Macroeconomic Policy Games*

Martin Bodenstein
Federal Reserve Board

Luca Guerrieri
Federal Reserve Board

Joe LaBriola**
UC Berkeley

September 17, 2016

Abstract

To facilitate the study of strategic interactions, we develop a toolbox that characterizes the welfare-maximizing cooperative Ramsey policies under full commitment and open-loop Nash games between policymakers. We adopt the timeless perspective. Two examples for the use of our toolbox offer novel results. The first example revisits the case of monetary policy coordination in a two-country model to confirm that our approach replicates well-known results in the literature and extends these results by highlighting their sensitivity to the choice of policy instruments. For the second example, a central bank and a macroprudential regulator are assigned distinct objectives in a model with financial frictions. Lack of cooperation can lead to large welfare losses even if technology shocks are the only source of fluctuations.

JEL classifications: E44, E61, F42.
Keywords: optimal policy, strategic interaction, welfare analysis, monetary policy cooperation, macroprudential regulation.

* The views expressed in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or of any other person associated with the Federal Reserve System. The toolbox and replication codes for the examples discussed in this paper are available from https://sites.google.com/site/martinbodenstein/ and from http://www.lguerrieri.com/games_code.zip.

** Contact information: Martin Bodenstein (corresponding author), Telephone +1 202 452 3579, E-mail Martin.R.Bodenstein@frb.gov; Luca Guerrieri, Telephone +1 202 452 2550, E-mail Luca.Guerrieri@frb.gov; Joe LaBriola: E-mail joelabriola@berkeley.edu.

1 Introduction

Both across countries and within countries regulators face the challenging task of finding the appropriate response to the actions of other regulators. This task has informed active research on the gains from monetary policy coordination across countries, as described in detail by Canzoneri and Henderson (1991). Strategic interactions also arise within a country when different regulators are assigned or pursue distinct objectives. For instance, the expansion and reorganization of regulatory responsibilities spurred by the Financial Crisis has been approached differently across countries. In the United States the Dodd-Frank Act substantially increased the macroprudential responsibilities of the Federal Reserve. In the United Kingdom, the Financial Services Act 2012 established an independent Financial Policy Committee as a subsidiary of the Bank of England, with some policymakers participating in both the Monetary and the Financial Policy Committee. By contrast, in the euro area monetary policy is strictly separated from macroprudential regulation, although both functions involve the European Central Bank. Other examples include the interaction between fiscal and monetary authorities or games between countries about improving global competitiveness by setting tariffs and taxes across countries.

To facilitate the study of strategic interactions between regulators, we develop a toolbox that characterizes the welfare-maximizing cooperative Ramsey policies under full commitment and open-loop Nash games from the timeless perspective. Our algorithm automates the analytical derivation of the conditions for an equilibrium under cooperative and open-loop Nash games. The algorithm has four main advantages: 1) it is fast; 2) it is widely applicable; 3) it avoids the error-prone manual derivation of the conditions for an equilibrium; and 4) it makes results easy to replicate. These characteristics open up the possibility to tackle questions practically infeasible with other approaches to setting up games in a DSGE setting as we showcase in our examples.

The toolbox is designed to extend Dynare, a convenient and popular modeling environment.[1] Our work augments the single regulator framework of Lopez-Salido and Levin (2004).[2] The general framework for the policy games that we consider distin-

[1] See Adjemian, Bastani, Karam, Juillard, Maih, Mihoubi, Perendia, Pfeifer, Ratto, and Villemot (2011).

[2] Given a characterization of the actions of private agents, the framework in Lopez-Salido and Levin (2004)

guishes between two groups of agents: the first group consists of private agents who incorporate the (expected) path of the policy instruments in their decisions; the second group consists of the policymakers, who determine policies taking into account the private sector's response to the implemented policies. Taking as input a set of equilibrium conditions given arbitrary rules for the reactions of the policy instruments, our toolbox replaces those rules with either the welfare-maximizing Ramsey policies or with the policies for the open-loop Nash game under the timeless perspective.[3]

To showcase the wide applicability of our toolbox, we consider two examples and provide new results regarding the gains from cooperative policies. The first example is a two-country monetary model that closely follows Benigno and Benigno (2006), and Corsetti, Dedola, and Leduc (2010). These authors characterize the optimal monetary policies under cooperation and the open-loop Nash game between two monetary policy authorities in a dynamic general equilibrium model with sticky prices. If we take a linear approximation to the policymakers' first-order conditions around the optimal deterministic steady state of the model, we confirm that our toolbox produces the same results as the linear-quadratic approach in Benigno and Benigno (2006) and Corsetti, Dedola, and Leduc (2010).

A key advantage of our toolbox is the automation of the analytical derivation of the cooperative and open-loop Nash policies, once the actions of the private agents are characterized. Beyond the replication of existing results, the rapidity and convenience of deploying our toolbox allows us to explore with ease different strategy spaces associated with alternative instruments. We find that the instrument typically selected for this kind of exercise, producer price inflation, would not be selected if regulators could enter a meta-game on instrument selection, prior to the formulation of their optimal strategies. They would instead choose real output, a finding related to the lower spillover effects abroad associated with the strategy space for this instrument. Following the linear-quadratic approach, each of the twenty-five combinations of instruments that we consider would involve a new set of long and tedious algebraic derivations.

facilitates the computation of the welfare-maximizing Ramsey policies for a single regulator that has one or several policy instruments.

[3] To obtain a recursive structure and to make the problem suitable for applying standard solution methods, we follow most of the literature in adopting the concept of optimality from a timeless perspective. See Benigno and Woodford (2012) for a discussion of this approach.

The second example considers the workhorse New Keynesian model with financial frictions of Gertler and Karadi (2011). An agency problem on financial intermediaries has two important effects. First, the problem inefficiently limits the provision of credit. Second, the agency problem also magnifies the reaction of the economy to shocks through familiar financial accelerator mechanisms. We extend the model of Gertler and Karadi (2011) to include a transfer tax between households and firms. Within that model, we consider a game between a financial regulator and a central bank – a question not previously explored. The policy instrument of the central bank is the inflation rate; the policy instrument of the financial regulator is the transfer tax. The objectives of the two regulators reflect the preferences of households, but in both cases include an extra term. The central bank has an objective biased towards stabilizing inflation. The financial regulator has an objective biased towards stabilizing the provision of credit. We characterize optimal cooperative Ramsey and open-loop Nash policies. Crucially, we constrain the choice of biases so that the cooperative policies with the skewed objectives come close to replicating the allocations under policies that maximize the welfare of the representative household. Nonetheless, the strategic interaction between regulators lead to large and persistent deviations from cooperative outcomes and imply substantial welfare losses.

To highlight the wide applicability and rapidity of our toolbox, we also consider how the introduction of altruistic objectives that would (at least partially) internalize the bias of the other regulator would affect the open-loop Nash equilibrium. Intuitively, we confirm that altruistic preferences move the Nash allocations closer to the cooperative allocations, even in the presence of biases.

The optimal control literature that focuses on DSGE models typically derives first-order conditions but does not check second-order conditions. Forward-looking variables in DSGE models complicate substantially the analysis of second-order conditions especially when considering fully nonlinear solutions. An exception is the work of Benigno and Woodford (2012), who derive second order-conditions for an optimal control problem in the case of a single planner under a linear-quadratic solution. Benigno and Woodford do not provide analogous derivations for the more involved case of the open-loop Nash problem considered here. Furthermore, the approach outlined in Benigno and Woodford (2012) is not directly applicable to the verification of optimality conditions under solutions from higher-order approximations, even for the case of a single regulator.

Our novel approach to checking second-order conditions relies on taking perturbations of the optimal solution in the direction of arbitrary policy rules. Our approach verifies that a convex combination of the optimal rule and an arbitrary policy rule does not improve the objective function of the regulator. This check applies both under cooperation, and under the open-loop Nash solution. After all, in the open-loop Nash case, we are interested in the best response of a regulator to the best response of the other regulator.

