)}\right)a_t\right]=\\&E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_t^B-\xi\right)\left(a_t+\Theta_t(\omega)b_t\right)\right].\end{aligned} \quad (A.12)$$

when $\omega = \overline{\omega}_t$. The left-hand side of equation (A.12) does not depend on ω, while the right-hand side of equation (A.12) increases in ω due to our assumption that $\Theta_t(\omega)$ is increasing in ω. Therefore, firms with $\omega > \overline{\omega}_t$ get higher expected profits from diverting funds than from lending. $\square$

Step 2: Firms with $\omega > \overline{\overline{\omega}}_t$ have higher expected profits from producing than from borrowing and defaulting.
Proof of step 2: Equation (32) can be described as

$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_t^B-\xi\right)\left(a_t+\Theta_t(\omega)b_t\right)\right] = E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_{t+1}(\omega)(a_t+b_t)-\rho_t b_t\right)\right] \quad (A.13)$$

when $\omega = \overline{\overline{\omega}}_t$. Taking the first derivative of both sides of equation (A.13) with respect to ω, we need to show that

$$E_t \left[\beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \left(R_t^B - \xi \right) \Theta_t'(\omega) b_t \right] < E_t \left[\beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \left(R_{t+1}'(\omega)(a_t + b_t) \right) \right] \tag{A.14}$$

for all $\omega > \overline{\overline{\omega}}_t$. According to this inequality, the expected profits from diverting funds grow less steeply than the expected profits from producing for all firms with the productivity level $\omega > \overline{\overline{\omega}}_t$. Together with the equalization of expected profits in equation (A.13) evaluated at $\omega = \overline{\overline{\omega}}_t$, this inequality implies that the expected profits from diverting funds are lower than the expected profits from producing for all $\omega > \overline{\overline{\omega}}_t$.

Let us show that condition (35) is sufficient to ensure inequality (A.14) for all $\omega > \overline{\overline{\omega}}_t$. Finding the derivative of $R_{t+1}'(\omega)$ from equation (13) and plugging it into equation (A.14), we get

$$E_t \left[\beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \left(R_t^B - \xi \right) \Theta_t'(\omega) b_t \right] < E_t \left[\beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \left(\frac{1}{Q_t} \alpha \, Z_{t+1} \left(\frac{H_{t+1}}{K_t} \right)^{1-\alpha} \right) (a_t + b_t) \right]. \tag{A.15}$$

Since $\Theta_t(\omega)$ is concave (so $\Theta_t'(\omega)$ is non-increasing in ω) and $\overline{\overline{\omega}}_t \geq \overline{\omega}_t$, then

$$E_t \left[\beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \left(R_t^B - \xi \right) \Theta_t'(\omega) b_t \right] \leq E_t \left[\beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \left(R_t^B - \xi \right) \Theta_t'(\overline{\omega}_t) b_t \right] \tag{A.16}$$

for all $\overline{\overline{\omega}}_t < \omega \leq 1$. Therefore, if

$$E_t \left[\beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \left(R_t^B - \xi \right) \Theta_t'(\overline{\omega}_t) b_t \right] < E_t \left[\beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \left(\frac{1}{Q_t} \alpha \, Z_{t+1} \left(\frac{H_{t+1}}{K_t} \right)^{1-\alpha} \right) (a_t + b_t) \right] \tag{A.17}$$

holds, then it implies

$$E_t \left[\beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \left(R_t^B - \xi \right) \Theta_t'(\omega) b_t \right] < E_t \left[\beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \left(R_{t+1}'(\omega)(a_t + b_t) \right) \right] \tag{A.18}$$

for all $\omega > \overline{\overline{\omega}}_t$. It establishes the sufficiency of the global slope condition in equation (35). □

Step 3: Firms with $\omega > \overline{\overline{\omega}}_t$ have higher expected profits from producing than from lending. *Proof of step 3:* It directly follows from the corollary of Proposition 2 and the two results shown in Steps 1 and 2. Since $\overline{\overline{\omega}}_t \geq \overline{\omega}_t$, the result of Step 1 implies that firms with $\omega > \overline{\overline{\omega}}_t$ have higher expected profits from diverting funds than from lending. Combining this implication with the result of Step 2 that establishes that firms with $\omega > \overline{\overline{\omega}}_t$ have higher expected profits from producing than from diverting funds, ensures that firms with $\omega > \overline{\overline{\omega}}_t$ have higher expected profits from producing than from lending.

To complete item 1 of the list above, combine Steps 2 and 3. □

Step 4: Firms with $\omega < \overline{\omega}_t$ have higher expected profits from borrowing and defaulting than from producing.

Proof of step 4: Since $\Theta_t(\omega)$ is concave and, for this part of the argument, we consider $\overline{\overline{\omega}}_t > \overline{\omega}_t$, we have

$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_t^B-\xi\right)\Theta_t'(\omega)b_t\right] \leq E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_t^B-\xi\right)\Theta_t'(\overline{\omega}_t)b_t,\right] \tag{A.19}$$

for each $\omega < \overline{\overline{\omega}}_t$. When $\overline{\overline{\omega}}_t = \overline{\omega}_t$, the case is degenerate: there is zero mass of diverting firms. In this situation, it suffices to verify that no firm has an incentive to deviate from producing or from lending. This condition has already been established in the preceding steps of the proof of this Proposition and in Proposition 1. Combining equation (A.19) with the global slope condition in equation (35), we establish that

$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_t^B-\xi\right)\Theta_t'(\omega)b_t\right] < E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_{t+1}'(\omega)(a_t+b_t)\right)\right] \tag{A.20}$$

for each $\omega < \overline{\overline{\omega}}_t$.

By definition of $\overline{\overline{\omega}}_t$, the expected payoff from diverting funds equals the expected payoff from producing at $\omega = \overline{\overline{\omega}}_t$. Equation (A.20) establishes that the slope of the production payoff strictly exceeds the slope of the diversion payoff for each ω. Therefore, for each $\omega < \overline{\overline{\omega}}_t$ the expected diversion payoff decreases more slowly than the expected production payoff, which implies that the expected payoff from diverting funds strictly exceeds the expected payoff from producing for all $\omega < \overline{\overline{\omega}}_t$.

To complete item 2 of the list above, we also need to combine the results of Steps 1 and 4 with the corollary of Proposition 2. Since $\overline{\overline{\omega}}_t \geq \overline{\omega}_t$, Steps 1 and 4 imply that the expected profits from borrowing and defaulting are higher than those from lending and from producing for all $\overline{\omega}_t < \omega < \overline{\overline{\omega}}_t$. □

B Structural Interpretation for the Diversion Technology

Our baseline specification adopts the parametric form $\Theta_t(\omega) = F_t\omega^{\psi}$ for the diversion technology. The objective of this section is to provide a decision-theoretic derivation that delivers exactly this functional form and pins down the qualitative dependence of F_t on the production cutoff $\overline{\overline{\omega}}_t$. We show that:

1. More productive firms can retain a larger fraction of the borrowed funds when they choose to default, justifying our assumption that $\Theta_t(\omega)$ is increasing in ω;

2. The diversion function $\Theta_t(\omega)$ is concave in ω;

3. The effectiveness of the diversion rate falls when more firms attempt to divert resources, justifying our normalization for F_t that varies endogenously and negatively with $\overline{\overline{\omega}}_t$.

The economy is populated by a continuum of firms and competitive lenders. Firms differ in productivity. After obtaining financing, firms choose between operating a productive project and diverting all available funds. We associate diversion with shifting resources into private assets or unobservable projects whose value depends on the firm's managerial ability (or as we call it organizational capital) that is linked to ω. Diversion leads to default and requires concealment services supplied by a separate sector. The supply of concealment services is fixed in the short run because labor is allocated to that sector before firm productivity is realized. Consequently, an increase in aggregate diversion activity raises the equilibrium price of concealment services and reduces the profitability of diversion.

The key mechanism relies on organizational capital. Firm productivity reflects both aggregate technology and firm-specific organizational capital. Organizational capital can be redeployed in diversion activities, and it can only be seized partially by creditors. As a result, more productive firms obtain larger private benefits from diversion. However, because diversion exhibits diminishing returns with respect to organizational capital, sufficiently productive firms prefer productive operation.

B.1 Environment

Consider a continuum of borrowing firms of mass 1. Firm i is indexed by productivity ω_i drawn from the cumulative distribution function $G(\omega)$ with the support $\left[\underline{\underline{\omega}}, \underline{\omega}\right]$.

Each firm has net worth a and borrows b. Total available resources equal

$$k = a + b. \tag{B.1}$$

As in the baseline model, lenders finance everyone identically and cannot condition contracts on productivity because firm-level production efficiency is not directly observed by investors and the claims of firms are not credible, so a and b are not firm-specific.

As in the baseline model, labor is committed before ω is realized, so a quantity h of labor is allocated equally to each borrowing firm.

The timing is:

1. Lenders extend credit.
2. Labor is allocated between production and concealment sectors.
3. Firm-specific productivity ω_i is realized.
4. Firms choose between production and diversion.
5. Output, repayment, and default occur.

B.2 Production

The production function of the firm i is given by

$$y_i = \omega_i z k^\alpha h^{1-\alpha}, \tag{B.2}$$

where z is the aggregate level of technology common to all firms and $0 < \alpha < 1$. The firm repays ρb, where ρ is the lending rate. The resulting payoff is

$$V_P(\omega_i) = \omega_i z k^\alpha h^{1-\alpha} - \rho b. \tag{B.3}$$

Define

$$A_P \equiv zk^{\alpha}h^{1-\alpha}. \tag{B.4}$$

Then

$$V_P(\omega_i) = A_P\omega_i - \rho b. \tag{B.5}$$

The value of productive operation is therefore linear in productivity as in the baseline model.

B.3 Concealment Services

Diversion requires concealment services. These services represent legal, financial, and organizational arrangements that facilitate hiding resources from creditors. We will measure concealment services as a fraction of the borrowed funds b.

A competitive concealment sector transforms aggregate labor employed in the diversion sector H_θ into concealment services Θ:

$$\Theta = A_\Theta H_\Theta^{\zeta}, \qquad 0 < \zeta < 1, \tag{B.6}$$

where A_Θ is the efficiency of the concealment technology.

Because labor is allocated before productivity is realized (so H_Θ is constant), we assume that lenders can control A_Θ to ensure that ex-post supply of concealment services is fixed (and corresponds to the constant θ in the baseline model):

$$\Theta_S = A_\Theta H_\Theta^{\zeta} = \theta. \tag{B.7}$$

Let q_Θ denote the market price of concealment services.

B.4 Diversion and Default

A diverting firm transfers all available resources away from production. Diversion automatically results in default. A diverting firm purchases concealment services $\Theta_i \equiv \Theta(\omega_i)$.

The private payoff from diversion is

$$V_D(\omega_i) = \omega_i^{\eta}(\Theta_i b)^{\gamma}a^{1-\gamma}, \tag{B.8}$$

where $0 < \gamma < 1$ and $0 < \eta < 1-\gamma$.[1] This payoff includes the private benefit from investing into unobserved projects minus the cost of concealment services.

The firm solves

$$\max_{\Theta_i} \left\{\omega_i^{\eta}(\Theta_i b)^{\gamma}a^{1-\gamma} - q_\Theta\Theta_i b\right\}. \tag{B.9}$$

The first-order condition is

$$\gamma\omega_i^{\eta}\Theta_i^{\gamma-1}\left(\frac{a}{b}\right)^{1-\gamma} = q_\Theta. \tag{B.10}$$

[1] Labor does not enter explicitly into this diversion payoff because concealment services are produced using labor, and the firm i purchases these services as a composite input Θ_i. Consequently, labor affects diversion indirectly through the equilibrium supply and price of concealment services.

Therefore, the optimal diversion

$$\Theta_i^* = \frac{a}{b}\left(\frac{\gamma\omega_i^\eta}{q_\Theta}\right)^{\frac{1}{1-\gamma}}. \tag{B.11}$$

From this equation, we can find that

$$\Theta'(\omega_i) \equiv \frac{\partial\Theta_i^*}{\partial\omega_i} > 0, \tag{B.12}$$

Substituting the optimal choice into the objective function yields

$$V_D(\omega_i) = \omega_i^{\frac{\eta}{1-\gamma}} q_\Theta^{-\frac{\gamma}{1-\gamma}} A_D, \tag{B.13}$$

where

$$A_D = (1-\gamma)\gamma^{\frac{\gamma}{1-\gamma}} a. \tag{B.14}$$

The diversion payoff is increasing in productivity and decreasing in the equilibrium price of concealment services.

B.5 Occupational Choice

Borrowing firms choose the activity that yields the larger payoff:

$$V(\omega_i) = \max\left\{V_P(\omega_i), V_D(\omega_i)\right\}. \tag{B.15}$$

Because $0 < \eta < 1-\gamma$, we have that $\frac{\eta}{1-\gamma} < 1$. Therefore, the diversion payoff V_D is concave in the firm-specific productivity.

Since V_P is linear in ω_i and V_D is concave in ω_i, there exists a unique threshold ω^* satisfying

$$A_P\omega^* - \rho b = (\omega^*)^{\frac{\eta}{1-\gamma}} q_\Theta^{-\frac{\gamma}{1-\gamma}} A_D. \tag{B.16}$$

It guarantees that firms with $\omega_i < \omega^*$ choose diversion, whereas firms with $\omega_i > \omega^*$ choose production.[2] Note the assumption of $0 < \gamma < 1$ is required to ensure the global optimality of Θ_i^*, while $0 < \eta < 1-\gamma$ ensures the concavity of $V_D(\omega_i)$ which is needed to establish the firm sorting described.

B.6 Concealment-Market Equilibrium

Only diverting firms demand concealment services. Aggregate demand is

$$\Theta_D(q_\Theta) = \int_{\underline{\omega}}^{\omega^*} \Theta^*(\omega, q_\Theta)\, dG(\omega). \tag{B.17}$$

[2] The parameter values and the support of the productivity distribution are chosen such that $\underline{\omega} < \omega^* < \underline{\underline{\omega}}$.

Plugging equation (B.11),

$$\Theta_D(q_\Theta) = \left[\gamma^{\frac{1}{1-\gamma}} \frac{a}{b}\right] q_\Theta^{-\frac{1}{1-\gamma}} \int_{\underline{\omega}}^{\omega^*} \omega^{\frac{\eta}{1-\gamma}} \, dG(\omega). \tag{B.18}$$

Market clearing requires

$$\Theta_D(q_\Theta) = \Theta_S = \theta. \tag{B.19}$$

This condition determines the equilibrium price.

B.7 Lenders

Lenders do not observe firm productivity when extending credit. If a firm produces, lenders receive ρb. If a firm diverts, default occurs and lenders recover $1-\theta$ fraction of b.

The lender revenues equal

$$\pi_L = \rho b(1 - G(\omega^*)) + G(\omega^*)b(1-\theta). \tag{B.20}$$

B.8 Main Results

The main results of the model are summarized in 3 propositions.

Proposition 1. *More productive diverting firms retain a larger fraction of the diverted funds. Moreover, the diversion function is concave in firm-specific productivity, i.e.*

$$\Theta'(\omega_i) \equiv \frac{\partial \Theta_i^*}{\partial \omega_i} > 0, \qquad \Theta''(\omega_i) \equiv \frac{\partial^2 \Theta_i^*}{\partial \omega_i^2} < 0. \tag{B.21}$$

Proof. It follows directly from equation (B.11) given that $\frac{\eta}{1-\gamma} < 1$. □

Proposition 2. *There exists a unique threshold ω^* such that firms with productivity ω below the threshold divert funds and firms with productivity above the threshold produce.*

Proof. See Section B.5 □

Proposition 3. *The effectiveness of diversion decreases endogenously when more firms attempt to divert (when ω^* increases).*

Proof. An increase in the mass of diverting firms increases aggregate demand for concealment services. Since supply is fixed ex post, market clearing requires a higher equilibrium price q_Θ.