The usefulness of our toolbox is not limited to solving the particular examples above. Following the approach in Dixit and Lambertini (2003), differences in objectives are fertile ground to explore the strategic interactions between policymakers. For instance, the solution under coordinated optimal monetary and fiscal policies explored in Schmitt-Grohe and Uribe (2004) could be readily extended for strategic interactions after allowing for small differences in the objectives of the monetary and fiscal authorities. More recent examples of stylized models that set the stage for strategic interactions between policymakers include Costinot, Lorenzoni, and Werning (2014), who illustrate the use of capital controls to manipulate the terms of trade, and Brunnermeier and Sannikov (2014), who show how capital controls may improve welfare in a model with financial frictions (but who do not consider a non-cooperative solution). Furthermore, our toolbox greatly facilitates the analysis of more fully articulated models. Examples include Bergin and Corsetti (2013), who introduce firm entry into a two-country model to study how the resulting production relocation externality influences monetary policy, and Fujiwara and Teranishi (2013), who allow for nominal rigidities in loan contracts. Finally, the optimal policy implications for models with numerous empirically relevant features such as consumption habits, capital accumulation, investment adjustment costs, incomplete financial markets, sticky wages, inefficient steady states.

The rest of the paper is organized as follows. Section 2 outlines the algorithm for calculating cooperative optimal policy and extends the algorithm to the calculation of optimal policies in open-loop Nash games. Section 3 applies the algorithm to an open-economy model where each country wishes to maximize welfare, and Section 4 considers the application of our algorithm to a model with a monetary authority and a macroprudential regulator. Section 5 concludes. An appendix provides instructions for the use of our toolbox as well as a more detailed description of our examples.

2 Equilibrium Definitions and Solution Algorithms

This section defines an equilibrium under cooperative Ramsey policies and under an open-loop Nash game. We discuss important computational issues and concepts as appropriate.

In maximizing the policy objectives subject to the structural equations of the private sector our toolbox employs a Lagrangian approach. The exact nonlinear first-order conditions that characterize the optimal policies under cooperation and the open-loop Nash game, respectively, are obtained by symbolic differentiation. Each system of equations is then approximated around its deterministic steady state using higher order perturbation methods. An alternative approach to characterizing optimal policies uses linear-quadratic (LQ) techniques. The LQ approach involves finding a purely quadratic approximation of each policymakers' objective function which is then optimized subject to a linear approximation of the structural equations of the model. Following Benigno and Woodford (2012), Levine, Pearlman, and Pierse (2008) and Debortoli and Nunes (2006) we show how the LQ approach relates to the approach underlying our numerical procedure and that the LQ approach delivers the same solution if the nonlinear output of our toolbox is approximated to the first order.

2.1 General Framework

Policy games distinguish between two groups of actors. We label the first group "private agents." Private agents take into account the (expected) path of the policy instruments. The second group consists of the policymakers who determine policies taking into account the private sector's response to the implemented policies. With more than one policymaker, strategic interaction between the policymakers can cause the outcomes of the dynamic game to deviate from the welfare-maximizing cooperative policy. For simplicity, we restrict the exposition to the case of two policymakers (or players). Furthermore, each policymaker is assumed to have only one instrument.

Let the $N \times 1$ vector of endogenous variables be denoted by x_t, which is partitioned as $x_t = (\tilde{x}_t', i_{1,t}, i_{2,t})'$. The variable $i_{j,t}$ is the policy instrument of player $j = [1, 2]$, respectively. The exogenous variables are captured by the vector ζ_t. For given sequences of the policy instruments $\{i_{1,t}, i_{2,t}\}_{t=0}^{\infty}$, the remaining $N-2$ endogenous variables need to satisfy the $N-2$ structural conditions that characterize an equilibrium

$$E_t g(\tilde{x}_{t-1}, \tilde{x}_t, \tilde{x}_{t+1}, i_{1,t}, i_{2,t}, \zeta_t) = 0. \tag{1}$$

We assume that the system of equations in g is differentiable up to the desired order of approximation. Without loss of generality and to facilitate changes in the set of policy instruments for our toolbox, the block of structural equations (1) contains two definitions relating the generic instrument variables $i_{1,t}$ and $i_{2,t}$ to the desired instruments in the model. For example, if player 1 uses the inflation rate $\pi_{1,t}$ as instrument as in Woodford (2003), then one of the equations in (1) simply reads $i_{1,t} - \pi_{1,t} = 0$.

To complete our framework, we need to describe how policies are determined. The intertemporal preferences of player j are given by $\mathcal{U}_j = E_0 \sum_{t=0}^{\infty} \beta^t U_j(\tilde{x}_{t-1}, \tilde{x}_t, \zeta_t)$ with the generic utility function $U_j(\tilde{x}_{t-1}, \tilde{x}_t, \zeta_t)$ required to be concave. Under cooperation, the two players maximise the joint welfare function $\omega_1 \mathcal{U}_1 + \omega_2 \mathcal{U}_2$ for given weights ω_1 and ω_2. We normalise the welfare weights to satisfy $\omega_1 + \omega_2 = 1$. Absent cooperation, each policymaker considers his own preferences only.

2.2 Definition of Equilibrium under Cooperation

The welfare-maximizing Ramsey policy with full commitment is derived from the maximization program

$$\begin{aligned}
&\max_{\{\tilde{x}_t, i_{1,t}, i_{2,t}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[\omega_1 U_1(\tilde{x}_{t-1}, \tilde{x}_t, \zeta_t) + \omega_2 U_2(\tilde{x}_{t-1}, \tilde{x}_t, \zeta_t)\right] \\
&s.t. \\
&E_t g(x_{t-1}, x_t, x_{t+1}, \zeta_t) = 0.
\end{aligned} \tag{2}$$

The first-order conditions for this problem can be obtained by differentiating the Lagrangian problem of the form

$$\mathcal{L}_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left[\omega_1 U_1(\tilde{x}_{t-1}, \tilde{x}_t, \zeta_t) + \omega_2 U_2(\tilde{x}_{t-1}, \tilde{x}_t, \zeta_t) + \lambda_t' g(x_{t-1}, x_t, x_{t+1}, \zeta_t)\right]. \tag{3}$$

The $(N-2) \times 1$ Lagrange multipliers associated with the private sector equilibrium conditions in (1) are denoted by λ_t for any $t \geq 0$.

Taking derivatives of $\mathcal{L}_0$ with respect to the N endogenous variables in x_t delivers N first order conditions. Additionally, taking derivatives with respect to λ_t delivers again the $N-2$ private sector conditions. In total, there are $2N-2$ conditions and $2N-2$ variables. Since the generic instruments $i_{1,t}$ and $i_{2,t}$ are added to the model equations through definitions of the form $i_{j,t} = \tilde{x}_t^j$ where $\tilde{x}_t^j$ is player j's actual policy instrument, taking derivatives with respect of $i_{1,t}$ and $i_{2,t}$ returns the Lagrange multipliers associated with these definitions. Here, we assume that λ_t^j is the Lagrange multiplier attached to the definition of player j's instrument. In sum, the Ramsey equilibrium process $\{\tilde{x}_t, i_{1,t}, i_{2,t}, \lambda_t\}_{t=0}^{\infty}$ satisfies

$$\begin{aligned}
&\sum_{j=1,2} \omega_j \{D_{\tilde{x}} U_j(\tilde{x}_{t-1}, \tilde{x}_t, \zeta_t) + \beta E_t D_{\tilde{x}^-} U_j(\tilde{x}_t, \tilde{x}_{t+1}, \zeta_{t+1})\} \\
&+\beta E_t \left\{\lambda_{t+1}' D_{\tilde{x}^-} g(x_t, x_{t+1}, x_{t+2}, \zeta_{t+1})\right\} + E_t \left\{\lambda_t' D_{\tilde{x}} g(x_{t-1}, x_t, x_{t+1}, \zeta_t)\right\} \\
&+\beta^{-1} \lambda_{t-1}' D_{\tilde{x}^+} g(x_{t-2}, x_{t-1}, x_t, \zeta_{t-1}) = 0 \qquad (4)\\
&\lambda_t^1 = 0 \qquad (5)\\
&\lambda_t^2 = 0 \qquad (6)\\
&E_t g(x_{t-1}, x_t, x_{t+1}, \zeta_t) = 0 \qquad (7)
\end{aligned}$$

at each date $t > 0$. The notation $D_{\tilde{x}}$ denotes the vector of partial derivatives of any functions with respect to the elements of $\tilde{x}_t$; likewise do $D_{\tilde{x}^-}$ and $D_{\tilde{x}^+}$ for derivatives with respect to $\tilde{x}_{t-1}$ and $\tilde{x}_{t+1}$, respectively. Following equations (5) and (6), the multipliers λ_t^1 and λ_t^2 need to be equal to zero for all $t \geq 0$. For $t = 0$, the set of equations in (4) is replaced by

$$\sum_{j=1,2}\omega_j\{D_{\tilde{x}}U_j(\tilde{x}_{-1},\tilde{x}_0,\zeta_t)+\beta E_0 D_{\tilde{x}^-}U_j(\tilde{x}_0,\tilde{x}_1,\zeta_1)\}+\beta E_0\left\{\lambda_1' D_{\tilde{x}^-}g(x_0,x_1,x_2,\zeta_1)\right\}$$
$$+E_0\left\{\lambda_0' D_{\tilde{x}}g(x_{-1},x_0,x_1,\zeta_t)\right\}=0.$$

This formulation of the problem implies that the first period is different from every other period because the choice of policies is not restricted by previous commitments. Although this system of equations can in general be solved, the equilibrium functions will not be time-invariant – something especially problematic given that the most frequently used solution methods (following a perturbation approach) require time-invariance. To avoid this problem, we follow most of the literature in adopting the concept of optimality from a *timeless perspective* which is discussed in great detail in Benigno and Woodford (2012).[4] In short, this concept requires an initial pre-commitment to suitably chosen values λ_{-1} at time 0 so that the first-order conditions (4) to (7) apply to all $t \geq 0$. Thus, the planner solves a modified optimization problem with additional constraints for time 0; equivalently, the planner's utility function in (2) is modified to reflect the initial commitments directly in the objective function

$$E_0\sum_{t=0}^{\infty}\beta^t\left[\omega_1 U_1(\tilde{x}_{t-1},\tilde{x}_t,\zeta_t)+\omega_2 U_2(\tilde{x}_{t-1},\tilde{x}_t,\zeta_t)\right]+\beta^{-1}\lambda_{-1}g\left(x_{-2},x_{-1},x_0\right). \tag{8}$$

The timeless perspective implies that the optimal deterministic steady state $(\bar{x},\bar{\lambda})$ needs to satisfy

$$\sum_{j=1,2}\omega_j\{D_{\tilde{x}}U_j(\bar{\tilde{x}},\bar{\tilde{x}},0)+\beta D_{\tilde{x}^-}U_j(\bar{\tilde{x}},\bar{\tilde{x}},0)\}$$
$$+\bar{\lambda}'\left(\beta D_{\tilde{x}^-}g(\bar{x},\bar{x},\bar{x},0)+D_{\tilde{x}}g(\bar{x},\bar{x},\bar{x},0)+\beta^{-1}D_{\tilde{x}^+}g(\bar{x},\bar{x},\bar{x},0)\right)=0 \tag{9}$$
$$\bar{\lambda}^1=0 \tag{10}$$
$$\bar{\lambda}^2=0 \tag{11}$$
$$E_t g(\bar{x},\bar{x},\bar{x},0)=0. \tag{12}$$

[4] In principle, the output of our toolbox can be used to compute a solution to the original problem. Yet, to make full use of the algorithms embedded in Dynare adopting the timeless perspective is key.

The problem stated in equations (9) to (12) is linear in the Lagrange multipliers. This feature can be exploited to obtain a reasonably accurate initial guess for computing the steady-state values of the Lagrange multipliers. For any initial guess of the steady-state values for the instruments i_1, i_2, we first find the vector $\tilde{x}$ that satisfies the equations for the private sector equilibrium (12). Because there are N first order conditions in (9) and $N-2$ Lagrange multipliers, the system allows multiple solutions, and we use linear regressions to obtain values for the Lagrange multipliers given the vector x. Re-interpreting equation (9), the dependent variables in our regressions are stacked in the vector $-\sum_{j=1,2}\omega_j\{D_{\tilde{x}}U_j(\bar{\tilde{x}},\bar{\tilde{x}},0)+\beta D_{\tilde{x}^-}U_j(\bar{\tilde{x}},\bar{\tilde{x}},0)\}$, the regression coefficients are the Lagrange multipliers $\bar{\lambda}$, and the explanatory variables are the matrix $\left(\beta D_{\tilde{x}^-}g(\bar{x},\bar{x},\bar{x},0)+D_{\tilde{x}}g(\bar{x},\bar{x},\bar{x},0)+\beta^{-1}D_{\tilde{x}^+}g(\bar{x},\bar{x},\bar{x},0)\right)$. The guess of the steady-state values of the instruments i_1, i_2 needs to change until all equations hold simultaneously, indicating that a solution for the steady-state has been found. Our toolbox implements these ideas to solve for the steady state numerically, relying on quasi-Newton methods, commonly available in Matlab, for solving the steady-state system of equations. As is familiar from the numerical literature, in the presence of multiple solutions, different initial guesses can be used to survey the possibility of multiple steady states. If multiple steady states are identified, the optimal steady-state must feature the highest value for the objective of the cooperative planner.

Equations (4) and (7) can now be replaced by a local approximation around the optimal steady state $\{\bar{x},\bar{\lambda}\}$ of desired order. The resulting system of (higher-order) difference equations can be solved by standard perturbation algorithms as further outlined in Section 2.5.

2.3 Definition of Open-loop Nash Equilibrium

To define an open-loop Nash equilibrium, let $\{i_{j,t,-t^*}\}_{t=0}^{\infty}$ denote the sequence of policy choices by player j before and after, but not including period t^*. An open-loop Nash equilibrium is a sequence $\left\{i_{j,t}^*\right\}_{t=0}^{\infty}$ with the property that for all t^*, i_{j,t^*}^* maximises player $j's$ objective function subject to the structural equations of the economy for given sequences $\left\{i_{j,t,-t^*}^*\right\}_{t=0}^{\infty}$ and $\left\{i_{-j,t}^*\right\}_{t=0}^{\infty}$, where $\left\{i_{-j,t}^*\right\}_{t=0}^{\infty}$ denotes the sequence of policy moves by all players other than player j. Each player's action is the best response to the other players' best responses.

With policymakers needing to specify a complete contingent plan at time 0 for their respective instrument variable $\{i_{j,t}\}_{t=0}^{\infty}$ for $j = [1, 2]$, under the open-loop equilibrium concept, the problem can be reinterpreted as a static game allowing us to recast each player's optimization problem as an optimal control problem given the policies of the remaining players. As under the static Nash equilibrium concept, player j restricts attention to his own objective function and the maximisation program is given by

$$\begin{aligned}
&\max_{\{\tilde{x}_t, i_{j,t}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t U_j(\tilde{x}_{t-1}, \tilde{x}_t, \zeta_t) \\
&s.t. \\
&E_t g(x_{t-1}, x_t, x_{t+1}, \zeta_t) = 0 \\
&\text{for given } \{i_{-j,t}\}_{t=0}^{\infty}.
\end{aligned} \tag{13}$$

The first-order conditions for each player are obtained from differentiating the Lagrangian of the form

$$\mathcal{L}_{j,0} = E_0 \sum_{t=0}^{\infty} \beta^t \left[U_j(\tilde{x}_{t-1}, \tilde{x}_t, \zeta_t) + \lambda'_{j,t} g(x_{t-1}, x_t, x_{t+1}, \zeta_t) \right] \tag{14}$$

for $j = [1, 2]$. Taking derivates of the $\mathcal{L}_{j,0}$ with respect to the $N-1$ choice variables $(\tilde{x}_t, i_{j,t})$, excluding the instrument of the other player, and the $N-2$ Lagrange multipliers $\lambda_{j,t}$ associated with the $N-2$ structural relationships yields $2N-3$ conditions for each player.