To formally show that q_Θ increases in equilibrium, notice that a higher ω^* increases the integral $\int_{\underline{\omega}}^{\omega^*} \omega^{\frac{\eta}{1-\gamma}} \, dG(\omega)$ in equation (B.18). Combining it with the market clearing condition in equation (B.19), it implies that q_Θ must increase since $-\frac{1}{1-\gamma} < 0$ for $0 < \gamma < 1$.

From equation (B.11), a rise in q_Θ decreases the optimal Θ_i^*. □

C Equilibrium Definition

This appendix is organized as follows. Sections C.1 and C.2 complete the derivations of the first-order conditions from the household and capital-producing-firm optimization problems, providing the steps omitted from the main text for brevity. Section C.3 presents the aggregation of firm-level decisions into aggregate variables and states a formal definition of competitive equilibrium. Sections C.4–C.9 derive the Beta-distribution closed forms for the aggregate equity return, aggregate output, average return on capital, aggregate equity-return market clearing, and the diversion scaling factor F_t. Finally, Section C.10 lists all equilibrium conditions in compact, Beta-distribution-specific notation.

C.1 Household Problem

The representative household maximizes

$$\max_{\left\{A_{t+\tau},C_{t+\tau},H_{t+\tau},B^H_{t+\tau}\right\}_{\tau=0}^{\infty}} E_t \sum_{\tau=0}^{\infty} \beta^\tau \left[\ln(C_{t+\tau}-\nu_{ct+\tau}) - \frac{\vartheta}{1+\nu}(H_{t+\tau})^{1+\nu}\right],$$

subject to

$$C_{t+\tau}+A_{t+\tau}+B^H_{t+\tau} = R^A_{t+\tau}A_{t+\tau-1}+W_{t+\tau}H_{t+\tau}+R^B_{t+\tau-1}B^H_{t+\tau-1}+\Pi_{t+\tau}+T_{t+\tau}+\Xi_{t+\tau},$$

reproduced from equations (1) and (2). The first-order conditions for assets,

$$-\lambda_{ct}+\beta E_t\left\{\lambda_{ct+1}R^A_{t+1}\right\}=0, \tag{C.1}$$

consumption,

$$\frac{1}{C_t-\nu_{ct}}=\lambda_{ct}, \tag{C.2}$$

labor,

$$-\vartheta H_t^\nu+\lambda_{ct}W_t=0, \tag{C.3}$$

and government bonds,

$$-\lambda_{ct}+\beta E_t\left\{\lambda_{ct+1}R^B_t\right\}=0, \tag{C.4}$$

follow from the stationarity conditions of the Lagrangian, where λ_{ct} is the multiplier on the budget constraint (2).

C.2 Capital-Producing Firms

Capital-producing firms solve

$$\max_{I^g_{t+i}} E_t \sum_{i=0}^{\infty}\beta^i\frac{\lambda_{ct+i}}{\lambda_{ct}}\left\{Q_{t+i}Z_{It+i}\left[1-\frac{\phi}{2}\left(\frac{I^g_{t+i}}{I^g_{t+i-1}}-1\right)^2\right]I^g_{t+i}-I^g_{t+i}\right\},$$

reproduced from equation (24). The first-order condition implies

$$0=E_t\left\{\begin{array}{c} Q_tZ_{It}\left[-\phi\left(\frac{I^g_t}{I^g_{t-1}}-1\right)\frac{1}{I^g_{t-1}}\right]I^g_t+Q_tZ_{It}\left[1-\frac{\phi}{2}\left(\frac{I^g_t}{I^g_{t-1}}-1\right)^2\right]-1\\ +\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}Q_{t+1}Z_{It+1}\phi\left(\frac{I^g_{t+1}}{I^g_t}-1\right)\left(\frac{I^g_{t+1}}{I^g_t}\right)^2\end{array}\right\} \tag{C.5}$$

C.3 Aggregation and Equilibrium

We proceed as follows: First, we link individual and aggregate variables, then we define a competitive equilibrium.

Since a mass $\mu(\overline{\omega}_t)$ of firms lend and the complement mass $1-\mu(\overline{\omega}_t)$ of firms borrow, the inter-firm market clears when

$$\int_0^{\overline{\omega}_t} l_t(\omega)\mu'(\omega)\mathrm{d}\omega = \int_{\overline{\omega}_t}^1 b_t(\omega)\mu'(\omega)\mathrm{d}\omega, \tag{C.6}$$

which by defining

$$L_t = \int_0^{\overline{\omega}_t} l_t(\omega)\mu'(\omega)\mathrm{d}\omega, \tag{C.7}$$

$$B_t = \int_{\overline{\omega}_t}^1 b_t(\omega)\mu'(\omega)\mathrm{d}\omega \tag{C.8}$$

translates into:

$$L_t = B_t. \tag{C.9}$$

Since each type of firm borrows the same amount, we have $a_t(\omega) = a_t$ for all ω and $b_t(\omega) = b_t$ for borrowing firms ($\omega > \overline{\omega}_t$). Using the definition $A_t = \int_0^1 a_t(\omega)\mu'(\omega)\mathrm{d}\omega$ and equation (C.8), we can relate individual to aggregate variables as follows:

$$a_t = A_t, \tag{C.10}$$

$$b_t = \frac{B_t}{1-\mu(\overline{\omega}_t)}. \tag{C.11}$$

Next, consider the aggregation of budget constraints of firms in each segment. For lending firms, $l_t(\omega) = a_t(\omega)$ for each ω. Aggregating over the relevant mass of firms, i.e.,

$$\int_0^{\overline{\omega}_t} l_t(\omega)\mu'(\omega)\mathrm{d}\omega = \int_0^{\overline{\omega}_t} a_t(\omega)\mu'(\omega)\mathrm{d}\omega, \tag{C.12}$$

results into

$$L_t = \mu(\overline{\omega}_t)A_t. \tag{C.13}$$

Let $d_t(\omega) = a_t(\omega) + \Theta_t(\omega)b_t(\omega)$ define the amount of resources diverted and invested in the outside option by the firm of productivity ω. Aggregating over the relevant mass of firms, i.e.,

$$\int_{\overline{\omega}_t}^{\overline{\overline{\omega}}_t} \left(a_t(\omega) + \Theta_t(\omega)b_t(\omega)\right)\mu'(\omega)\mathrm{d}\omega = \int_{\overline{\omega}_t}^{\overline{\overline{\omega}}_t} d_t(\omega)\mu'(\omega)\mathrm{d}\omega, \tag{C.14}$$

and using the aggregation identity for firms' bond holdings,

$$D_t = \int_{\overline{\omega}_t}^{\overline{\overline{\omega}}_t} d_t(\omega)\mu'(\omega)\mathrm{d}\omega, \tag{C.15}$$

result into

$$D_t = \left(\mu(\overline{\overline{\omega}}_t) - \mu(\overline{\omega}_t)\right) A_t + \frac{\mu(\overline{\overline{\omega}}_t) - \mu(\overline{\omega}_t)}{1 - \mu(\overline{\omega}_t)} \theta B_t. \tag{C.16}$$

For producing firms, $a_t(\omega) + b_t(\omega) = b_t^{tot}(\omega)$. Aggregating over the relevant mass of firms, i.e.,

$$\int_{\overline{\overline{\omega}}_t}^{1} \left(a_t(\omega) + b_t(\omega)\right) \mu'(\omega)\mathrm{d}\omega = \int_{\overline{\overline{\omega}}_t}^{1} b_t^{tot}(\omega)\mu'(\omega)\mathrm{d}\omega, \tag{C.17}$$

and defining

$$B_t^{tot} = \int_{\overline{\overline{\omega}}_t}^{1} b_t^{tot}(\omega)\mu'(\omega)\mathrm{d}\omega. \tag{C.18}$$

result into

$$B_t^{tot} = \left(1 - \mu(\overline{\overline{\omega}}_t)\right) A_t + \frac{1 - \mu(\overline{\overline{\omega}}_t)}{1 - \mu(\overline{\omega}_t)} B_t. \tag{C.19}$$

In equilibrium, all firms that produce raise the same amount of financing. Therefore $k_t(\omega) = \tilde{k}_t$ is uniform across firms. Because labor is also committed before ω is observed, the labor allocation (11) gives

$$h_{t+1}(\omega) = \tilde{h}_{t+1}, \tag{C.20}$$

uniform across producing firms. Individual output is therefore

$$y_{t+1}(\omega) = Z_{t+1}\,\omega\,\tilde{k}_t^{\alpha}\,\tilde{h}_{t+1}^{1-\alpha}, \tag{C.21}$$

linear in ω. Aggregate capital and labor are

$$K_t = \left(1 - \mu(\overline{\overline{\omega}}_t)\right) \tilde{k}_t, \tag{C.22}$$

$$H_{t+1} = \int_{\overline{\overline{\omega}}_t}^{1} h_{t+1}(\omega)\,\mu'(\omega)\,\mathrm{d}\omega = \left(1 - \mu(\overline{\overline{\omega}}_t)\right) \tilde{h}_{t+1}. \tag{C.23}$$

Integrating individual outputs and eliminating $\tilde{k}_t$ and $\tilde{h}_{t+1}$ using (C.22)–(C.23) yields

$$Y_{t+1} = Z_{t+1}\,K_t^{\alpha}\,H_{t+1}^{1-\alpha} \frac{\int_{\overline{\overline{\omega}}_t}^{1} \omega\,\mu'(\omega)\,\mathrm{d}\omega}{1 - \mu(\overline{\overline{\omega}}_t)}. \tag{C.24}$$

Individual producing firms borrow to finance the purchase of capital, i.e., $b_t^{tot}(\omega) = Q_t k_t(\omega)$, so aggregating over this mass of firms results into the aggregate constraint

$$B_t^{tot} = Q_t K_t. \tag{C.25}$$

Let R_t define the average return on capital that producing firms receive, with the relevant density $\frac{\mu'(\omega)}{1-\mu(\overline{\overline{\omega}}_{t-1})}$:

$$R_t = \int_{\overline{\overline{\omega}}_{t-1}}^{1} R_t(\omega)\frac{\mu'(\omega)}{1-\mu(\overline{\overline{\omega}}_{t-1})}\mathrm{d}\omega. \tag{C.26}$$

Substituting $R_t(\omega) = (\kappa_t/Q_{t-1})\,\omega + (1-\delta)Q_t/Q_{t-1}$ from equation (13) and noting that $\kappa_t\, E[\omega \mid \omega \geq \overline{\overline{\omega}}_{t-1}] = \alpha Y_t/K_{t-1}$ (which follows from equation (C.24) applied at period t):

$$R_t = \frac{\alpha\, Y_t}{Q_{t-1}\, K_{t-1}} + \frac{(1-\delta)\, Q_t}{Q_{t-1}}. \tag{C.27}$$

The $E[\omega]$ factors cancel in the weighted average: the distribution-specific terms that appear in individual returns $R_t(\omega)$ drop out of the aggregate return, leaving a clean expression in observable aggregates. The aggregate equity return to households is defined as

$$R_t^A = \int_0^1 R_t^A(\omega)\mu'(\omega)\mathrm{d}\omega. \tag{C.28}$$

The following subsection derives that this equation can be used to get that

$$R_t^A A_{t-1} = B_{t-1}^{tot} R_t + (R_{t-1}^B - \xi)D_{t-1} + (1-\theta)\,\frac{\mu(\overline{\overline{\omega}}_{t-1}) - \mu(\overline{\omega}_{t-1})}{1-\mu(\overline{\omega}_{t-1})} B_{t-1}. \tag{C.29}$$

This equation says that the funds paid to the household are equal to the returns from lending to producing firms net of intermediation costs, plus the returns from diverting funds, and including the funds that were lent to diverting firms and recovered.

The goods market clears:

$$Y_t = C_t + I_t^g. \tag{C.30}$$

Now we are ready to define a competitive equilibrium. The equilibrium is an allocation

$$\left\{C_t, H_t, Y_t, K_t, B_t^{tot}, I_t^n, I_t^g, \Pi_t, T_t, B_t^G, B_t^H, D_t, \Xi_t, \overline{\omega}_t, \overline{\overline{\omega}}_t, Z_t, Z_{It}, \nu_{ct}, F_t, L_t, A_t, B_t\right\}_{t=0}^{\infty},$$

together with the sequence of prices $\left\{\lambda_{ct}, R_t^A, W_t, R_t^B, Q_t, R_t, \rho_t\right\}_{t=0}^{\infty}$ satisfying equations (C.1), (C.2), (C.3), (C.4), (3), (5), (22), (12), (21), (23), (C.5), (25), (26), (27), (28), (30), (31), (32), (33), (39), (C.9), (C.13), (C.16), (C.19), (C.24), (C.25), (C.27), (C.29), and (C.30), together with the slope conditions defined in Appendix A, given initial conditions K_0, I_0^g, B_0^{tot}, B_0^G, B_0^H, D_0, $\overline{\omega}_0$, $\overline{\overline{\omega}}_0$, A_0, Z_0, $Z_{I,0}$, ν_{c0}, R_0^B, Q_0, and the exogenous processes $\{\varepsilon_t^z, \varepsilon_{It}, \varepsilon_{\nu t}\}$.[3]

[3] To express equations (32) and (33) in terms of the aggregate variables A_t, B_t, and F_t, we use equations (C.10), (C.11), and (38) to substitute for a_t, b_t, and $\Theta_t(\omega)$, respectively.

C.4 Aggregate Equity Return

By definition

$$R_t^A = \int_0^{\overline{\omega}_{t-1}} R_t^A(\omega)\mu'(\omega)\mathrm{d}\omega + \int_{\overline{\omega}_{t-1}}^{\overline{\overline{\omega}}_{t-1}} R_t^A(\omega)\mu'(\omega)\mathrm{d}\omega + \int_{\overline{\overline{\omega}}_{t-1}}^{1} R_t^A(\omega)\mu'(\omega)\mathrm{d}\omega =$$

$$\int_0^{\overline{\omega}_{t-1}} \left(\rho_{t-1}\frac{1-\mu(\overline{\overline{\omega}}_{t-1})}{1-\mu(\overline{\omega}_{t-1})} + \frac{\mu(\overline{\overline{\omega}}_{t-1})-\mu(\overline{\omega}_{t-1})}{1-\mu(\overline{\omega}_{t-1})}(1-\theta)\right)\mu'(\omega)\mathrm{d}\omega +$$

$$\int_{\overline{\omega}_{t-1}}^{\overline{\overline{\omega}}_{t-1}} \frac{\left(R_{t-1}^B-\xi\right)\left(a_{t-1}(\omega)+\Theta_t(\omega)b_{t-1}(\omega)\right)}{a_{t-1}(\omega)}\mu'(\omega)\mathrm{d}\omega +$$

$$\int_{\overline{\overline{\omega}}_{t-1}}^{1} \frac{R_t(\omega)\left(a_{t-1}(\omega)+b_{t-1}(\omega)\right)-\rho_{t-1}b_{t-1}(\omega)}{a_{t-1}(\omega)}\mu'(\omega)\mathrm{d}\omega, \quad \text{(C.31)}$$

where we use equations (20), (17), and (15) to size the equity returns on each segment of firms.