Notice that the full set of $4N-6$ equations includes the $N-2$ structural equations twice. Since in equilibrium all players face the same values of the non-policy variables $\tilde{x}_t$, an interior Nash equilibrium $\{\tilde{x}^*_t, i^*_{1,t}, i^*_{2,t}, \lambda^*_{1,t}, \lambda^*_{2,t}\}_{t=0}^{\infty}$ satisfies the following $3N-4$ conditions for $t > 0$

$$\begin{aligned}
&D_{\tilde{x}} U_1(\tilde{x}^*_{t-1}, \tilde{x}^*_t, \zeta_t) + \beta E_t D_{\tilde{x}^-} U_1(\tilde{x}^*_t, \tilde{x}^*_{t+1}, \zeta_{t+1}) + \beta E_t \left\{ \lambda^{*'}_{1,t+1} D_{\tilde{x}^-} g(x^*_t, x^*_{t+1}, x^*_{t+2}, \zeta_{t+1}) \right\} \\
&+ E_t \left\{ \lambda^{*'}_{1,t} D_{\tilde{x}} g(x^*_{t-1}, x^*_t, x^*_{t+1}, \zeta_t) \right\} + \beta^{-1} \lambda^{*'}_{1,t-1} D_{\tilde{x}^+} g(x^*_{t-2}, x^*_{t-1}, x^*_t, \zeta_{t-1}) = 0 && (15) \\
&\lambda^{1*'}_{1,t} = 0 && (16) \\
&D_{\tilde{x}} U_2(\tilde{x}^*_{t-1}, \tilde{x}^*_t, \zeta_t) + \beta E_t D_{\tilde{x}^-} U_2(\tilde{x}^*_t, \tilde{x}^*_{t+1}, \zeta_{t+1}) + \beta E_t \left\{ \lambda^{*'}_{2,t+1} D_{\tilde{x}^-} E_t g(x^*_t, x^*_{t+1}, x^*_{t+2}, \zeta_{t+1}) \right\}
\end{aligned}$$

$$+E_t\left\{\lambda_{2,t}^{*\prime} D_{\tilde{x}} g(x_{t-1}^*, x_t^*, x_{t+1}^*, \zeta_t)\right\} + \beta^{-1}\lambda_{2,t-1}^{*\prime} D_{\tilde{x}^+} g(x_{t-2}^*, x_{t-1}^*, x_t^*, \zeta_{t-1}) = 0 \quad (17)$$

$$\lambda_{2,t}^{2*\prime} = 0 \quad (18)$$

$$E_t g(x_{t-1}^*, x_t^*, x_{t+1}^*, \zeta_t) = 0. \quad (19)$$

In a fashion similar to the case of cooperation, the first-order conditions with respect to $i_{1,t}$ and $i_{2,t}$ imply the restriction that the Lagrange multipliers associated with the definition of the policy instruments — here $\lambda_{1,t}^{1*\prime}$ and $\lambda_{2,t}^{2*\prime}$ for players 1 and 2, respectively — are zero.

Adopting the timeless perspective is again key to obtaining time-invariant decision rules. The optimal response of each player given the policies of the other player derived from the optimal control problem at time 0 is not necessarily time consistent. Last, the deterministic steady state is found as for the cooperative case by exploiting the linearity of the system (15)-(19) in the $2N-4$ Lagrange multipliers.

2.4 Relationship to Linear-Quadratic Approach

An alternative approach to solve optimal policy problems uses LQ techniques. In the case of a single decision maker, the LQ approach involves finding a purely quadratic approximation of the policymaker's objective function which is then optimized subject to a linear approximation of the structural equations of the model. Benigno and Woodford (2012) and Levine, Pearlman, and Pierse (2008) and Debortoli and Nunes (2006) discuss necessary and sufficient conditions for a "correct LQ approximation" to the optimization problem stated in equation (2) to exist. Adopting the timeless perspective is shown to be one of the necessary conditions. In contrast to the early literature the approach followed here does not require the steady state of the model to be efficient.[5]

Appendix B shows that, to a first-order approximation, the output of our toolbox is equivalent to that of the LQ approach. The appendix also gives a roadmap for constructing the LQ matrices from the output of our toolbox.

[5] Rotemberg and Woodford (1998) popularized this approach in economics. To gain tractability they assumed the steady state to satisfy certain efficiency conditions.

2.5 Solution Algorithms

For all the examples demonstrating the use of our toolbox, we apply a perturbation approach to approximating the model solution. When reporting impulse response functions for alternative shocks, we use a first-order approximation. When reporting welfare results, we use a "true" second-order approximation by following the pruning algorithm in Kim, Kim, Schaumburg, and Sims (2008). Pruning keeps the approximation constant at the second-order by avoiding the accumulation of higher-order terms. Moreover, pruning ensures that the Blanchard-Kahn conditions for stability and local uniqueness for the first-order of approximation apply to the second-order, too.

To compute the welfare costs of suboptimal policies, we draw initial conditions for the state variables from the ergodic distribution associated with the optimal cooperative solution. This procedure is motivated by the fact that the planners/players in our examples have objective functions that are conditional on initial states. By sampling the initial states for any suboptimal policies from the ergodic space of the optimal policies we avoid spurious welfare reversals that could otherwise occur.[6]

3 Monetary Policy in an Open-Economy Model

We first illustrate our toolbox for the workhorse two-country model of monetary economics laid out in Benigno and Benigno (2006) and Corsetti, Dedola, and Leduc (2010). The model features two countries, each specialized in the production of one type of goods in different varieties. Each household produces exactly one variety and engages in monopolistic competition with all other households. Time-invariant subsidies offset the monopoly distortions in the steady state. A household chooses its nominal price to maximize its utility; as in Calvo (1983) the household can adjust the price at future dates with a fixed probability. Export prices are set in the currency of the producer. Shocks to technology affect the marginal product of labor, whereas a markup shock influences how much prices exceed the marginal cost of production. Finally, goods trade freely across borders and international financial markets are frictionless and complete.

[6] See Kim and Kim (2015) for examples of how conditional or unconditional objectives can lead to different optimal policies.

Benigno and Benigno (2006) and Corsetti, Dedola, and Leduc (2010) derive the optimal monetary policy under commitment from the timeless perspective using the LQ approach for the case of cooperation. Producer price inflation is the policy instrument in both countries. Under cooperation, the objective is an equally weighted average of the welfare of the representative agents in the two countries. When policymakers do not cooperate, strategic interaction generally leads to welfare inferior outcomes: the failure to account for the international spillovers of domestic policies causes foreign policymakers to adopt policies that in turn negatively impact the domestic country in the open-loop Nash equilibrium. Thus, there are gains from cooperation.

Appendix C covers the problems faced by the various agents in the model and reports the conditions (equations 36-60) that characterize the private-sector equilibrium in the model of Corsetti, Dedola, and Leduc (2010) which generalizes the one in Benigno and Benigno (2006) by allowing for home bias in consumption. The appendix also shows how to cast the model in a form suitable for the application of our toolbox.

To bolster confidence in our toolbox, we proceed by showing that it reproduces the results derived by Benigno and Benigno (2006) and Corsetti, Dedola, and Leduc (2010). We then turn to the novel aspects of our analysis. First, we introduce checks to assess the optimality of the computed equilibria under cooperation and in the open-loop Nash game. Second, we explore the impact of the policy instrument choice for the gains from cooperation. The literature has almost exclusively restricted the policy instrument to be producer price inflation in both countries.[7] Expanding the strategy space to include many more candidate instruments is easily accomplished with our toolbox. Thus, we are in a position to set up an extension of the usual game in the form of a meta-game that lets planners choose their instruments prior to choosing optimal strategies for the selected instrument.

3.1 Optimal Policy with and without Cooperation

The output of our toolbox matches well-known results in the literature. In the face of technology shocks, the welfare-maximising policy under cooperation replicates the flexible price allocations for the two-country model laid out above. As in closed economy

[7] An exception is Coenen, Lombardo, Smets, and Straub (2007), who consider the money supply as instruments.

models, the "divine coincidence" applies for "efficient shocks" – see Blanchard and Galí (2007). Accordingly, technology shocks move quantities and prices in the same direction relative to the flexible price economy and the central bank does not face a trade-off between inflation and output gap stabilisation.

A different picture emerges when the economy experiences a markup, an "inefficient disturbance." As would be the case in an analogous closed economy model, the cooperating policymakers cannot perfectly stabilize the economy. In response to a positive cost-push shock, the output gap turns negative, whereas inflation is positive.

If policymakers do not cooperate across borders, prices and quantities will in general differ from those under cooperation. Each country has the ability to influence the terms of trade through its monetary policy stance and the (open-loop) Nash equilibrium does not replicate the flexible-price allocations even for efficient shocks.[8]

Figures 1 and 2 confirm the findings of previous papers as outlined above for our toolbox. They show the responses to a positive technology shock and a cost-push shock under the welfare-maximizing cooperative policy and under an open-loop Nash game. As in Benigno and Benigno (2006) and Corsetti, Dedola, and Leduc (2010), Figure 1 shows that output price inflation is perfectly stabilized under the cooperative policy, and that the output response coincides with its counterpart in a flexible price model (not shown) for both countries after a technology shock. In the open-loop Nash game, inflation and output gaps are not perfectly stabilized. In that case, terms-of-trade movements affect the objectives of the foreign policymaker, and those effects are not fully internalized by the home policymaker.

Under the cost-push shock in Figure 2, neither policy completely stabilizes output price inflation and the output gaps.[9] As shown in Corsetti, Dedola, and Leduc (2010), the home country's real exchange rate appreciates and its terms of trade improve by more under the open-loop Nash policies than under the cooperative policy, resulting in larger spillover effects.[10]

[8] A necessary condition for the gains from cooperation to disappear in response to a technology shock is that the intratemporal and intertemporal elasticities of substitution be equal, which is not a feature of our calibration.

[9] The efficient output level does not move at all in response to a technology shock. Hence, any movements in actual output are equivalent to movements in the output gap.

[10] To further assess the reliability of our toolbox, we confirmed that its output under a first-order approximation coincides with the results produced by the LQ approach in Benigno and Benigno (2006) and Corsetti, Dedola, and Leduc (2010). Appendix C.3 reconciles the notation in Corsetti, Dedola, and Leduc (2010) with ours. The toolbox

Previous explorations of the gains from cooperation for monetary policy in an open economy setting restricted the strategy space to certain families of instrument rules. A prominent example is Obstfeld and Rogoff (2002). We confirmed that if the two countries in our model were to use simple interest rate rules responding to the lagged interest rate and to producer price inflation, the resulting cooperative and Nash allocations would be very similar to the optimal policies under cooperation and the open-loop Nash game, respectively.[11] This finding, however, is not general. It is driven by the apt choice of variables that enter the interest rate rule. Relatedly, when the analysis of strategic interactions is dependent on the particular family of interest rate rules considered, any results of this analysis could be overturned by including additional terms in the rules. The more general policies automatically set up by our toolbox avoid this shortcoming.

3.2 Assessing the Optimality of Policy Choices

The optimal control literature that focuses on DSGE models typically does not go beyond the derivation of first-order optimality conditions. An exception is the work of Benigno and Woodford (2012), who derive second order-conditions for an optimal control problem in the case of a single planner under a LQ solution. Benigno and Woodford do not provide analogous derivations for the more involved case of the open-loop Nash problem considered here. Furthermore, the approach outlined in Benigno and Woodford (2012) is not directly applicable to the verification of optimality conditions under solutions from higher-order approximations even for the case of a single regulator.

Our novel approach to checking second-order conditions relies on taking perturbations of the optimal solution in the direction of arbitrary policy rules. Our approach verifies that a convex combination of the optimal rule and an arbitrary policy rule does not improve on the objective function of the regulator. This check applies both under

that accompanies this paper provides a routine that lines up our results with those in Benigno and Benigno (2006) and Corsetti, Dedola, and Leduc (2010).

[11] In the case of the Nash game, we identified the solution by alternatively optimizing the parameters for the instrument rule (governing the weights on the lagged interest rate and on inflation) of one regulator keeping the other rule constant at the previously optimized parameters. We stopped the iteration at a fixed point (consistent with the definition of a Nash equilibrium). In all cases, the optimized simple rules featured a very high weight on interest rate smoothing and responded strongly to inflation.

cooperation, and under the open-loop Nash solution. After all, in the open-loop Nash case, we are interested in the best response of a regulator to the best response of the other regulator.

Practically, we stack the necessary conditions for an equilibrium for the optimal control problem (either for the cooperative or the competitive case) with the conditions for an equilibrium for the analogous economy governed by the arbitrary policy rule. All the endogenous xvariables for the two stacked models remain distinct in order to track numerically the optimal policy for the particular instrument of choice. With this approach we can check the payoffs associated with any convex combination of the optimal policy and the arbitrary instrument rule (as long as the instrument rule does not lead to a violation of the Blanchard-Kahn conditions). Optimality requires intuitively that the value of the objective function of the regulator be reduced if any non-zero weight is attached to the arbitrary policy rule.

In performing this check, it is important to recognize that under the timeless perspective, the objective function of the regulator is modified relative to the the original objective to ensure time-invariant decision rules in equilibrium.[12] Our toolbox provides tools to stack the necessary conditions for an equilibrium for our test, as well as to size the change in the objective function consistent with the timeless perspective – essentially the value of promises made by the regulator before the initial period.

An example is provided in Figure 3, for which we considered a policy rule that sets producer price inflation to its steady-state value in the home country and the optimal cooperative policy in the foreign country. Namely, let the instruments under the cooperative policy set to be π_t and π_t^*, i.e., domestic and foreign producer price inflation. The suboptimal policy sets producer price inflation in the home country as $\pi_t^s = \upsilon^s \pi + (1 - \upsilon^s)\, \pi_t$, where the parameter υ^s governs the convex combination. The foreign country follows $\pi_t^{s*} = \pi_t^*$.

The top panel plots the difference between conditional welfare under the optimal cooperative policy and the suboptimal policy for different values of υ^s. As indicated by the welfare difference being minimized at $\upsilon^s = 0$, the arbitrary rule considered cannot

[12] Recall that under the timeless perspective the utility function maximized by the planner is given by equation (8). In particular, the switch to an alternative policy could break previous commitments made under the timeless perspective and therefore outperform the optimal policy if the welfare criterion is not taken to be (8).

improve the optimal cooperative policy.

The bottom panel of Figure 3 reports results analogous to those for the top panel for the open-loop Nash game. In this case, we check whether the home country can improve upon the optimal strategy in the Nash game by also assigning weight to an arbitrary policy rule. With the foreign country setting the foreign inflation rate following the optimal strategy from the open-loop Nash game, the home country prefers the (optimal) strategy from the open-loop Nash game to any convex combination of that strategy and the arbitrary rule. (Given symmetry, identical results obtain when the roles of the home and foreign country are reversed.)

Though we fall short of providing a fully analytical sufficient statistic for optimality, our check can go a long way towards ensuring that indeed the solution identified by the analytical first-order conditions has key characteristics of the optimal solution by exploring combinations of the optimal policies and a plethora of suboptimal rules.