Simplifying

$$R_t^A = \rho_{t-1}\frac{1-\mu(\overline{\overline{\omega}}_{t-1})}{1-\mu(\overline{\omega}_{t-1})}\mu(\overline{\omega}_{t-1}) + \frac{\mu(\overline{\overline{\omega}}_{t-1})-\mu(\overline{\omega}_{t-1})}{1-\mu(\overline{\omega}_{t-1})}(1-\theta)\mu(\overline{\omega}_{t-1}) +$$

$$\int_{\overline{\omega}_{t-1}}^{\overline{\overline{\omega}}_{t-1}} \frac{(R_{t-1}^B-\xi)d_{t-1}(\omega)}{a_{t-1}}\mu'(\omega)\mathrm{d}\omega + \int_{\overline{\overline{\omega}}_{t-1}}^{1} \frac{R_t(\omega)(a_{t-1}+b_{t-1})}{a_{t-1}}\mu'(\omega)\mathrm{d}\omega - \int_{\overline{\overline{\omega}}_{t-1}}^{1} \frac{\rho_{t-1}b_{t-1}}{a_{t-1}}\mu'(\omega)\mathrm{d}\omega =$$

$$\rho_{t-1}\left(1-\mu(\overline{\overline{\omega}}_{t-1})\right)\frac{b_{t-1}}{a_{t-1}} + \left(\mu(\overline{\overline{\omega}}_{t-1})-\mu(\overline{\omega}_{t-1})\right)(1-\theta)\frac{b_{t-1}}{a_{t-1}} + \frac{(R_{t-1}^B-\xi)}{A_{t-1}}D_{t-1} +$$

$$\left(1-\mu(\overline{\overline{\omega}}_{t-1})\right)\frac{\left(A_{t-1}+\frac{B_{t-1}}{1-\mu(\overline{\omega}_{t-1})}\right)}{A_{t-1}}\int_{\overline{\overline{\omega}}_{t-1}}^{1} R_t(\omega)\mu'(\omega)\mathrm{d}\omega - \rho_{t-1}\left(1-\mu(\overline{\overline{\omega}}_{t-1})\right)\frac{b_{t-1}}{a_{t-1}} =$$

$$(1-\theta)\frac{\mu(\overline{\overline{\omega}}_{t-1})-\mu(\overline{\omega}_{t-1})}{1-\mu(\overline{\omega}_{t-1})}\frac{B_{t-1}}{A_{t-1}} + \frac{(R_{t-1}^B-\xi)}{A_{t-1}}D_{t-1} +$$

$$\left(1-\mu(\overline{\overline{\omega}}_{t-1})\right)\frac{\left(A_{t-1}+\frac{B_{t-1}}{1-\mu(\overline{\omega}_{t-1})}\right)}{A_{t-1}}\int_{\overline{\overline{\omega}}_{t-1}}^{1} R_t(\omega)\frac{\mu'(\omega)}{1-\mu(\overline{\overline{\omega}}_{t-1})}\mathrm{d}\omega =$$

$$(1-\theta)\frac{\mu(\overline{\overline{\omega}}_{t-1})-\mu(\overline{\omega}_{t-1})}{1-\mu(\overline{\omega}_{t-1})}\frac{B_{t-1}}{A_{t-1}} + \frac{(R_{t-1}^B-\xi)}{A_{t-1}}D_{t-1} + \frac{B_{t-1}^{tot}}{A_{t-1}}R_t, \quad \text{(C.32)}$$

where we use $a_{t-1}(\omega) = a_{t-1}$ and $b_{t-1}(\omega) = b_{t-1}$ for all ω and equations (C.10), (C.11), (C.14), and (C.19). Moreover, we use that

$$\frac{b_{t-1}}{a_{t-1}} = \frac{\mu_{\eta_1,\eta_2}(\overline{\omega}_{t-1})}{1-\mu_{\eta_1,\eta_2}(\overline{\omega}_{t-1})} = \frac{B_{t-1}}{A_{t-1}}\frac{1}{1-\mu_{\eta_1,\eta_2}(\overline{\omega}_{t-1})} \quad \text{(C.33)}$$

by combining equations (C.10), (C.11), and (C.9).

We can re-write equation (C.32) as follows:

$$R_t^A A_{t-1} = B_{t-1}^{tot} R_t + (R_{t-1}^B - \xi) D_{t-1} + (1-\theta) \frac{\mu(\overline{\overline{\omega}}_{t-1}) - \mu(\overline{\omega}_{t-1})}{1 - \mu(\overline{\omega}_{t-1})} B_{t-1}. \tag{C.34}$$

C.5 Diversion Function: Limiting Case

Consider

$$I(\overline{\omega}_t, \overline{\overline{\omega}}_t) = \int_{\overline{\omega}_t}^{\overline{\overline{\omega}}_t} \Theta_t(\omega) \frac{\mu'(\omega)}{\mu(\overline{\overline{\omega}}_t) - \mu(\overline{\omega}_t)} \mathrm{d}\omega. \tag{C.35}$$

Here we seek to find $\lim\limits_{\overline{\overline{\omega}}_t \to \overline{\omega}_t} I(\overline{\omega}_t, \overline{\overline{\omega}}_t)$. We can write $I(\overline{\omega}_t, \overline{\overline{\omega}}_t)$ as a ratio, i.e.,

$$I(\overline{\omega}_t, \overline{\overline{\omega}}_t) = \frac{\int_{\overline{\omega}_t}^{\overline{\overline{\omega}}_t} \Theta_t(\omega) \mu'(\omega) \mathrm{d}\omega}{\int_{\overline{\omega}_t}^{\overline{\overline{\omega}}_t} \mu'(\omega) \mathrm{d}\omega}. \tag{C.36}$$

Both numerator and denominator go to zero as $\overline{\overline{\omega}}_t \to \overline{\omega}_t$. Applying L'Hopital's rule

$$\lim_{\overline{\overline{\omega}}_t \to \overline{\omega}_t} = \frac{\int_{\overline{\omega}_t}^{\overline{\overline{\omega}}_t} \Theta_t(\omega) \mu'(\omega) \mathrm{d}\omega}{\int_{\overline{\omega}_t}^{\overline{\overline{\omega}}_t} \mu'(\omega) \mathrm{d}\omega} = \lim_{\overline{\overline{\omega}}_t \to \overline{\omega}_t} \frac{\Theta_t(\overline{\overline{\omega}}_t) \mu'(\overline{\overline{\omega}}_t)}{\mu'(\overline{\overline{\omega}}_t)} = \Theta_t(\overline{\omega}_t), \tag{C.37}$$

as long as $\mu'(\overline{\omega}_t) \neq 0$ (a condition satisfied for our baseline calibration).

C.6 Beta Distribution: Aggregate Output

Plugging the Beta(η_1, η_2) density into equation (C.24), the key integral is

$$\int_{\overline{\overline{\omega}}_t}^{1} \omega \mu'_{\eta_1,\eta_2}(\omega)\, \mathrm{d}\omega = \int_{\overline{\overline{\omega}}_t}^{1} \frac{\omega^{\eta_1}(1-\omega)^{\eta_2 - 1}}{B(\eta_1, \eta_2)}\, \mathrm{d}\omega. \tag{C.38}$$

Multiplying and dividing by $B(\eta_1 + 1, \eta_2)$ identifies the integrand as a Beta$(\eta_1 + 1, \eta_2)$ density kernel:

$$\begin{aligned}\int_{\overline{\overline{\omega}}_t}^{1} \omega \mu'_{\eta_1,\eta_2}(\omega)\, \mathrm{d}\omega &= \frac{B(\eta_1+1, \eta_2)}{B(\eta_1, \eta_2)} \int_{\overline{\overline{\omega}}_t}^{1} \frac{\omega^{\eta_1}(1-\omega)^{\eta_2-1}}{B(\eta_1+1, \eta_2)}\, \mathrm{d}\omega \\ &= c_\alpha \left(1 - \mu_{\eta_1+1,\eta_2}(\overline{\overline{\omega}}_t)\right), \end{aligned} \tag{C.39}$$

where $c_\alpha = B(\eta_1+1,\eta_2)/B(\eta_1,\eta_2) = \eta_1/(\eta_1+\eta_2)$ as defined in (40). Therefore

$$E\Big[\omega \,\Big|\, \omega \geq \overline{\overline{\omega}}_t\Big] = c_\alpha \,\frac{1-\mu_{\eta_1+1,\eta_2}(\overline{\overline{\omega}}_t)}{1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t)}, \tag{C.40}$$

and substituting into (C.24):

$$Y_{t+1} = c_\alpha \,\frac{1-\mu_{\eta_1+1,\eta_2}(\overline{\overline{\omega}}_t)}{1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t)} \cdot Z_{t+1}\, K_t^\alpha\, H_{t+1}^{1-\alpha}. \tag{C.41}$$

C.7 Beta Distribution: Average Return on Capital

The average return on capital follows directly from the general result (C.27), which no longer contains distribution-specific terms. For completeness, we verify it from the individual return. Substituting $R_t(\omega) = (\kappa_t/Q_{t-1})\,\omega + (1-\delta)Q_t/Q_{t-1}$ and using the Beta-distribution result for $E[\omega|\omega \geq \overline{\overline{\omega}}_{t-1}]$ from Appendix C.6:

$$\begin{aligned} R_t &= \frac{\kappa_t}{Q_{t-1}}\, E\Big[\omega \,\Big|\, \omega \geq \overline{\overline{\omega}}_{t-1}\Big] + \frac{(1-\delta)Q_t}{Q_{t-1}} \\ &= \frac{\kappa_t}{Q_{t-1}} \cdot c_\alpha \,\frac{1-\mu_{\eta_1+1,\eta_2}(\overline{\overline{\omega}}_{t-1})}{1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_{t-1})} + \frac{(1-\delta)Q_t}{Q_{t-1}}. \end{aligned} \tag{C.42}$$

Since $\kappa_t\, E[\omega \mid \omega \geq \overline{\overline{\omega}}_{t-1}] = \alpha Y_t/K_{t-1}$ (from equation (C.41) applied at period t), this simplifies to

$$R_t = \frac{\alpha\, Y_t}{Q_{t-1}\, K_{t-1}} + \frac{(1-\delta)\, Q_t}{Q_{t-1}}, \tag{C.43}$$

confirming (C.27). The distribution-specific ratio cancels because both Y_t and $\kappa_t\, E[\omega]$ contain the same Beta survival-function ratio.

C.8 Beta Distribution: Aggregate Equity Return

Plugging the functional form of the Beta distribution into equation (C.32), we get:

$$R_t^A = (1-\theta)\,\frac{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_{t-1}) - \mu_{\eta_1,\eta_2}(\overline{\omega}_{t-1})}{1-\mu_{\eta_1,\eta_2}(\overline{\omega}_{t-1})}\,\frac{B_{t-1}}{A_{t-1}} + \frac{(R_{t-1}^B - \xi)}{A_{t-1}} D_{t-1} + \frac{B_{t-1}^{tot}}{A_{t-1}} R_t, \tag{C.44}$$

where R_t is given by equation (C.43).

C.9 Beta Distribution: Diversion Function

Expressing F_t from equation (39):

$$\theta = \frac{1}{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t) - \mu_{\eta_1,\eta_2}(\overline{\omega}_t)} \int_{\overline{\omega}_t}^{\overline{\overline{\omega}}_t} \omega^{\psi} F_t \frac{\omega^{\eta_1-1}(1-\omega)^{\eta_2-1}}{B(\eta_1,\eta_2)} \mathrm{d}\omega =$$

$$\frac{1}{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t) - \mu_{\eta_1,\eta_2}(\overline{\omega}_t)} \int_{\overline{\omega}_t}^{\overline{\overline{\omega}}_t} F_t \frac{\omega^{\eta_1+\psi-1}(1-\omega)^{\eta_2-1}}{\Gamma(\eta_1)\Gamma(\eta_2)} \Gamma(\eta_1+\eta_2) \mathrm{d}\omega =$$

$$\frac{1}{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t) - \mu_{\eta_1,\eta_2}(\overline{\omega}_t)} \int_{\overline{\omega}_t}^{\overline{\overline{\omega}}_t} F_t \frac{\omega^{\eta_1+\psi-1}(1-\omega)^{\eta_2-1}}{\Gamma(\eta_1+\psi)\Gamma(\eta_2)} \Gamma(\eta_1+\psi+\eta_2) \frac{\Gamma(\eta_1+\eta_2)\Gamma(\eta_1+\psi)}{\Gamma(\eta_1)\Gamma(\eta_1+\psi+\eta_2)} \mathrm{d}\omega =$$

$$\frac{F_t}{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t) - \mu_{\eta_1,\eta_2}(\overline{\omega}_t)} \frac{\Gamma(\eta_1+\eta_2)\Gamma(\eta_1+\psi)}{\Gamma(\eta_1)\Gamma(\eta_1+\psi+\eta_2)} \left(\mu_{\eta_1+\psi,\eta_2}(\overline{\overline{\omega}}_t) - \mu_{\eta_1+\psi,\eta_2}(\overline{\omega}_t)\right) \quad \text{(C.45)}$$

Therefore,

$$F_t = \frac{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t) - \mu_{\eta_1,\eta_2}(\overline{\omega}_t)}{\mu_{\eta_1+\psi,\eta_2}(\overline{\overline{\omega}}_t) - \mu_{\eta_1+\psi,\eta_2}(\overline{\omega}_t)} \frac{\Gamma(\eta_1)\Gamma(\eta_1+\psi+\eta_2)}{\Gamma(\eta_1+\eta_2)\Gamma(\eta_1+\psi)} \theta \quad \text{(C.46)}$$

C.9.1 Limiting Case as $\overline{\overline{\omega}}_t \to \overline{\omega}_t$

In the limiting case as $\overline{\overline{\omega}}_t \to \overline{\omega}_t$, we can use the result of Appendix C.5

$$\lim_{\overline{\overline{\omega}}_t \to \overline{\omega}_t} \int_{\overline{\omega}_t}^{\overline{\overline{\omega}}_t} \Theta_t(\omega) \frac{\mu'(\omega)}{\mu(\overline{\overline{\omega}}_t) - \mu(\overline{\omega}_t)} \mathrm{d}\omega = \Theta_t(\overline{\omega}_t) = \overline{\omega}_t^{\psi} F_t. \quad \text{(C.47)}$$

Equating this result with θ, we find that, in the limiting case as $\overline{\overline{\omega}}_t \to \overline{\omega}_t$,

$$F_t = \frac{\theta}{\overline{\omega}_t^{\psi}}. \quad \text{(C.48)}$$

C.10 Equilibrium Conditions

This subsection lists the 29 equilibrium conditions that constitute a competitive equilibrium and identifies the derivation or earlier statement from which each is obtained.

Asset Euler equation. Equation (C.1) from Appendix C.1, rearranged to place λ_{ct} on the left-hand side:

$$\lambda_{ct} = \beta E_t \left\{ \lambda_{ct+1} R_{t+1}^{A} \right\}. \quad \text{(C.49)}$$

Consumption Euler equation. Equation (C.2) from Appendix C.1:

$$\beta E_t \left\{ \lambda_{ct+1} R_t^{B} \right\} = \frac{1}{C_t - \nu_{ct}}. \quad \text{(C.50)}$$

Labor supply. Compact form obtained by combining equations (C.3), (C.4), and (C.2) from Appendix C.1. Substituting $\beta E_t\{\lambda_{ct+1}R_t^B\} = \lambda_{ct}$ from (C.4) into (C.3) gives $-\vartheta H_t^\nu + \lambda_{ct}W_t = 0$. Applying $\lambda_{ct} = 1/(C_t - \nu_{ct})$ from (C.2) then yields:

$$W_t = \vartheta(C_t - \nu_{ct})H_t^\nu. \tag{C.51}$$

Bond Euler equation. Equation (C.4) from Appendix C.1, rearranged to place λ_{ct} on the left-hand side:

$$\lambda_{ct} = \beta E_t\left\{\lambda_{ct+1}R_t^B\right\}. \tag{C.52}$$

Aggregate output. Beta-distribution specialization of equation (C.24) from Appendix C.3, with time indices shifted by one period. The integral $\int_{\overline{\overline{\omega}}_{t-1}}^{1} \omega\, \mu'_{\eta_1,\eta_2}(\omega)\, \mathrm{d}\omega$ is evaluated in Appendix C.6 by rewriting $\omega \cdot \mu'_{\eta_1,\eta_2}(\omega)$ as a scalar multiple of the Beta(η_1+1,η_2) density kernel, using $c_\alpha = B(\eta_1+1,\eta_2)/B(\eta_1,\eta_2) = \eta_1/(\eta_1+\eta_2)$ from (40):

$$Y_t = c_\alpha\, \frac{1-\mu_{\eta_1+1,\eta_2}(\overline{\overline{\omega}}_{t-1})}{1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_{t-1})} \cdot Z_t K_{t-1}^\alpha H_t^{1-\alpha}. \tag{C.53}$$

Capital financing. Aggregate counterpart of the individual firm budget constraint (8) from Section 4.2.1. Derived in Appendix C.3 as equation (C.25) by integrating $b_t^{tot}(\omega) = Q_t k_t(\omega)$ over the mass of producing firms:

$$B_t^{tot} = Q_t K_t. \tag{C.54}$$

Labor demand. Aggregate form of the individual firm's optimal labor condition (11). Because $h_{t+1}(\omega) = \tilde{h}_{t+1}$ is uniform across producing firms, the aggregate wage equals the aggregate marginal product of labor:

$$W_t = (1-\alpha)\,\frac{Y_t}{H_t}. \tag{C.55}$$

Average return on capital. Equation (C.27) from Appendix C.3, verified for the Beta distribution in Appendix C.7. The distribution-specific terms cancel (see (C.27)), yielding the compact expression:

$$R_t = \frac{\alpha\, Y_t}{Q_{t-1}\, K_{t-1}} + \frac{(1-\delta)}{Q_{t-1}} Q_t. \tag{C.56}$$

Investment supply. Equation (21) from Section 4.3:

$$I_t^n = Z_{It}\left[1 - \frac{\phi}{2}\left(\frac{I_t^g}{I_{t-1}^g} - 1\right)^2\right] I_t^g. \tag{C.57}$$

Capital accumulation. Equation (23) from Section 4.3:

$$K_t = I_t^n + (1-\delta)K_{t-1}. \tag{C.58}$$

Capital-firm profits. Equation (25) from Section 4.3, the zero-profit condition for competitive capital-producing firms:

$$\Pi_t = Q_t Z_{It}\left[1 - \frac{\phi}{2}\left(\frac{I_t^g}{I_{t-1}^g} - 1\right)^2\right] I_t^g - I_t^g. \tag{C.59}$$

Government budget constraint. Equation (26) from Section 4.4:

$$T_t = B_t^G - R_{t-1}^B B_{t-1}^G. \tag{C.60}$$

Bond market identity. Equation (27) from Section 4.4:

$$B_t^G = B_t^H + D_t. \tag{C.61}$$

Household bond holdings. Equation (28) from Section 4.4. By assumption, government bonds are sold only to firms, so households hold none:

$$B_t^H = 0. \tag{C.62}$$

Haircut rebate to households. Equation (30) from Section 4.4:

$$\Xi_t = \xi D_{t-1}. \tag{C.63}$$

Lower cutoff condition. Equation (31) from Section 5.1. The expected return from producing at the marginal lending firm (productivity $\overline{\omega}_t$) equals the expected return from the outside option:

$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(\frac{\kappa_{t+1}}{Q_t}\,\overline{\omega}_t + \frac{(1-\delta)}{Q_t}Q_{t+1}\right)\right] = E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_t^B - \xi\right)\right]. \tag{C.64}$$

Aggregate lending. Equation (C.13) from Appendix C.3, with the generic cumulative distribution function μ replaced by its Beta-distribution counterpart μ_{η_1,η_2}:

$$L_t = \mu_{\eta_1,\eta_2}(\overline{\omega}_t)A_t. \tag{C.65}$$

Loan market clearing. Equation (C.9) from Appendix C.3:

$$B_t = L_t. \tag{C.66}$$

Aggregate diversion. Equation (C.16) from Appendix C.3, with μ replaced by μ_{η_1,η_2} throughout:

$$D_t = \left(\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t) - \mu_{\eta_1,\eta_2}(\overline{\omega}_t)\right)A_t + \frac{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t) - \mu_{\eta_1,\eta_2}(\overline{\omega}_t)}{1-\mu_{\eta_1,\eta_2}(\overline{\omega}_t)}\theta B_t. \tag{C.67}$$

Aggregate total borrowing. Equation (C.19) from Appendix C.3, with μ replaced by μ_{η_1,η_2} throughout:

$$B_t^{tot} = \left(1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t)\right)A_t + \frac{1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t)}{1-\mu_{\eta_1,\eta_2}(\overline{\omega}_t)}B_t. \tag{C.68}$$

Equity return market clearing. Equation (C.29) from Appendix C.3, with μ replaced by μ_{η_1,η_2} throughout. The general form is derived in Appendix C.4 by aggregating the per-firm equity returns (20), (17), and (15) across the three firm segments; Appendix C.8 verifies the Beta-specific version:

$$R_t^A A_{t-1} = B_{t-1}^{tot}R_t + (R_{t-1}^B - \xi)D_{t-1} + (1-\theta)\,\frac{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_{t-1}) - \mu_{\eta_1,\eta_2}(\overline{\omega}_{t-1})}{1-\mu_{\eta_1,\eta_2}(\overline{\omega}_{t-1})}B_{t-1}. \tag{C.69}$$

Goods market clearing. Equation (C.30) from Appendix C.3:

$$Y_t = C_t + I_t^g. \tag{C.70}$$

Loan rate equation. Aggregate Beta-distribution form of equation (32) from Section 5.1. Individual-firm variables are replaced using $a_t = A_t$ from (C.10) and $b_t = B_t/(1-\mu_{\eta_1,\eta_2}(\overline{\omega}_t))$ from (C.11); the diversion function $\Theta_t(\overline{\omega}_t)b_t$ is substituted using the functional form (38), giving $F_t\overline{\omega}_t^{\psi}B_t/(1-\mu_{\eta_1,\eta_2}(\overline{\omega}_t))$, where F_t is defined in equation (C.77) below:

$$\begin{aligned}\left(\rho_t\frac{1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t)}{1-\mu_{\eta_1,\eta_2}(\overline{\omega}_t)}+(1-\theta)\frac{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t)-\mu_{\eta_1,\eta_2}(\overline{\omega}_t)}{1-\mu_{\eta_1,\eta_2}(\overline{\omega}_t)}\right)A_t = \\ \left(R_t^B-\xi\right)\left(A_t+F_t\overline{\omega}_t^{\psi}\frac{B_t}{1-\mu_{\eta_1,\eta_2}(\overline{\omega}_t)}\right).\end{aligned} \tag{C.71}$$

Upper cutoff condition. Aggregate Beta-distribution form of equation (33) from Section 5.1. The same substitutions as for the loan rate equation above are applied: a_t, b_t, and $\Theta_t(\overline{\overline{\omega}}_t)$ are replaced using (C.10), (C.11), and (38) respectively:

$$\begin{aligned}E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_t^B-\xi\right)\left(A_t+F_t\overline{\overline{\omega}}_t^{\psi}\frac{B_t}{1-\mu_{\eta_1,\eta_2}(\overline{\omega}_t)}\right)\right] = \\ E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(\left(\frac{1}{Q_t}\alpha\ Z_{t+1}\left(\frac{H_{t+1}}{K_t}\right)^{1-\alpha}\overline{\overline{\omega}}_t+\frac{(1-\delta)}{Q_t}Q_{t+1}\right)\left(A_t+\frac{B_t}{1-\mu_{\eta_1,\eta_2}(\overline{\omega}_t)}\right)-\right.\right. \\ \left.\left.\rho_t\frac{B_t}{1-\mu_{\eta_1,\eta_2}(\overline{\omega}_t)}\right)\right].\end{aligned} \tag{C.72}$$

Capital firm first-order condition (Tobin's q). Equation (C.5) from Appendix C.2, derived from the capital-producing firm's optimization problem (24) stated in Section 4.3:

$$0 = E_t\left\{\begin{array}{c} Q_tZ_{It}\left[-\phi\left(\frac{I_t^g}{I_{t-1}^g}-1\right)\frac{1}{I_{t-1}^g}\right]I_t^g+Q_tZ_{It}\left[1-\frac{\phi}{2}\left(\frac{I_t^g}{I_{t-1}^g}-1\right)^2\right]-1 \\ +\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}Q_{t+1}Z_{It+1}\phi\left(\frac{I_{t+1}^g}{I_t^g}-1\right)\left(\frac{I_{t+1}^g}{I_t^g}\right)^2 \end{array}\right\}. \tag{C.73}$$

Aggregate TFP process. Equation (5) from Section 4.2:

$$\log Z_t = \rho_z \log Z_{t-1} + \varepsilon_t^z. \tag{C.74}$$

Investment-specific technology process. Equation (22) from Section 4.3:

$$\ln(Z_{It}) = \rho_I \ln(Z_{It-1}) + \varepsilon_{It}. \tag{C.75}$$

Consumption demand shock process. Equation (3) from Section 4.1:

$$\nu_{ct} = \rho_\nu \nu_{ct-1} + \varepsilon_{\nu t}. \tag{C.76}$$

Diversion scaling factor (F_t). Beta-distribution closed form of the integral definition (39) from Section 5.1. The derivation in Appendix C.9 rewrites $\omega^{\psi}\mu'_{\eta_1,\eta_2}(\omega)$ as a scalar multiple of the Beta$(\eta_1+\psi,\eta_2)$ density kernel, using $\Gamma(\eta_1+\psi)/\Gamma(\eta_1)$ to convert the normalizing constants:

$$F_t = \frac{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}_t) - \mu_{\eta_1,\eta_2}(\overline{\omega}_t)}{\mu_{\eta_1+\psi,\eta_2}(\overline{\overline{\omega}}_t) - \mu_{\eta_1+\psi,\eta_2}(\overline{\omega}_t)} \frac{\Gamma(\eta_1)\Gamma(\eta_1+\psi+\eta_2)}{\Gamma(\eta_1+\eta_2)\Gamma(\eta_1+\psi)}\theta \tag{C.77}$$

and ω follows the Beta distribution with parameters η_1 and η_2, i.e.,

$$\mu'_{\eta_1,\eta_2}(\omega) = \frac{\omega^{\eta_1-1}(1-\omega)^{\eta_2-1}}{B(\eta_1,\eta_2)},$$

where $B(\eta_1,\eta_2) = \frac{\Gamma(\eta_1)\Gamma(\eta_2)}{\Gamma(\eta_1+\eta_2)}$ and Γ is the Gamma function.

D Steady-State Conditions

We derived 29 equilibrium conditions for 29 endogenous variables:

$\lambda_{ct}, R_t^A, C_t, H_t, W_t, R_t^B, Y_t, \overline{\omega}_t, \overline{\overline{\omega}}_t, Z_t, Z_{It}, \nu_{ct}, K_t, B_t^{tot}, Q_t, R_t, I_t^n, I_t^g, \Pi_t, T_t, B_t^G, B_t^H, D_t, \Xi_t, L_t, A_t, B_t, \rho_t,$ and F_t.

Next, we consider a strategy for finding the non-stochastic steady state. The strategy will involve guessing the values of $\overline{\omega}$, $\overline{\overline{\omega}}$, and R and iterating to a fixed point.

Given our guesses, we can find

$$F = \frac{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}) - \mu_{\eta_1,\eta_2}(\overline{\omega})}{\mu_{\eta_1+\psi,\eta_2}(\overline{\overline{\omega}}) - \mu_{\eta_1+\psi,\eta_2}(\overline{\omega})} \frac{\Gamma(\eta_1)\Gamma(\eta_1+\psi+\eta_2)}{\Gamma(\eta_1+\eta_2)\Gamma(\eta_1+\psi)}\theta \tag{D.1}$$

Continue by fixing

$$H = H^{ss} = 1. \tag{D.2}$$

We can support any choice of H by choosing ϑ appropriately. For example, from the intratemporal condition (C.51) evaluated in steady state, we can find that $\vartheta = \frac{W}{C\,H^{\nu}}$ supports $H = 1$ in the steady state.

From equation (C.52), we can see that in the steady state

$$R^B = \frac{1}{\beta}. \tag{D.3}$$

Similarly, from equation (C.49)

$$R^A = \frac{1}{\beta}. \tag{D.4}$$

From equation (C.74)

$$Z = 1. \tag{D.5}$$

From equation (C.75)

$$Z_I = 1. \tag{D.6}$$

From equation (C.76)

$$\nu_c = 0. \tag{D.7}$$

From equation (C.73)

$$Q = 1. \tag{D.8}$$

From equation (C.56) in the steady state ($Q = 1$, $Z = 1$) and using (C.53) to substitute $\alpha Y/K = \alpha\,\Phi\,(H/K)^{1-\alpha}$ (where Φ is the steady-state TFP prefactor (42) evaluated at $\overline{\overline{\omega}}$):

$$R = \alpha\,\Phi\left(\frac{H}{K}\right)^{1-\alpha} + 1 - \delta,$$

we can solve for K:

$$K = H\left[\frac{\alpha\,\Phi}{R-(1-\delta)}\right]^{\frac{1}{1-\alpha}}, \tag{D.9}$$

where $\Phi = c_\alpha\,(1-\mu_{\eta_1+1,\eta_2}(\overline{\overline{\omega}}))/(1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}))$ from (42). Combining equation (C.54) and our result (D.8), we get

$$B^{tot} = K. \tag{D.10}$$

From equation (C.53)

$$Y = c_\alpha\,\frac{1-\mu_{\eta_1+1,\eta_2}(\overline{\overline{\omega}})}{1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}})}\cdot K^\alpha H^{1-\alpha} = \Phi\,K^\alpha H^{1-\alpha}. \tag{D.11}$$

From equation (C.55)

$$W = (1-\alpha)\,\frac{Y}{H}. \tag{D.12}$$

From equation (C.58)

$$I^n = \delta K. \tag{D.13}$$

From equation (C.57)

$$I^g = I^n. \tag{D.14}$$

Therefore, from equation (C.70), we can find

$$C = Y - I^g. \tag{D.15}$$

Therefore, from equation (C.50), we can find

$$\lambda_c = \frac{1}{C}. \tag{D.16}$$

Combining equations (C.65) and (C.66) and plugging the result into equation (C.68),

$$B^{tot} = (1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}))\,A + \frac{1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}})}{1-\mu_{\eta_1,\eta_2}(\overline{\omega})}\mu_{\eta_1,\eta_2}(\overline{\omega})A$$

we can find A

$$A = \frac{B^{tot}}{1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}) + \frac{1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}})}{1-\mu_{\eta_1,\eta_2}(\overline{\omega})}\mu_{\eta_1,\eta_2}(\overline{\omega})}. \tag{D.17}$$