3.3 Exploring the Strategy Space

Exploiting the flexibility of our toolbox, we can easily analyze how the choice of instruments impacts the outcomes of the open-loop Nash game. Suppose that at the first stage policymakers choose the policy instrument from a given set of instruments. At the second stage of the game, each policymaker chooses the optimal strategy given his choice of instrument taking the strategy of the other policymaker as given. To determine the optimal choice of instruments, we need to recompute and solve the first-order conditions of the open-loop Nash game described in equation (13) for all possible combinations of the instruments included in the set of instruments. An exhaustive exploration of the strategy space for the open-loop Nash game has not been undertaken, thus far; the LQ approach followed in Benigno and Benigno (2006) is simply to cumbersome for this pursuit.

In principle, any variable that enters the model can be taken as instrument in problem (13). For ease of presentation, we restrict attention to the following five instruments: producer price inflation (π_t), consumer price inflation ($\pi_{C,t}$), real output (Y_t), nominal output (P_tY_t), or the change in the nominal exchange rate (e_t/e_{t-1}). In total, we allow twenty-five instrument combinations, a number that strikes a balance between comprehensiveness and easy of exposition. In this set of instruments we omitted the nominal interest rate since we found that any combination of instruments involving the nominal interest rate leads to equilibrium indeterminacy in the open-loop Nash game.

Table 1 reports the gains from cooperation for each of the twenty-five combinations of instruments relative to the gains from cooperation under the baseline specification of both countries choosing producer price inflation as the instrument — the specification in Benigno and Benigno (2006), Corsetti, Dedola, and Leduc (2010). Notice that, since we translate the gains from cooperation in terms of a consumption subsidy levied in the home country, Table 1 is not symmetric across the diagonal entries.[13] Strikingly, the baseline specification does not imply comparatively large or small gains from cooperation; a finding that stresses how arbitrary this instrument choice is. If both countries adopt real output as the instrument, the outcome of the open-loop Nash game is much closer to the outcomes under cooperation as evidenced by the much reduced welfare gains from cooperation in this case. The largest gains from cooperation obtain if policymakers play the open-loop Nash game using the growth rate of the nominal exchange rate as instrument. In comparison to the best scenario of both countries formulating their strategies in terms of real output, the welfare losses are 150 times bigger!

Table 1 focuses on overall welfare implications, but the first stage game in which each country chooses its instrument may not result in the combination of instruments associated with the most desirable outcomes. For the instruments considered here, we confirmed that the home country maximizes its own expected utility by opting for real output as the instrument, irrespective of the foreign country's choice. Likewise, the foreign country maximizes its expected utility by choosing real output as its policy instrument. Thus, real output in both countries is a Nash equilibrium choice at the first stage and leads to outcomes that are closest to those under cooperation.

To shed additional light on the role of the policy instruments, Figure 4 plots the impulse responses after a cost push shock in the home country when both countries adopt 1) producer price inflation, 2) real output, and 3) the change in the nominal exchange rate as the instrument. The response of the real exchange rate can be viewed as a gauge of the international spillover effects — not internalized by each policymaker.

[13] Notice also that if we were to assign all the gains to the foreign country, the resulting table would be the mirror image of Table 1.

The smallest spillover effects, and consequently the smallest gains from cooperation occur when real output is used as the instrument in both countries. Remarkably, when real output is the policy instrument, the foreign country is almost insulated from the shock similar to the case of full cooperation in Figure 2. When policymakers use the change in the nominal exchange rate as instrument, they cannot stabilize the economy as effectively as under the other two instruments as exemplified by the larger response of output.

4 Macroprudential Regulation Model

Our toolbox can also be applied to policy games in a closed economy. We lay out a policy game between a central bank and a financial regulator in the model of Gertler and Karadi (2011). That model features two types of rigidities. Allocations are skewed by nominal rigidities as well as by financial frictions. Non-financial firms are prevented from issuing equity to households directly and have to rely on financial intermediaries, referred to as "banks," in order to raise funds. Due to an agency problem, however, banks are limited in their ability to attract deposits and issue credit to non-financial firms. Accordingly, credit is under-supplied, and the reactions to shocks are amplified by a familiar financial-accelerator mechanism.

The only, but crucial, modification that we introduce to the setup of Gertler and Karadi (2011) is a lump-sum tax charged on banks and rebated to households. This is the powerful instrument used by the financial regulator in our policy game, while inflation is the instrument used by the central bank.[14] Appendix D covers the problems faced by the various agents in the model and reports the conditions (equations 77-99) that characterize the private-sector equilibrium. The appendix also reviews the model calibration. In brief, we stay close the calibration choices in Gertler and Karadi (2011) with two exceptions: 1) for ease of exposition, we simplify the stochastic structure to include technology shocks only; and 2) we impose that the interest rates on deposits and on loans to non-financial firms coincide in the steady state. This second exception implies that the steady-state allocations are efficient and that distortions only open up

[14] Similar to the case of the two-country model, the open-loop Nash equilibrium is indeterminate when the nominal interest rate is used as policy instrument.

in response to shocks.

4.1 Analyzing the Gains from Cooperation

Figure 5 shows the responses to a contraction in technology under alternative policies. The shock considered brings down technology by 1 percent in the first quarter. Subsequently, technology follows its auto-regressive process.

We first consider the cooperative policy between the two regulators that maximize the utility of the representative household (see equation 77 in the appendix). The solid lines in Figure 5 denote the responses for this case. The instruments are so powerful that, for a technology shock, the policymakers replicate the allocations that obtain in the analogous frictionless model. Due to the financial friction, absent intervention from the financial regulator, banks are undercapitalized after the contractionary technology shock. An infusion of cash into the banks (i.e., a negative bank tax) can prop up their equity position and expand lending next period. At the same time, nominal rigidities call for a slight increase in the policy interest rate to prevent inflation from rising inefficiently. Notice that the welfare-maximizing cooperative policy completely stabilizes the expected spread between the bank return on investment and its cost of funding (the loan rate $E_t R^s_{t+1}$ minus the deposit R_t) in all periods. The same policy also achieves full inflation stabilization.

With identical objectives for the two regulators, the open-loop Nash and cooperative policies coincide. However, in practice, different regulators are assigned or pursue different objectives. We assume objectives for the two regulators that are biased versions of the preferences of the representative agent. Apart from incorporating terms that reflect utility from consumption, C_t, and leisure, L_t, the objective of the central bank also incorporates a term that reflects an inflation stabilization bias (where π_t is inflation and $\bar{\pi}$ is its steady state value):

$$Obj_{cb} = E_0 \sum_{t=0}^{\infty} \beta^t \left[\log(C_t - \gamma C_{t-1}) - \chi_0 \frac{L_t^{1+\chi}}{1+\chi} - \mu_{cb}(\pi_t - \bar{\pi})^2 \right], \tag{20}$$

where the parameter $\mu_{cb} = 5$ in our benchmark calibration governs the extent of the inflation bias. Analogously, the objective of the macroprudential regulator is given by

$$Obj_{mpr} = E_0 \sum_{t=0}^{\infty} \beta^t \left[\log(C_t - \gamma C_{t-1}) - \chi_0 \frac{L_t^{1+\chi}}{1+\chi} - \mu_{mpr} \left((R_t^s - \bar{R}^s) - (R_{t-1} - \bar{R}) \right)^2 \right], \tag{21}$$

where the parameter $\mu_{mpr} = 4$ in our benchmark calibration governs the extent of the bias towards stabilizing the interest rate spread for banks, the term $\left((R_t^s - \bar{R}^s) - (R_{t-1} - \bar{R}) \right)^2$. For the baseline calibration, this particular formulation of biased objectives yields minor differences relative to the welfare-maximizing cooperative policies (as quantified below).[15]

As can be seen from Figure 5, the differences between the cooperative policies with biased and unbiased objectives are relatively minor. The bias implies that the macroprudential regulator is overzealous in stabilizing the interest rate spread for banks when the shock occurs. Conversely, the central bank accepts small deviations from full stabilization of inflation. Similarly, all other allocations remain close to their counterparts under the welfare-maximizing cooperative policies with biased objectives.