Therefore, from the combination of equations (C.65) and (C.66), we can find

$$B = \mu_{\eta_1,\eta_2}(\overline{\omega})A. \tag{D.18}$$

From equation (C.66)

$$L = B. \tag{D.19}$$

From equation (C.67)

$$D = \left(\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}) - \mu_{\eta_1,\eta_2}(\overline{\omega})\right) A + \frac{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}) - \mu_{\eta_1,\eta_2}(\overline{\omega})}{1 - \mu_{\eta_1,\eta_2}(\overline{\omega})} \theta B. \tag{D.20}$$

From equation (C.62)

$$B^H = 0. \tag{D.21}$$

Therefore, from equation (C.61)

$$B^G = D. \tag{D.22}$$

From equation (C.60)

$$T = B^G - R^B B^G, \tag{D.23}$$

From equation (C.63)

$$\Xi = \xi D. \tag{D.24}$$

From equation (C.59)

$$\Pi = 0. \tag{D.25}$$

From equation (C.71)

$$\left(\rho \frac{1 - \mu_{\eta_1,\eta_2}(\overline{\overline{\omega}})}{1 - \mu_{\eta_1,\eta_2}(\overline{\omega})} + (1-\theta) \frac{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}) - \mu_{\eta_1,\eta_2}(\overline{\omega})}{1 - \mu_{\eta_1,\eta_2}(\overline{\omega})}\right) A = \left(R^B - \xi\right) \left(A + \frac{F \overline{\omega}^{\psi} B}{1 - \mu_{\eta_1,\eta_2}(\overline{\omega})}\right),$$

we can find

$$\rho = \frac{1-\mu_{\eta_1,\eta_2}(\overline{\omega})}{1-\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}})} \left(\left(R^B - \xi\right) \left(1 + \frac{F\overline{\omega}^{\psi} \mu_{\eta_1,\eta_2}(\overline{\omega})}{(1-\mu_{\eta_1,\eta_2}(\overline{\omega}))}\right) - (1-\theta) \frac{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}) - \mu_{\eta_1,\eta_2}(\overline{\omega})}{1-\mu_{\eta_1,\eta_2}(\overline{\omega})} \right), \tag{D.26}$$

where we use equation (D.18) to substitute for $\frac{B}{A}$.
We have 3 equations to verify our 3 guesses

1. From equation (C.64), verify the guess for $\overline{\omega}$

$$\alpha \left(\frac{H}{K}\right)^{1-\alpha} \overline{\omega} + 1 - \delta = R^B - \xi,$$

which can be re-written as

$$0 = -1 + \frac{R^B - \xi - (1-\delta)}{\alpha \left(\frac{H}{K}\right)^{1-\alpha} \overline{\omega}}. \tag{D.27}$$

2. From equation (C.72), verify the guess for $\overline{\overline{\omega}}$

$$\left(R^B - \xi\right) \left(A + F\overline{\overline{\omega}}^{\psi} \frac{B}{1 - \mu_{\eta_1,\eta_2}(\overline{\omega})}\right) =$$
$$\left(\alpha \left(\frac{H}{K}\right)^{1-\alpha} \overline{\overline{\omega}} + 1 - \delta\right) \left(A + \frac{B}{1 - \mu_{\eta_1,\eta_2}(\overline{\omega})}\right) - \rho \frac{B}{1 - \mu_{\eta_1,\eta_2}(\overline{\omega})},$$

which can be re-written as

$$0 = -1 + \left(\alpha \ \left(\frac{H}{K}\right)^{1-\alpha} \overline{\overline{\omega}} + 1 - \delta\right) \frac{1}{\rho} \left(\frac{1 - \mu_{\eta_1,\eta_2}(\overline{\omega})}{\mu_{\eta_1,\eta_2}(\overline{\omega})} + 1\right) - \left(R^B - \xi\right) \frac{1}{\rho} \left(\frac{1 - \mu_{\eta_1,\eta_2}(\overline{\omega})}{\mu_{\eta_1,\eta_2}(\overline{\omega})} + F\overline{\overline{\omega}}^{\psi}\right), \quad \text{(D.28)}$$

where we use equation (D.18) to substitute for $\frac{A}{B}$

3. From equation (C.69), verify the guess for R

$$R^A A = B^{tot} R + (R^B - \xi) D + (1-\theta) \frac{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}) - \mu_{\eta_1,\eta_2}(\overline{\omega})}{1 - \mu_{\eta_1,\eta_2}(\overline{\omega})} B,$$

which can be re-written as

$$0 = -1 + \frac{B^{tot} R}{R^A A} + \frac{(R^B - \xi) D}{R^A A} + (1-\theta) \frac{\mu_{\eta_1,\eta_2}(\overline{\overline{\omega}}) - \mu_{\eta_1,\eta_2}(\overline{\omega})}{(1 - \mu_{\eta_1,\eta_2}(\overline{\omega})) R^A A} B. \quad \text{(D.29)}$$

E Data and Model Counterparts

We are interested in the segment for the producing firms. To calculate the 90^{th} percentile for that segment, we need to rescale the density so that it integrates to 1 over the segment. For the 10^{th} percentile, we solve for ω_{10} from

$$\int_{\overline{\overline{\omega}}}^{\omega_{10}} \frac{\mu'_{\eta_1,\eta_2}(\omega)}{1 - \mu_{\eta_1,\eta_2}(\overline{\overline{\omega}})} \mathrm{d}\omega = 0.10. \quad \text{(E.30)}$$

Working on the derivation:

$$\frac{1}{1 - \mu_{\eta_1,\eta_2}(\overline{\overline{\omega}})} \left(\mu_{\eta_1,\eta_2}(\omega_{10}) - \mu_{\eta_1,\eta_2}(\overline{\overline{\omega}})\right) = 0.10.$$

Hence,

$$\omega_{10} = \mu^{-1}_{\eta_1,\eta_2} \left[0.10 \left(1 - \mu_{\eta_1,\eta_2}(\overline{\overline{\omega}})\right) + \mu_{\eta_1,\eta_2}(\overline{\overline{\omega}})\right], \quad \text{(E.31)}$$

where $\mu^{-1}_{\eta_1,\eta_2}$ is the quantile function of the Beta(η_1, η_2) distribution on $[0, 1]$.

Analogously for the 90^{th} percentile, the expression will be

$$\omega_{90} = \mu^{-1}_{\eta_1,\eta_2} \left[0.90 \left(1 - \mu_{\eta_1,\eta_2}(\overline{\overline{\omega}})\right) + \mu_{\eta_1,\eta_2}(\overline{\overline{\omega}})\right]. \quad \text{(E.32)}$$

Therefore, our target ratio is

$$\frac{\omega_{90}}{\omega_{10}} = \frac{\mu^{-1}_{\eta_1,\eta_2} \left[0.90 \left(1 - \mu_{\eta_1,\eta_2}(\overline{\overline{\omega}})\right) + \mu_{\eta_1,\eta_2}(\overline{\overline{\omega}})\right]}{\mu^{-1}_{\eta_1,\eta_2} \left[0.10 \left(1 - \mu_{\eta_1,\eta_2}(\overline{\overline{\omega}})\right) + \mu_{\eta_1,\eta_2}(\overline{\overline{\omega}})\right]}. \quad \text{(E.33)}$$

F Canonical Representative-Firm RBC Model

In this appendix we show that equilibrium conditions of our baseline real model are equivalent to those of a canonical RBC model for the special case in which production is efficient, i.e. $\bar{\omega} = \bar{\bar{\omega}} = 1$ and $\xi = 0$.

We describe the canonical RBC model before tackling the equivalence proof.

F.1 The canonical RBC model

The model adopts the canonical RBC decentralization: the household directly owns the capital stock and rents it to the firm, rather than holding equity issued by firms as in the baseline. Capital-producing firms with investment adjustment costs are kept unchanged.

F.1.1 Firm's Problem

In each period t the representative goods-producing firm rents capital K_{t-1} from the household at the competitive rental rate r_t^K and hires labor H_t at the competitive wage W_t. It produces output

$$Y_t = Z_t K_{t-1}^{\alpha} H_t^{1-\alpha} \tag{F.1}$$

and maximizes static profit $\Pi_t^f = Y_t - r_t^K K_{t-1} - W_t H_t$. With constant returns to scale, equilibrium profits are zero and the first-order conditions give

$$W_t = (1-\alpha)\frac{Y_t}{H_t}, \tag{F.2}$$

$$r_t^K = \alpha \frac{Y_t}{K_{t-1}}. \tag{F.3}$$

F.1.2 Capital Producers' Problem

Competitive capital-producing firms solve exactly the same problem as in the baseline model (Section 4.3). They purchase I_t^g units of the consumption good, install

$$I_t^n = Z_{It}\left[1 - \frac{\phi}{2}\left(\frac{I_t^g}{I_{t-1}^g} - 1\right)^2\right] I_t^g \tag{F.4}$$

units of new capital, and sell it at the shadow price Q_t (Tobin's q), earning profits

$$\Pi_t = Q_t Z_{It}\left[1 - \frac{\phi}{2}\left(\frac{I_t^g}{I_{t-1}^g} - 1\right)^2\right] I_t^g - I_t^g. \tag{F.5}$$

Profit maximization subject to the investment technology yields

$$0 = E_t \left\{ \begin{array}{c} Q_t Z_{It}\left[-\phi\left(\frac{I_t^g}{I_{t-1}^g} - 1\right)\frac{1}{I_{t-1}^g}\right] I_t^g + Q_t Z_{It}\left[1 - \frac{\phi}{2}\left(\frac{I_t^g}{I_{t-1}^g} - 1\right)^2\right] - 1 \\ +\beta \frac{\lambda_{ct+1}}{\lambda_{ct}} Q_{t+1} Z_{It+1} \phi \left(\frac{I_{t+1}^g}{I_t^g} - 1\right)\left(\frac{I_{t+1}^g}{I_t^g}\right)^2 \end{array} \right\}. \tag{F.6}$$

F.1.3 Household's Problem

The representative household has the same KPR preferences as in the baseline model. It directly owns the capital stock K_{t-1}, rents it to the representative firm, receives the profits of capital-producing firms, and chooses consumption C_t, labor H_t, and the future capital stock K_t, purchasing the installed capital I_t^n from capital producers at price Q_t. The household solves

$$\max_{\{C_{t+\tau},H_{t+\tau},I^n_{t+\tau},K_{t+\tau}\}_{\tau=0}^{\infty}} E_0 \sum_{\tau=0}^{\infty} \beta^\tau \left[\ln(C_{t+\tau} - \nu_{c,t+\tau}) - \frac{\vartheta}{1+\nu}(H_{t+\tau})^{1+\nu}\right], \tag{F.7}$$

subject to the flow budget constraint

$$C_t + Q_t I_t^n = W_t H_t + r_t^K K_{t-1} + \Pi_t \tag{F.8}$$

and the capital accumulation equation

$$K_t = I_t^n + (1-\delta)K_{t-1}, \tag{F.9}$$

where the household purchases the installed capital I_t^n produced through the capital producers' technology (F.4) at the price Q_t, and Π_t denotes the profits of capital-producing firms rebated to the household.

Let λ_{ct} be the Lagrange multiplier on (F.8) and Q_t the multiplier (in units of the consumption good) on (F.9). With KPR log utility in consumption, the first-order conditions are:

$$\lambda_{ct} = \frac{1}{C_t - \nu_{ct}}, \tag{F.10}$$

$$W_t = \vartheta(C_t - \nu_{ct})H_t^\nu, \tag{F.11}$$

$$\lambda_{ct} = \beta E_t\{\lambda_{ct+1} R_{t+1}\}, \tag{F.12}$$

where the gross return on capital is

$$R_{t+1} \equiv \frac{r^K_{t+1} + (1-\delta)Q_{t+1}}{Q_t} = \frac{\alpha Z_{t+1}(H_{t+1}/K_t)^{1-\alpha} + (1-\delta)Q_{t+1}}{Q_t}. \tag{F.13}$$

Gross investment I_t^g is chosen by the capital producers, whose optimality condition (F.6) determines the price Q_t of installed capital.

F.1.4 Equilibrium Conditions

The equilibrium consists of sequences for $\{Y_t, C_t, H_t, W_t, K_t, I_t^n, I_t^g, R_t, Q_t, \Pi_t, \lambda_{ct}, Z_t, Z_{It}\}$ satisfying the following conditions:

$$\lambda_{ct} = \beta E_t\{\lambda_{ct+1} R_{t+1}\}, \tag{F.14}$$

$$\lambda_{ct} = \frac{1}{C_t - \nu_{ct}}, \tag{F.15}$$

$$W_t = \vartheta(C_t - \nu_{ct}) H_t^{\nu}, \tag{F.16}$$

$$Y_t = Z_t K_{t-1}^{\alpha} H_t^{1-\alpha}, \tag{F.17}$$

$$W_t = (1-\alpha)\frac{Y_t}{H_t}, \tag{F.18}$$

$$R_t = \frac{1}{Q_{t-1}} \alpha\, Z_t \left(\frac{H_t}{K_{t-1}}\right)^{1-\alpha} + \frac{(1-\delta)}{Q_{t-1}} Q_t, \tag{F.19}$$

$$I_t^n = Z_{It}\left[1 - \frac{\phi}{2}\left(\frac{I_t^g}{I_{t-1}^g} - 1\right)^2\right] I_t^g, \tag{F.20}$$

$$K_t = I_t^n + (1-\delta)K_{t-1}, \tag{F.21}$$

$$\Pi_t = Q_t Z_{It}\left[1 - \frac{\phi}{2}\left(\frac{I_t^g}{I_{t-1}^g} - 1\right)^2\right] I_t^g - I_t^g, \tag{F.22}$$

$$0 = E_t\left\{\begin{array}{c} Q_t Z_{It}\left[-\phi\left(\frac{I_t^g}{I_{t-1}^g} - 1\right)\frac{1}{I_{t-1}^g}\right] I_t^g + Q_t Z_{It}\left[1 - \frac{\phi}{2}\left(\frac{I_t^g}{I_{t-1}^g} - 1\right)^2\right] - 1 \\ +\beta \frac{\lambda_{ct+1}}{\lambda_{ct}} Q_{t+1} Z_{It+1} \phi\left(\frac{I_{t+1}^g}{I_t^g} - 1\right)\left(\frac{I_{t+1}^g}{I_t^g}\right)^2 \end{array}\right\}, \tag{F.23}$$

$$Y_t = C_t + I_t^g, \tag{F.24}$$

$$\log Z_t = \rho_z \log Z_{t-1} + \varepsilon_t^z, \tag{F.25}$$

$$\ln(Z_{It}) = \rho_I \ln(Z_{It-1}) + \varepsilon_{It}, \tag{F.26}$$

$$\nu_{ct} = \rho_\nu \nu_{ct-1} + \varepsilon_t^{\nu}. \tag{F.27}$$

Equations (F.14)–(F.16) are the household's first-order conditions. Under KPR preferences, equation (F.15) gives $\lambda_{ct} = 1/(C_t - \nu_{ct})$ and equation (F.16) gives $W_t = \vartheta(C_t - \nu_{ct})H_t^{\nu}$, so a wealth effect on labor is present. Equation (F.27) is the consumption preference shock process. Equations (F.17)–(F.19) describe goods production and factor returns. Equations (F.20)–(F.22) govern investment and capital accumulation. Equation (F.23) is the capital producers' optimality condition (Tobin's q equation). Equations (F.24)–(F.25) are the goods market clearing condition and the TFP process, respectively. Equation (F.26) is the investment-efficiency shock process.

The household budget constraint (F.8) is not listed among the equilibrium conditions because it is redundant: by Walras' law, goods-market clearing (F.24) together with the factor-income identity $W_t H_t + r_t^K K_{t-1} = Y_t$ (which follows from CRS via equations (F.17)–(F.18)) implies $C_t + I_t^g = W_t H_t + r_t^K K_{t-1} + \Pi_t = W_t H_t + r_t^K K_{t-1}$, so the budget constraint forces $\Pi_t = 0$. This holds exactly only in the non-stochastic steady state, where $Q_t = 1$ and equation (F.22) confirms $\Pi = 0$; away from steady state, the household budget is simply dropped from the system as the redundant equation by Walras' law.

F.2 Equivalence with the Baseline Real Model

We show that the equilibrium conditions of the RBC model (Section F.1.4) are equivalent to those of the baseline real model (Appendix C.10) when $\bar{\omega}_t = \bar{\bar{\omega}}_t = 1$ for all t, in the sense that each RBC condition (F.14)–(F.27) coincides with the corresponding baseline condition once the financial variables absent from the RBC model are eliminated by substitution. We derive a key limit first and then proceed in three steps.

A key limit. The TFP prefactor $\Phi(\bar{\bar{\omega}})$ defined in equation (42) satisfies $\Phi(\bar{\bar{\omega}}) \to 1$ as $\bar{\bar{\omega}} \to 1$. To see this, note that for any shape parameter $\eta > 0$ the Beta c.d.f. satisfies

$$1 - \mu_{\eta,\eta_2}(\omega) \ \sim\ \frac{(1-\omega)^{\eta_2}}{B(\eta,\eta_2)\,\eta_2} \qquad \text{as } \omega \to 1,$$

so both survival functions in Φ share the same $(1-\omega)^{\eta_2}$ tail and their ratio converges to the ratio of Beta normalisation constants:

$$\lim_{\bar{\bar{\omega}}\to 1} c_\alpha \frac{1-\mu_{\eta_1+1,\eta_2}(\bar{\bar{\omega}})}{1-\mu_{\eta_1,\eta_2}(\bar{\bar{\omega}})} \ =\ c_\alpha \cdot \frac{B(\eta_1,\eta_2)}{B(\eta_1+1,\eta_2)} \ =\ c_\alpha \cdot \frac{1}{c_\alpha} \ =\ 1, \tag{F.28}$$

and therefore $\Phi(\bar{\bar{\omega}}) \to 1$.

Step 1: Financial variables degenerate. At $\bar{\omega}_t = \bar{\bar{\omega}}_t = 1$ the CDF satisfies $\mu_{\eta_1,\eta_2}(1) = 1$, so the measures of the three firm segments collapse:

- Equation (C.62): $B^H_t = 0$.
- Equation (C.67): $D_t = \big(\mu(1) - \mu(1)\big)A_t + \frac{\mu(1)-\mu(1)}{1-\mu(1)}\theta B_t = 0$.
- Equations (C.61)–(C.60): $B^G_t = 0,\ T_t = 0$.
- Equation (C.63): $\Xi_t = \xi D_{t-1} = 0$.
- Equations (C.65)–(C.66): $L_t = B_t = \mu(1)A_t = A_t$.

For B^{tot}_t, equation (C.68) reads $B^{tot}_t = (1-\mu(\bar{\bar{\omega}}_t))A_t + \frac{1-\mu(\bar{\bar{\omega}}_t)}{1-\mu(\bar{\omega}_t)}B_t$. Since $\bar{\omega}_t = \bar{\bar{\omega}}_t$, the second ratio equals unity for every $\bar{\omega}_t < 1$; taking the limit as $\bar{\omega}_t \to 1$ (and using $B_t = A_t$) gives

$$B^{tot}_t = A_t. \tag{F.29}$$

Combined with equation (C.54) this yields $A_t = Q_t K_t$.

Substituting $D_{t-1} = 0$, $B^{tot}_{t-1} = A_{t-1}$, and $\mu(\bar{\bar{\omega}}_{t-1}) - \mu(\bar{\omega}_{t-1}) = 0$ into equation (C.69):

$$R^A_t A_{t-1} = A_{t-1} R_t \ \Longrightarrow\ R^A_t = R_t. \tag{F.30}$$

Step 2: Real conditions. Table A.1 lists each RBC equilibrium condition alongside the Appendix B condition(s) from which it follows at $\bar{\omega}_t = \bar{\bar{\omega}}_t = 1$.

Step 3: Remaining baseline conditions are vacuous or determine financial variables only.

- *Government block* ((C.60)–(C.63)): shown in Step 1 to give $T_t = \Xi_t = B^G_t = B^H_t = 0$, trivially consistent.
- *Aggregation* ((C.65)–(C.68) and (C.69)): used in Step 1 to derive $B^{tot}_t = A_t = Q_t K_t$ and $R^A_t = R_t$, both already absorbed into conditions (F.14) and (F.19).

Table A.1: Correspondence between the RBC model's equilibrium conditions (Section F.1.4) and the baseline model's conditions (Appendix C.10) at $\bar{\omega}_t = \bar{\bar{\omega}}_t = 1$.

RBC eq.	Condition	Baseline source
(F.14)	$\lambda_{ct} = \beta E_t\{\lambda_{ct+1} R_{t+1}\}$	Substitute $R^A_t = R_t$ from (F.30) into (C.49).
(F.15)	$\lambda_{ct} = 1/(C_t - \nu_{ct})$	Equations (C.50)–(C.52): both give $\lambda_{ct} = \beta E_t\{\lambda_{ct+1} R^B_t\} = 1/(C_t - \nu_{ct})$.
(F.16)	$W_t = \vartheta(C_t - \nu_{ct})H^\nu_t$	Identical to (C.51).
(F.17)	$Y_t = Z_t K^\alpha_{t-1} H^{1-\alpha}_t$	Equation (C.53) with the limit (F.28) at $\bar{\bar{\omega}}_t = 1$.
(F.18)	$W_t = (1-\alpha)Y_t/H_t$	Identical to (C.55).
(F.19)	$R_t = \frac{1}{Q_{t-1}}\alpha Z_t\left(\frac{H_t}{K_{t-1}}\right)^{1-\alpha} + (1-\delta)\frac{Q_t}{Q_{t-1}}$	Equation (C.56); substitute (C.53) with $\Phi_t = 1$ (from (F.28)) to expand Y_t.
(F.20)	$I^n_t = Z_{It}[1 - \frac{\phi}{2}(\cdot)^2]I^g_t$	Identical to (C.57).
(F.21)	$K_t = I^n_t + (1-\delta)K_{t-1}$	Identical to (C.58).
(F.22)	$\Pi_t = Q_t Z_{It}[1 - \frac{\phi}{2}(\cdot)^2]I^g_t - I^g_t$	Identical to (C.59).
(F.23)	Tobin's q equation	Identical to (C.73).
(F.24)	$Y_t = C_t + I^g_t$	Identical to (C.70).
(F.25)	$\log Z_t = \rho_z \log Z_{t-1} + \varepsilon^z_t$	Identical to (C.74).
(F.26)	$\ln(Z_{It}) = \rho_I \ln(Z_{It-1}) + \varepsilon_{It}$	Identical to (C.75).
(F.27)	$\nu_{ct} = \rho_\nu \nu_{ct-1} + \varepsilon^\nu_t$	Identical to (C.76).

Note: "Identical" means the equation is structurally unchanged between the two models. Equations (C.60)–(C.63) (government block), (C.64) (lower cutoff), (C.67)–(C.68) (aggregation), (C.69) (equity return), (C.71)–(C.72) (cutoffs and loan rate), (C.76)–(C.77) (preference shock, diversion shock, and diversion scaling) are used only to determine financial variables absent from the RBC model or become vacuously satisfied at $\bar{\omega}_t = \bar{\bar{\omega}}_t = 1$; see Step 3 below.

- *Bond Euler equation* (C.52): $\lambda_{ct} = \beta E_t\{\lambda_{ct+1} R_t^B\}$ pins the financial variable R_t^B given the real allocation (which determines λ_{ct} through (F.15)).
- *Lower cutoff* (C.64): at $\bar{\omega}_t = 1$, the left-hand side equals $E_t[\beta \frac{\lambda_{ct+1}}{\lambda_{ct}} R_{t+1}] = 1$ (by (F.14)), so the condition becomes $\lambda_{ct} = \beta E_t\{\lambda_{ct+1}(R_t^B - \xi)\}$. This must hold simultaneously with the bond Euler (C.52), which gives $\lambda_{ct} = \beta E_t\{\lambda_{ct+1} R_t^B\}$. Subtracting yields $\xi\,\beta E_t\{\lambda_{ct+1}\} = 0$; since $\lambda_{ct+1} > 0$, this requires $\xi = 0$. The full RBC equivalence therefore requires both $\bar{\omega}_t = \bar{\bar{\omega}}_t = 1$ and $\xi = 0$. Under these conditions the lower cutoff degenerates to an identity and imposes no further restriction on real quantities.
- *Upper cutoff and loan rate* ((C.71)–(C.72)): at $\bar{\omega}_t = \bar{\bar{\omega}}_t = 1$ these conditions degenerate to equalities between ρ_t and $R_t^B - \xi$, jointly pinning the financial variables R_t^B and ρ_t without restricting real quantities.

Hence every condition determining a real quantity in the RBC model corresponds to a baseline condition, and the baseline conditions that do not appear in the RBC model either become trivially satisfied or serve only to determine financial variables. The equivalence holds when $\bar{\omega}_t = \bar{\bar{\omega}}_t = 1$ and $\xi = 0$; the haircut parameter ξ must vanish because the bond Euler (C.52) and the lower cutoff (C.64) are jointly consistent at $\bar{\omega}_t = 1$ only in that case. □

F.3 Steady-State Conditions

We solve for the non-stochastic steady state sequentially. For every variable that also appears in the baseline model we verify that the steady-state value coincides with the expression in Appendix D evaluated at $\bar{\omega} = \bar{\bar{\omega}} = 1$; the equivalence of equilibrium conditions established in Section F.2 implies that any verification of a dynamic condition in steady state automatically checks the corresponding Appendix C condition.

Sequential derivation. From equation (F.25),

$$Z = 1. \tag{F.31}$$

From equation (F.26),

$$Z_I = 1. \tag{F.32}$$

From equation (F.23) with $Z_I = 1$,

$$Q = 1. \tag{F.33}$$

Matches Appendix D (D.5)–(D.8)*.*
From equation (F.14),

$$R = \frac{1}{\beta}. \tag{F.34}$$

Matches Appendix D (D.4) *(the baseline gives $R^A = 1/\beta$; here $R_t^A = R_t$ by* (F.30)*).*
Fix $H = H^{ss} = 1$ and choose $\vartheta = W/(C\,H^{\nu})$ to support this normalisation. From equation (F.19) in steady state ($Z = Q = 1$),

$$R = \alpha\left(\frac{H}{K}\right)^{1-\alpha} + (1-\delta), \tag{F.35}$$

so

$$K = H\left[\frac{R-(1-\delta)}{\alpha}\right]^{1/(\alpha-1)}. \tag{F.36}$$

Matches Appendix D (D.9): the prefactor Φ in (D.9) equals 1 at $\bar{\bar{\omega}} = 1$ by (F.28).
From equation (F.17),

$$Y = K^{\alpha} H^{1-\alpha}. \tag{F.37}$$

Matches Appendix D (D.11) at $\bar{\bar{\omega}} = 1$ by (F.28).
From equation (F.18),

$$W = (1-\alpha)\frac{Y}{H}. \tag{F.38}$$

Matches Appendix D (D.12).
From equations (F.20)–(F.21) in steady state,

$$I^n = \delta K, \qquad I^g = I^n. \tag{F.39}$$

Matches Appendix D (D.13)–(D.14).
From equation (F.24),

$$C = Y - I^g. \tag{F.40}$$

Matches Appendix D (D.15).
From equation (F.15),

$$\lambda_c = \frac{1}{C}. \tag{F.41}$$

Matches Appendix D (D.16).
From equation (F.22) with $Q = 1$ and $I^g = I^n$,

$$\Pi = 0. \tag{F.42}$$

Matches Appendix D (D.25).

G Model Without Strategic Default

This appendix develops a variant of the baseline model in which the outside option is unavailable. Without an outside option, no firm has an incentive to divert borrowed funds, so strategic default is impossible. The two-cutoff structure of the baseline model—which is needed to demarcate lenders, strategic defaulters, and producers—collapses to a single cutoff that separates lenders from firms that borrow and produce. The resulting model is structurally simpler than the baseline and provides a useful intermediate case between the full model and the frictionless canonical RBC model of Appendix F: financial intermediation and capital misallocation are still present, but the default margin is shut down.

The appendix proceeds as follows. Section G.1 describes which elements of the baseline model change and which are preserved. Section G.2 restates the firm problems. Section G.3 derives the single cutoff condition and the equilibrium loan rate. Section G.5 lists all equilibrium conditions. Section G.6 derives the non-stochastic steady state.

G.1 Changes Relative to the Baseline Model

The following building blocks of the baseline model are *unchanged*: the household problem (Section 4.1 and Appendix C.1); the capital-producing firm problem (Section 4.3 and Appendix C.2); the aggregate technology shock process (equation (5)); the investment-specific technology process (equation (22)); and the consumption preference shock process (equation (3)).

The following elements are *removed or simplified*:

No outside option. Firms cannot divert borrowed funds into the government bond. Parameters ξ (the haircut on the outside-option return) and θ (the average diversion share) play no role. The firm-specific diversion function $\Theta_t(\omega)$, its scaling factor F_t, and the aggregate diversion D_t are all undefined. Because no funds are ever diverted, the haircut rebate to households is $\Xi_t = 0$ for all t.

Two firm segments instead of three. Since strategic default is impossible, the inter-firm intermediation market sorts firms into only two groups depending on the realization of idiosyncratic productivity ω:

1. Firms with $\omega < \overline{\omega}_t$ that lend their equity in the inter-firm market.

2. Firms with $\omega \geq \overline{\omega}_t$ that borrow and produce.

The defaulter segment $\overline{\omega}_t \leq \omega \leq \overline{\overline{\omega}}_t$ of the baseline model is absent. The upper cutoff $\overline{\overline{\omega}}_t$ does not arise.

Simplified government sector. Because no firm uses the government bond as an outside option, aggregate firm holdings of government bonds are $D_t = 0$. Retaining the assumption from the baseline that households do not hold government bonds, $B_t^H = 0$, the government budget constraint (equation (27)) implies $B_t^G = 0$ and hence $T_t = 0$ for all t. The government sector drops out of the model entirely. The risk-free rate R_t^B nonetheless remains well defined as the shadow rate implied by the household's bond Euler equation.

Loan rate. Because all inter-firm loans are repaid with certainty, the inter-firm loan market is free of default risk. To preserve the financial friction from the baseline model, we retain $\xi > 0$ as a parameter that separates lenders and borrowers via a single cutoff $\omega = \overline{\omega}_t$, defined by condition (31) as in the baseline. In the present specification the default margin is switched off, so the second cutoff is not operational; this design allows us to isolate the role of the default margin. In the version of the model without an outside option, $\xi > 0$ functions solely as an intermediation-cost parameter that governs the intermediation spread in the inter-firm lending market, rather than also acting as a haircut on the outside option as in the baseline specification. Competition in the inter-firm market drives

$$\rho_t = R_t^B - \xi. \tag{G.1}$$

Keeping ξ at its baseline-calibrated value facilitates direct comparison across models.