By contrast, an open-loop Nash game with the same biased objectives yields outcomes that are drastically different. To understand the extent of these differences, consider the side effects of a policy that, in reaction to a decline in technology, pushes up the equity positions of banks. Higher equity positions allow banks to expand credit, push up investment, and boost aggregate demand. In the presence of nominal rigidities, this expansion in demand leads to higher resource utilization and higher marginal costs of production, which cause inflation to rise. In reaction to the same decline in technology, the central bank wants to curb the inflationary effects of the shock and increase policy rates. However, higher policy rates bring up the cost of funding for banks, and by reducing profitability ultimately reduce the amount of funds available to support lending.

Accordingly, as the macroprudential regulator recognizes that the central bank intends to move rates up, he counteracts that action by recapitalizing banks even more

[15] In analyzing the strategic interaction between fiscal and monetary policy Dixit and Lambertini (2003) assume the central bank to be more aggressive about inflation stabilization than the representative agent (and the fiscal authority) in order to obtain different objective functions for the fiscal and monetary authorities. Our formulation is more general, but reduces to the idea captured in Dixit and Lambertini (2003) for $\mu_{mpr} = 0$.

(shown as a negative movement of the tax in Figure 5). In turn, the central bank will have an incentive to increase policy interest rates by more, realizing that the macroprudential regulator will step up the recapitalization of banks. Effectively, the different biases in the objectives push each regulator to discount the reverberations of his own actions onto the objectives of the other regulator. Ultimately, as shown in Figure 5, the strategic interactions lead to an excessive recapitalization of banks, unnecessarily aggressive tightening in monetary policy, and stark deviations from the allocations under the welfare-maximizing cooperative policies, which imply substantial welfare losses.

The top panel of Figure 6 confirms that the welfare losses from adopting biased objectives are small for cooperative policies for a broad range of the parameters that govern the biases. The panel's abscissae measure the parameter governing the bias of the macroprudential regulator towards stabilizing credit spreads. The panel's ordinates measure the welfare loss relative to allocations obtained from cooperative policies with unbiased preferences (expressed in terms of a proportional consumption tax that would leave the households indifferent between cooperative policies with and without biased objectives). The chart shows multiple contours of the tax schedule for different values of the parameter governing the bias of the central bank towards stabilizing inflation.

By contrast, the bottom panel of Figure 6 shows that the welfare gains from cooperative policies increase substantially with the bias towards spread stabilization. With biased objectives, the welfare cost of open-loop Nash policies relative to the welfare maximizing policies can be orders of magnitude higher than the losses from allowing for biased objectives under cooperative policies (relative to the case of unbiased objectives). Notice also that these welfare costs are orders of magnitudes larger than the welfare costs of business cycles reported in Lucas (2003). Notably, the cost of open-loop Nash policies decreases in the bias of the central bank. This feature is easy to understand. The optimal cooperative policy entails complete inflation stabilization in response to technology shocks. Consequently, a more pronounced bias towards inflation stabilization fosters allocations more closely aligned with those of the cooperative policy.

4.2 Altruistic Objectives

To showcase the flexibility of our toolbox, we also consider how the introduction of altruistic objectives that (at least partially) internalize the bias of the other regulator affect the open-loop Nash equilibrium. For this exercise, we modify the objective functions of the two regulators as follows:

$$\begin{aligned} Obj_{cb} &= E_0 \sum_{t=0}^{\infty} \beta^t \Bigg[\log(C_t - \gamma C_{t-1}) - \chi_0 \frac{L_t^{1+\chi}}{1+\chi} \\ &\quad -(1-\omega_{cb})\mu_{cb}(\pi_t - \bar{\pi})^2 - \omega_{cb}\left((R_t^s - \bar{R}^s) - (R_{t-1} - \bar{R})\right)^2 \Bigg], \end{aligned}$$

$$\begin{aligned} Obj_{mpr} &= E_0 \sum_{t=0}^{\infty} \beta^t \Bigg[\log(C_t - \gamma C_{t-1}) - \chi_0 \frac{L_t^{1+\chi}}{1+\chi} \\ &\quad -(1-\omega_{mpr})\mu_{mpr}\left((R_t^s - \bar{R}^s) - (R_{t-1} - \bar{R})\right)^2 - \omega_{mpr}\mu_{cb}(\pi_t - \bar{\pi})^2 \Bigg], \end{aligned}$$

where the parameters ω_{cb} and ω_{mpr} govern the extent of the altruism of each regulator towards the bias of the other regulator. In Figure 7, we explore how variation in ω_{cb} and ω_{mpr} affect the welfare costs of open-loop Nash games for an intermediate calibration of the bias parameters that sets $\mu_{cb} = 10$ and $\mu_{mpr} = 4$. The top panel shows that the costs of biased objectives remain small for all the alternative levels of the altruism parameters considered. The bottom panel shows the welfare costs of the open-loop Nash game relative to allocations from an unbiased cooperative policy. Intuitively, we confirm that higher values of the altruism parameters move the Nash allocations closer to the cooperative allocations.

Our results point to two implications for the design of institutional arrangements. Bringing different regulatory functions under the same institution fosters the recognition of alternative objectives and avoids potentially large welfare losses from strategic interactions. When this solution is politically not feasible, our results argue for devising altruistic objectives for each regulator as a way to minimize the welfare-reducing impact of strategic behavior.

5 Conclusions

Studying strategic interactions between policymakers has a long tradition in macroeconomics. A popular approach is to solve the problem using linear-quadratic techniques. Purely quadratic objective functions are derived for each policymaker; the first order conditions of the problem are then obtained by optimizing the quadratic objectives subject to linear approximations of the structural economic relationships. Unfortunately, this approach becomes laborious and potentially error-prone for larger models, limiting the range of analysis that can be tackled.

A more direct approach is to obtain the first-order conditions of the problem by using the nonlinear structural equations of the model and the nonlinear objective functions assigned to the policymakers. Our toolbox fully automates this procedure using symbolic differentiation. The quadratic approximations to the policymakers' objective functions can in principle be retrieved from the output of our toolbox. Changes to an existing model such as allowing for cooperation between policymakers instead of playing out an open-loop Nash game or changing the policy instruments assigned to the policymakers imply a new set of first order conditions that is easily generated by our toolbox.

We apply the toolbox introduced in this paper to the well-known case of monetary policy coordination in a two-country model. The flexibility of our toolbox allows us to easily replicate the results in the literature and move beyond them. We show that alternative instruments change the strategy space in important ways. In particular, if players were allowed a choice of instruments before a choice of strategies, they would favor real output over producer price inflation, the instrument typically considered by papers that have studied monetary policy coordination.

We also apply the toolbox to address strategic interactions between a macroprudential regulator and a central bank in a model with financial frictions. The analysis points to potentially large welfare losses stemming from the lack of cooperation between policymakers, even if technology shocks are the only source of fluctuations. The flexibility of our toolbox allows us to easily vary the objectives of the policymakers. Intuitively, we confirm that when a regulator's objetive encompasses the objective of the other regulator in an altruistic fashion, the allocations under the open-loop Nash policies move closer to those of a cooperative policy.