The household budget constraint simplifies to

$$C_t + A_t = R_t^A A_{t-1} + W_t H_t + \Pi_t, \tag{G.2}$$

dropping the terms B_t^H, $R_{t-1}^B B_{t-1}^H$, T_t, and Ξ_t that appear in equation (2) of the baseline.

G.2 Firm Problems

G.2.1 Firms Choosing to Produce ($\omega \geq \overline{\omega}_t$)

The problem of a producing firm is identical to that in Section 4.2.1. Each producing firm finances its capital purchase by combining household equity $a_t(\omega) = a_t$ with an inter-firm loan $b_t(\omega) = b_t$, so that total financing is $b_t^{tot}(\omega) = a_t + b_t = Q_t k_t(\omega)$. The first-order conditions for labor and capital are equations (12) and (13) of the baseline model, reproduced here for convenience:

$$W_t = (1-\alpha)\frac{Y_t}{H_t}, \tag{G.3}$$

$$R_{t+1}(\omega) = \frac{\kappa_{t+1}}{Q_t}\,\omega + \frac{(1-\delta)}{Q_t}Q_{t+1}, \tag{G.4}$$

where $\kappa_{t+1} \equiv \alpha Z_{t+1}(H_{t+1}/K_t)^{1-\alpha}$ is the common factor defined in equation (13) of Section 4.2.1. The return paid to households by a producing firm is

$$R_{t+1}^A(\omega) = \frac{R_{t+1}(\omega)(a_t + b_t) - \rho_t\, b_t}{a_t}, \tag{G.5}$$

identical to equation (15) of the baseline model.

G.2.2 Firms Choosing to Lend ($\omega < \overline{\omega}_t$)

The problem of a lending firm simplifies substantially relative to Section 4.2.3. Because all borrowers repay at rate ρ_t, the lending firm faces no default loss. Its problem is

$$\max_{l_t(\omega)|\omega} E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\Big(\rho_t\, l_t(\omega) - R_{t+1}^A(\omega)\, a_t(\omega)\Big)\right], \tag{G.6}$$

subject to $l_t(\omega) \leq a_t(\omega)$. Since revenues are increasing in $l_t(\omega)$, the constraint holds with equality: $l_t(\omega) = a_t(\omega)$. The zero-profit condition that holds under every state of nature is

$$R_{t+1}^A(\omega) = \rho_t \qquad \text{for all } \omega < \overline{\omega}_t. \tag{G.7}$$

Compare this to the baseline equation (20): the two terms involving the fraction of defaulting borrowers vanish, leaving a simple pass-through of the loan rate to households.

G.3 The Single Cutoff Point and the Loan Rate

With only lenders and producers, a single cutoff $\overline{\omega}_t$ characterizes the equilibrium sorting of firms. A firm with productivity exactly at $\overline{\omega}_t$ is indifferent between lending—earning ρ_t on its equity—and borrowing to produce—earning $R_{t+1}(\overline{\omega}_t)$ on total funds net of loan repayments. Formally,

$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\,\rho_t\right] = E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(R_{t+1}(\overline{\omega}_t)\frac{a_t+b_t}{a_t} - \rho_t\frac{b_t}{a_t}\right)\right].$$

Rearranging and collecting terms in ρ_t:

$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\,\rho_t\right] = E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\,R_{t+1}(\overline{\omega}_t)\right]. \tag{G.8}$$

Substituting $\rho_t = R_t^B - \xi$ from equation (G.1) and expanding $R_{t+1}(\overline{\omega}_t)$ using equation (13), the *single cutoff condition* is

$$\boxed{E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(\frac{\kappa_{t+1}}{Q_t}\,\overline{\omega}_t + \frac{(1-\delta)}{Q_t}Q_{t+1}\right)\right] = E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\,(R_t^B-\xi)\right].} \tag{G.9}$$

This condition is *identical* to the baseline lower-cutoff condition (31): ξ enters in exactly the same way in both models, which preserves a clean mapping between the two calibrations.

The following proposition establishes that equation (G.9) together with (G.1) is sufficient to support the two-segment equilibrium.

Proposition 4. *Suppose $R_{t+1}(\omega)$ is strictly increasing in ω and $\rho_t = R_t^B - \xi$ with $\xi \geq 0$. Assuming an interior solution exists (i.e., there is a $\overline{\omega}_t \in (0,1)$ satisfying equation (G.9)), then equation (G.9) defines a unique cutoff $\overline{\omega}_t$ such that*

1. *firms with $\omega < \overline{\omega}_t$ strictly prefer to lend;*
2. *firms with $\omega \geq \overline{\omega}_t$ weakly prefer to borrow and produce.*

Proof. Equation (G.8) equates the expected discounted return from lending, ρ_t (independent of ω), with the expected discounted return from producing at $\overline{\omega}_t$. Since $R_{t+1}(\omega)$ is strictly increasing in ω, the return from producing is strictly higher than the return from lending for all $\omega > \overline{\omega}_t$ and strictly lower for all $\omega < \overline{\omega}_t$. Because there is no outside option, no firm defaults strategically; the only alternatives are lending and producing, and the sorting follows directly from the monotonicity of $R_{t+1}(\omega)$. □

Remark. The baseline model requires two conditions (equations (31), (32)) and a verification that no firm deviates to the outside option (Proposition 3). Here a single condition (G.9) suffices because the outside option is unavailable and the only relevant comparison is lending versus producing.

G.4 Aggregation and Equilibrium Definition

With two firm segments instead of three, the aggregation of individual decisions into aggregate variables simplifies relative to Appendix C.3. The steps below follow the same procedure; we highlight the equations that change.

Inter-firm loan market clearing. A mass $\mu_{\eta_1,\eta_2}(\overline{\omega}_t)$ of firms lend and a mass $1-\mu_{\eta_1,\eta_2}(\overline{\omega}_t)$ borrow. The aggregate supply and demand of inter-firm loans are

$$L_t = \int_0^{\overline{\omega}_t} l_t(\omega)\, \mu'(\omega)\, \mathrm{d}\omega = \mu_{\eta_1,\eta_2}(\overline{\omega}_t)\, A_t, \tag{G.10}$$

$$B_t = \int_{\overline{\omega}_t}^{1} b_t(\omega)\, \mu'(\omega)\, \mathrm{d}\omega, \tag{G.11}$$

and market clearing requires $L_t = B_t$. Individual borrowing is therefore $b_t = B_t/\Big(1 - \mu_{\eta_1,\eta_2}(\overline{\omega}_t)\Big)$.

Aggregate total borrowing. Every producing firm uses total funds $b_t^{tot}(\omega) = a_t + b_t$. Integrating over the producing segment:

$$B_t^{tot} = \Big(1 - \mu_{\eta_1,\eta_2}(\overline{\omega}_t)\Big) A_t + B_t. \tag{G.12}$$

Compare this with the baseline equation (C.19): when $\overline{\overline{\omega}}_t = \overline{\omega}_t$ the two expressions coincide, confirming that the no-default model is the special case in which the defaulter segment has zero mass.

Aggregate equity return. Using equations (G.7) and (G.5) and integrating over both segments:

$$\begin{aligned} R_t^A A_{t-1} &= \int_0^{\overline{\omega}_{t-1}} \rho_{t-1}\, \mu'(\omega)\, \mathrm{d}\omega\, A_{t-1} + \int_{\overline{\omega}_{t-1}}^{1} \Big[R_t(\omega)(A_{t-1} + b_{t-1}) - \rho_{t-1} b_{t-1}\Big] \mu'(\omega)\, \mathrm{d}\omega \\ &= \rho_{t-1} L_{t-1} + R_t B_{t-1}^{tot} - \rho_{t-1} B_{t-1} \\ &= R_t\, B_{t-1}^{tot}, \end{aligned} \tag{G.13}$$

where the last equality uses loan market clearing $L_{t-1} = B_{t-1}$. This is the simplification of the baseline equation (C.29) obtained by setting $D_{t-1} = 0$ and $\theta = 0$: without diversion losses, all capital returns flow through to households intact.

Aggregate output and average return on capital. With $\omega_i \geq \overline{\omega}_t$ for every producing firm, output and capital returns aggregate to the same Beta-distribution expressions as in

Appendix C, replacing the baseline $\overline{\overline{\omega}}_t$ with $\overline{\omega}_t$ throughout:

$$Y_t = c_\alpha \frac{1-\mu_{\eta_1+1,\eta_2}(\overline{\omega}_{t-1})}{1-\mu_{\eta_1,\eta_2}(\overline{\omega}_{t-1})} \cdot Z_t K_{t-1}^{\alpha} H_t^{1-\alpha}, \tag{G.14}$$

$$R_t = \frac{\alpha\, Y_t}{Q_{t-1}\, K_{t-1}} + \frac{(1-\delta)}{Q_{t-1}}\, Q_t. \tag{G.15}$$

A competitive equilibrium of the no-default model is an allocation

$$\Big\{C_t,\, H_t,\, Y_t,\, K_t,\, B_t^{tot},\, I_t^n,\, I_t^g,\, \Pi_t,\, \overline{\omega}_t,\, Z_t,\, Z_{It},\, \nu_{ct},\, L_t,\, A_t,\, B_t\Big\}_{t=0}^{\infty}$$

together with a sequence of prices $\{\lambda_{ct},\, R_t^A,\, W_t,\, R_t^B,\, Q_t,\, R_t,\, \rho_t\}_{t=0}^{\infty}$ satisfying the 22 equations listed in the next subsection, given initial conditions K_0, I_0^g, B_0^{tot}, $\overline{\omega}_0$, A_0, B_0, Z_0, $Z_{I,0}$, ν_{c0}, R_0^B, Q_0, and exogenous processes $\{\varepsilon_t^z,\, \varepsilon_{It},\, \varepsilon_{\nu t}\}$.

G.5 Equilibrium Conditions

This subsection lists the 22 equilibrium conditions of the no-default model, identifying the source of each. Equations that are *identical* to their baseline counterparts in Appendix C.10 are noted as such; the remaining equations are new or simplified.

Asset Euler equation. Identical to baseline equation (C.49):

$$\lambda_{ct} = \beta\, E_t\Big\{\lambda_{ct+1}\, R_{t+1}^A\Big\}. \tag{G.16}$$

Consumption Euler equation. Identical to baseline equation (C.50):

$$\beta\, E_t\Big\{\lambda_{ct+1}\, R_t^B\Big\} = \frac{1}{C_t - \nu_{ct}}. \tag{G.17}$$

Labor supply. Identical to baseline equation (C.51):

$$W_t = \vartheta\,(C_t - \nu_{ct})\, H_t^{\nu}. \tag{G.18}$$

Bond Euler equation. Identical to baseline equation (C.52):

$$\lambda_{ct} = \beta\, E_t\Big\{\lambda_{ct+1}\, R_t^B\Big\}. \tag{G.19}$$

Aggregate output. Beta-distribution form of equation (G.14). The single cutoff $\overline{\omega}_{t-1}$ replaces the baseline $\overline{\overline{\omega}}_{t-1}$:

$$Y_t = c_\alpha \frac{1-\mu_{\eta_1+1,\eta_2}(\overline{\omega}_{t-1})}{1-\mu_{\eta_1,\eta_2}(\overline{\omega}_{t-1})} \cdot Z_t\, K_{t-1}^{\alpha}\, H_t^{1-\alpha}. \tag{G.20}$$

Capital financing. Identical to baseline equation (C.54):

$$B_t^{tot} = Q_t\, K_t. \tag{G.21}$$

Labor demand. Identical to baseline equation (C.55):

$$W_t = (1-\alpha)\,\frac{Y_t}{H_t}. \tag{G.22}$$

Average return on capital. Equation (G.15): the $E[\omega]$ factors cancel between output and average return, giving the same simplified form as baseline equation (C.56):

$$R_t = \frac{\alpha\, Y_t}{Q_{t-1}\, K_{t-1}} + \frac{(1-\delta)}{Q_{t-1}}\, Q_t. \tag{G.23}$$

Investment supply. Identical to baseline equation (C.57):

$$I_t^n = Z_{It}\left[1 - \frac{\phi}{2}\left(\frac{I_t^g}{I_{t-1}^g} - 1\right)^2\right] I_t^g. \tag{G.24}$$

Capital accumulation. Identical to baseline equation (C.58):

$$K_t = I_t^n + (1-\delta)\, K_{t-1}. \tag{G.25}$$

Capital-firm profits. Identical to baseline equation (C.59):

$$\Pi_t = Q_t\, Z_{It}\left[1 - \frac{\phi}{2}\left(\frac{I_t^g}{I_{t-1}^g} - 1\right)^2\right] I_t^g - I_t^g. \tag{G.26}$$

Capital-firm first-order condition (Tobin's q). Identical to baseline equation (C.73):

$$0 = E_t\left\{\begin{array}{c} Q_t Z_{It}\left[-\phi\left(\frac{I_t^g}{I_{t-1}^g} - 1\right)\frac{1}{I_{t-1}^g}\right] I_t^g + Q_t Z_{It}\left[1 - \frac{\phi}{2}\left(\frac{I_t^g}{I_{t-1}^g} - 1\right)^2\right] - 1 \\ +\beta\frac{\lambda_{ct+1}}{\lambda_{ct}} Q_{t+1} Z_{It+1}\,\phi\left(\frac{I_{t+1}^g}{I_t^g} - 1\right)\left(\frac{I_{t+1}^g}{I_t^g}\right)^2 \end{array}\right\}. \tag{G.27}$$

Single cutoff condition. Equation (G.9) from Section G.3, reproduced here. The expected discounted return from producing at the marginal lending firm equals the inter-firm lending rate $\rho_t = R_t^B - \xi$. This replaces the two baseline conditions (C.64) and (C.71)–(C.72):

$$E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left(\frac{\kappa_{t+1}}{Q_t}\,\overline{\omega}_t + \frac{(1-\delta)\, Q_{t+1}}{Q_t}\right)\right] = E_t\left[\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\,(R_t^B - \xi)\right]. \tag{G.28}$$

Aggregate lending. Equation (G.10), with $\overline{\omega}_t$ in place of the baseline $\overline{\omega}_t$ (same label, but here it is the *only* cutoff):

$$L_t = \mu_{\eta_1,\eta_2}(\overline{\omega}_t)\, A_t. \tag{G.29}$$

Loan market clearing. Identical to baseline equation (C.66):

$$B_t = L_t. \tag{G.30}$$

Aggregate total borrowing. Equation (G.12), the two-segment simplification of the baseline equation (C.68):

$$B_t^{tot} = \left(1 - \mu_{\eta_1,\eta_2}(\overline{\omega}_t)\right) A_t + B_t. \tag{G.31}$$

Equity return market clearing. Equation (G.13), the simplification of the baseline equation (C.69) with $D_{t-1} = 0$ and $\theta = 0$. All capital returns flow to households without any diversion losses:

$$R_t^A A_{t-1} = R_t B_{t-1}^{tot}. \tag{G.32}$$

Goods market clearing. Identical to baseline equation (C.70):

$$Y_t = C_t + I_t^g. \tag{G.33}$$

Loan rate. Equation (G.1):

$$\rho_t = R_t^B - \xi. \tag{G.34}$$

Aggregate TFP process. Identical to baseline equation (C.74):

$$\log Z_t = \rho_z \log Z_{t-1} + \varepsilon_t^z. \tag{G.35}$$

Investment-specific technology process. Identical to baseline equation (C.75):

$$\ln(Z_{It}) = \rho_I \ln(Z_{It-1}) + \varepsilon_{It}. \tag{G.36}$$

Consumption demand shock process. Identical to baseline equation (C.76):

$$\nu_{ct} = \rho_\nu \nu_{ct-1} + \varepsilon_{\nu t}. \tag{G.37}$$

Table A.2 summarizes the 22 equations and their relationship to the baseline model.

G.6 Steady-State Conditions

We solve for the non-stochastic steady state sequentially. The strategy mirrors Appendix F.3 for the canonical RBC model and Appendix D for the baseline model; it is considerably simpler than for the baseline because the government sector is absent and the loan rate is pinned directly by the risk-free rate.

The 22 endogenous variables in steady state are

$$\lambda_c,\ R^A,\ C,\ H,\ W,\ R^B,\ Y,\ \overline{\omega},\ Z,\ Z_I,\ \nu_c,\ K,\ B^{tot},\ Q,\ R,\ I^n,\ I^g,\ \Pi,\ L,\ A,\ B,\ \rho.$$

The solution uses one free initial guess: the single cutoff $\overline{\omega}$.

Step 1: Exogenous variables and prices that do not depend on $\overline{\omega}$.

From equations (G.35), (G.36), (G.37):

$$Z = 1, \qquad Z_I = 1, \qquad \nu_c = 0. \tag{G.38}$$

From equation (G.27) with $Z_I = 1$ and no investment growth:

$$Q = 1. \tag{G.39}$$

Table A.2: Equilibrium conditions of the no-default model and their relation to the baseline model (Appendix C.10)

#	Condition	No-default eq.	Relation to baseline model
1	Asset Euler	(G.16)	Identical to (C.49)
2	Consumption Euler	(G.17)	Identical to (C.50)
3	Labor supply	(G.18)	Identical to (C.51)
4	Bond Euler	(G.19)	Identical to (C.52)
5	Aggregate output	(G.20)	Baseline (C.53) with $\overline{\overline{\omega}}_{t-1} \to \overline{\omega}_{t-1}$
6	Capital financing	(G.21)	Identical to (C.54)
7	Labor demand	(G.22)	Identical to (C.55)
8	Average return on capital	(G.23)	Baseline (C.56) with $\overline{\overline{\omega}}_t \to \overline{\omega}_t$
9	Investment supply	(G.24)	Identical to (C.57)
10	Capital accumulation	(G.25)	Identical to (C.58)
11	Capital-firm profits	(G.26)	Identical to (C.59)
12	Tobin's q	(G.27)	Identical to (C.73)
13	Single cutoff	(G.28)	Replaces (C.64), (C.71), (C.72); identical to (C.64)
14	Aggregate lending	(G.29)	Baseline (C.65), same form
15	Loan market clearing	(G.30)	Identical to (C.66)
16	Aggregate total borrowing	(G.31)	Baseline (C.68) with $\overline{\overline{\omega}}_t \to \overline{\omega}_t$
17	Equity return MC	(G.32)	Baseline (C.69) with $D_{t-1} = 0$, $\theta = 0$
18	Goods market clearing	(G.33)	Identical to (C.70)
19	Loan rate	(G.34)	Simplified from (C.71); same ξ, no default term
20	TFP process	(G.35)	Identical to (C.74)
21	IST process	(G.36)	Identical to (C.75)
22	Consumption shock	(G.37)	Identical to (C.76)

Note: The baseline model has 29 endogenous variables and 29 equations (Appendix C.10). Removing the outside option eliminates seven variables ($\overline{\overline{\omega}}_t$, F_t, D_t, Ξ_t, T_t, B_t^G, B_t^H) and replaces three cutoff/loan-rate conditions by two simpler ones (the single cutoff (G.28) and the no-default loan rate (G.34)), yielding 22 equations in 22 unknowns.

From the bond Euler equation (G.19) in steady state:

$$R^B = \frac{1}{\beta}. \tag{G.40}$$

From equation (G.34):

$$\rho = R^B - \xi = \frac{1}{\beta} - \xi. \tag{G.41}$$

Step 2: Real allocation given $\overline{\omega}$.

Because no default occurs, the equity return market clearing condition (G.32) in steady state gives $R^A A = R B^{tot}$. Since the asset Euler equation (G.16) implies $R^A = 1/\beta$ and the financial aggregation (shown in Step 3 below) implies $A = B^{tot}$, we obtain

$$R = R^A = \frac{1}{\beta}. \tag{G.42}$$

This result also obtains in the baseline model (Appendix D, equation (D.4)) and the canonical RBC model (Appendix F.3, equation (F.34)).

With $R = 1/\beta$, $Q = 1$, and $Z = 1$, the simplified equation (G.23) in steady state gives

$$\frac{1}{\beta} = \frac{\alpha Y}{K} + (1-\delta) = \alpha\,\Phi^{nd}\left(\frac{H}{K}\right)^{1-\alpha} + (1-\delta),$$

where $\Phi^{nd} = \Phi^{nd}(\overline{\omega})$ is the no-default TFP prefactor (equation (G.14) with $Z = 1$, $H = 1$). Setting $H = H^{ss} = 1$ (choosing $\vartheta = W/(C\,H^{\nu})$ to support this normalization) and solving for K:

$$K = \left[\frac{\alpha\,\Phi^{nd}}{1/\beta - (1-\delta)}\right]^{1/(1-\alpha)}. \tag{G.43}$$

Compare with Appendix D equation (D.9): the formula is identical in structure, with $\Phi^{nd}(\overline{\omega})$ in place of $\Phi(\overline{\overline{\omega}})$.

The remaining real variables follow directly:

$$B^{tot} = Q\,K = K, \tag{G.44}$$

$$Y = \Phi^{nd}\,K^{\alpha} H^{1-\alpha}, \tag{G.45}$$

$$W = (1-\alpha)\,\frac{Y}{H}, \tag{G.46}$$

$$I^n = \delta\,K, \tag{G.47}$$

$$I^g = I^n = \delta\,K, \tag{G.48}$$

$$C = Y - I^g, \tag{G.49}$$

$$\lambda_c = \frac{1}{C}, \tag{G.50}$$

$$\Pi = 0. \tag{G.51}$$

Step 3: Financial variables given $\overline{\omega}$.

From the aggregate total borrowing equation (G.31) and the lending and market-clearing equations (G.29)–(G.30):

$$B^{tot} = \Big(1 - \mu_{\eta_1,\eta_2}(\overline{\omega})\Big) A + B,$$
$$B = L = \mu_{\eta_1,\eta_2}(\overline{\omega})\, A.$$

Substituting the second into the first and using (G.44):

$$K = B^{tot} = \Big(1 - \mu_{\eta_1,\eta_2}(\overline{\omega})\Big) A + \mu_{\eta_1,\eta_2}(\overline{\omega})\, A = A, \tag{G.52}$$

confirming that household equity equals total capital financing, and hence $R^A = R$ as used in Step 2.

Aggregate lending and borrowing are then

$$B = \mu_{\eta_1,\eta_2}(\overline{\omega})\, A = \mu_{\eta_1,\eta_2}(\overline{\omega})\, K, \tag{G.53}$$
$$L = B. \tag{G.54}$$

Step 4: Verifying the cutoff $\overline{\omega}$.

The steady-state cutoff condition follows from equation (G.28) with $Q = 1$, $Z = 1$, and the stochastic discount factor evaluated in steady state. Writing $R_{t+1}(\overline{\omega})$ from equation (13) at $Z = Q = 1$:

$$\kappa\,\overline{\omega} + (1 - \delta) = \rho = R^B - \xi = \frac{1}{\beta} - \xi, \tag{G.55}$$

where $\kappa \equiv \alpha(H/K)^{1-\alpha}$ is the steady-state value of κ_{t+1} (see equation (13)). This condition states that the return from producing at the marginal lender equals the inter-firm lending rate ρ. Together with equation (G.43), this forms a scalar fixed-point problem in $\overline{\omega}$ alone, solvable by one-dimensional root finding.

Iterative algorithm.

1. Guess $\overline{\omega} \in (0, 1)$.
2. Compute K from equation (G.43) and Y from equation (G.45).
3. Check whether equation (G.55) holds. Specifically, evaluate the residual

$$\mathcal{R}(\overline{\omega}) \;\equiv\; \kappa(\overline{\omega})\,\overline{\omega} + (1 - \delta) - \left(\frac{1}{\beta} - \xi\right), \tag{G.56}$$

   where $\kappa(\overline{\omega}) = \alpha(H/K(\overline{\omega}))^{1-\alpha}$, and update $\overline{\omega}$ until $\mathcal{R}(\overline{\omega}) = 0$.
4. Recover all remaining variables from Steps 2 and 3.

Unlike for the baseline model (Appendix D), which requires guessing *three* values ($\overline{\omega}$, $\overline{\overline{\omega}}$, R) and iterating on a three-dimensional fixed point, the no-default steady state requires guessing only $\overline{\omega}$, since $R = 1/\beta$ is determined analytically.

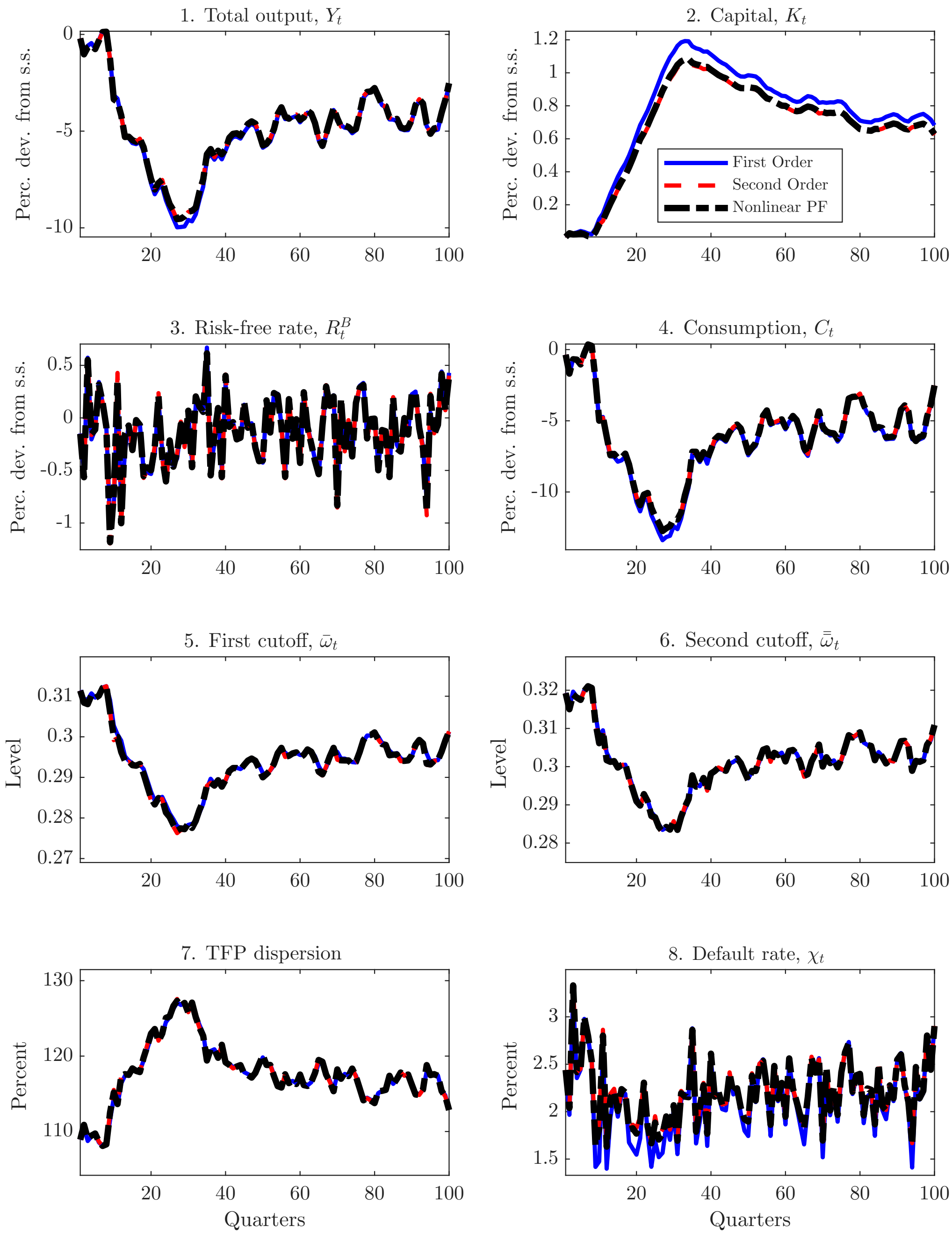
Comparison with the canonical RBC model. The frictionless canonical RBC model of Appendix F is recovered in the joint limit $\xi \to 0$ and $\overline{\omega} \to 1$. As $\xi \to 0$, the fixed-point condition (G.55) forces $\overline{\omega} \to 1$: the two equations become consistent only when the cutoff approaches unity, so that production is concentrated in the most productive firms (those with ω arbitrarily close to 1). In that limit the TFP prefactor $\Phi^{nd}(\overline{\omega}) \to 1$ by the limit established in equation (F.28) of Appendix F, and equations (G.43)–(G.51) coincide exactly with the RBC steady-state equations (F.36)–(F.42). For $\xi > 0$, the no-default model provides an intermediate case between the baseline model (which adds strategic default on top of the same ξ) and the frictionless RBC model.

H Comparing Alternative Solution Methods

In the main body of the paper, all model calculations are based on an exact second-order perturbation solution. We show here a comparison between alternative solution methods, a first-order solution and a solution based on the fully nonlinear perfect-foresight shooting algorithm based on Fair and Taylor (1983) as implemented in Adjemian et al. (2026). We set the precision of the nonlinear solution close to machine precision. We have generalized the implementation of Adjemian et al. (2026) by taking the first guess of the solution paths from the first-order perturbation solution.

In a random sample, such as the ones that we simulate for our SMM estimation, the accumulation of deviations from the linearization point can render the first-order solution inaccurate. To compare the alternative solution methods, we draw a random sample including all the shocks with parameters set as described in Section 6. We find it reassuring that in Figure A.1, the second-order perturbation solution is closely consistent with the fully-nonlinear solution. Conditional on a realized shock sequence, the perfect-foresight path captures the full nonlinearity of the model's transition but abstracts from the stochastic risk terms that the second-order solution prices; the comparison therefore validates the second-order solution against nonlinearity in the deterministic transition rather than against all risk-adjustment terms. We prefer the second-order perturbation solution based on computational speed. By contrast, a noticeable gap opens up between the first-order perturbation solution and the perfect foresight solution. Note that, in the figure, the path from the first-order solution and from the perfect-foresight solution are shown in deviation from the non-stochastic steady state. For the second-order solution, the paths are shown in deviation from the stochastic fixed point.

Figure A.1: Comparing the evolution of a random sample across solutions methods



Note: "Omega bar" refers to $\overline{\omega}_t$ and "omega double bar" refers to $\overline{\overline{\omega}}_t$. For the first-order perturbation solution and the nonlinear perfect-foresight solution, the paths start from the non-stochastic steady state and are also in deviation from that point (where relevant). For the second-order solution the paths start from the stochastic steady state and are also in deviation from that point (where relevant).

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