References

Adjemian, S., H. Bastani, F. Karam, M. Juillard, J. Maih, F. Mihoubi, G. Perendia, J. Pfeifer, M. Ratto, and S. Villemot (2011). Dynare: Reference manual version 4. Dynare working papers, CEPREMAP. [2]

Benigno, G. and P. Benigno (2006). Designing targeting rules for international monetary policy cooperation. *Journal of Monetary Economics 53*(3), 473–506. [3, 13, 14, 15, 16, 18, 19, 11, 17]

Benigno, P. and M. Woodford (2012). Linear-quadratic approximation of optimal policy problems. *Journal of Economic Theory 147*(1), 1–42. [3, 4, 5, 6, 9, 12, 16, 10]

Bergin, P. R. and G. Corsetti (2013, August). International Competitiveness and Monetary Policy: Strategic Policy and Coordination with a Production Relocation Externality. NBER Working Papers 19356, National Bureau of Economic Research, Inc. [5]

Blanchard, O. and J. Galí (2007, 02). Real Wage Rigidities and the New Keynesian Model. *Journal of Money, Credit and Banking 39*(s1), 35–65. [15]

Brunnermeier, M. K. and Y. Sannikov (2014). International credit flows, pecuniary externalities, and capital controls. Technical report. [5]

Calvo, G. A. (1983, September). Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics 12*(3), 383–398. [13, 12]

Canzoneri, M. B. and D. W. Henderson (1991, December). *Monetary Policy in Interdependent Economies: A Game-Theoretic Approach*. MIT Press Books. The MIT Press. [2]

Coenen, G., G. Lombardo, F. Smets, and R. Straub (2007). International Transmission and Monetary Policy Cooperation. In *International Dimensions of Monetary Policy*, NBER Chapters, pp. 157–192. National Bureau of Economic Research, Inc. [14, 13]

Corsetti, G., L. Dedola, and S. Leduc (2010). Optimal Monetary Policy in Open Economies. In B. M. Friedman and M. Woodford (Eds.), *Handbook of Monetary Economics*, Volume 3 of *Handbook of Monetary Economics*, Chapter 16, pp. 861–933. Elsevier. [3, 13, 14, 15, 16, 19, 1, 11, 17, 18]

Costinot, A., G. Lorenzoni, and I. Werning (2014). A Theory of Capital Controls as Dynamic Terms-of-Trade Manipulation. *Journal of Political Economy 122*(1), 77 – 128. [5]

Debortoli, D. and R. Nunes (2006). On Linear Quadratic Approximations. MPRA Paper 544, University Library of Munich, Germany. [6, 12]

Dixit, A. and L. Lambertini (2003, December). Interactions of Commitment and Discretion in Monetary and Fiscal Policies. *American Economic Review 93*(5), 1522–1542. [5, 22]

Fujiwara, I. and Y. Teranishi (2013, October). Financial Stability in Open Economies. CAMA Working Papers 2013-71, Centre for Applied Macroeconomic Analysis, Crawford School of Public Policy, The Australian National University. [5]

Gertler, M. and P. Karadi (2011, January). A model of unconventional monetary policy. *Journal of Monetary Economics 58*(1), 17–34. [4, 20, 19, 21, 28]

Kim, J. and S. Kim (2015). Conditional versus unconditional utility as welfare criterion: Two examples. Mimeo, Korea University. [13]

Kim, J., S. Kim, E. Schaumburg, and C. A. Sims (2008). Calculating and using second-order accurate solutions of discrete time dynamic equilibrium models. *Journal of Economic Dynamics and Control 32*, 3397–3414. [13]

Levine, P., J. Pearlman, and R. Pierse (2008). Linear-quadratic approximation, external habit and targeting rules. *Journal of Economic Dynamics and Control 32*(10), 3315–3349. [6, 12, 10]

Lopez-Salido, D. and A. T. Levin (2004). Optimal monetary policy with endogenous capital accumulation. 2004 Meeting Papers 826, Society for Economic Dynamics. [2, 4]

Lucas, R. E. (2003). Macroeconomic Priorities. *American Economic Review 93*(1), 1–14. [23]

Obstfeld, M. and K. Rogoff (2002). Global Implications of Self-Oriented National Monetary Rules. *The Quarterly Journal of Economics 117*(2), 503–535. [16]

Rotemberg, J. J. and M. Woodford (1998). An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy: Expanded Version. NBER Technical Working Papers 0233, National Bureau of Economic Research, Inc. [12]

Schmitt-Grohe, S. and M. Uribe (2004, February). Optimal fiscal and monetary policy under sticky prices. *Journal of Economic Theory 114*(2), 198–230. [5]

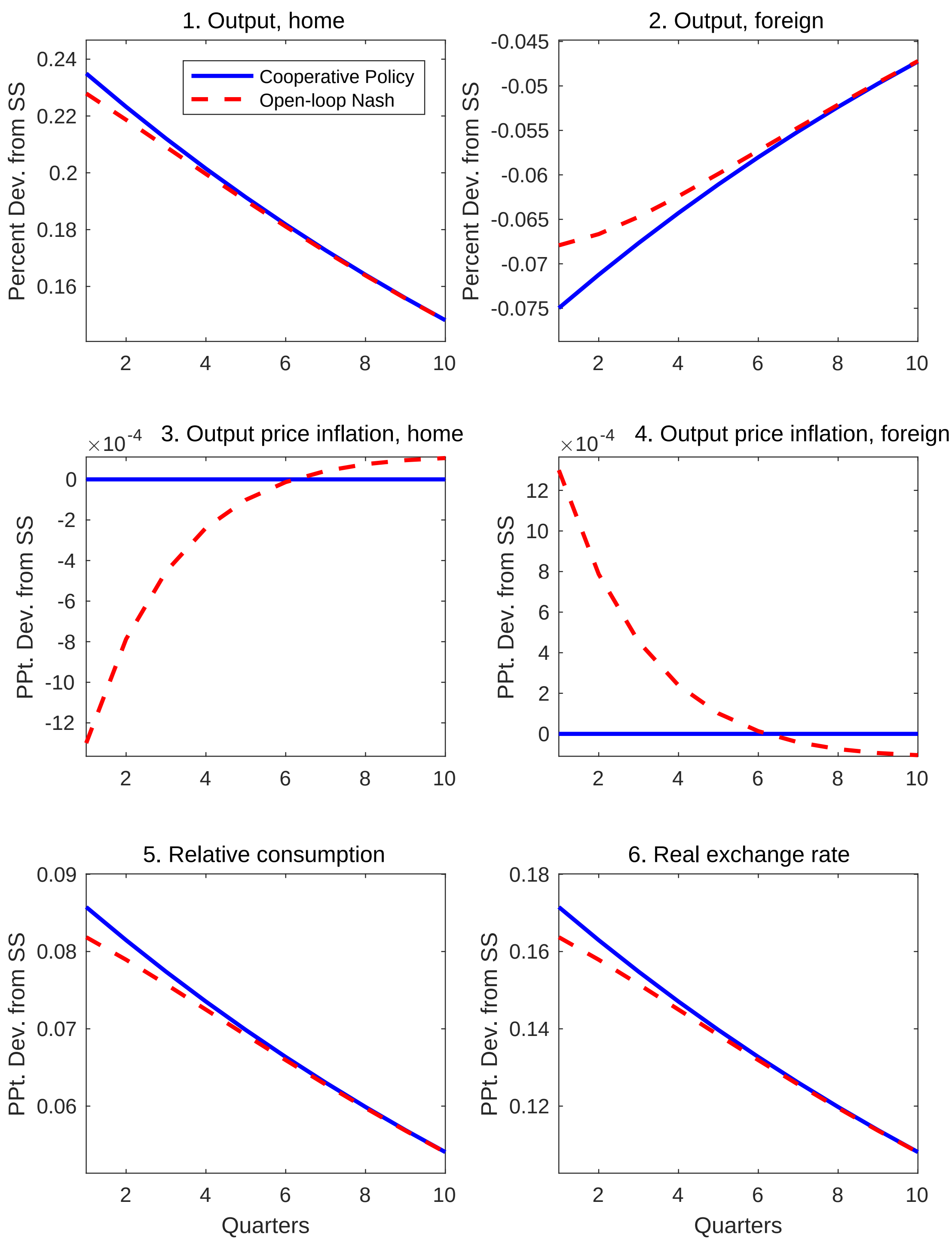
Woodford, M. (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press. [7]

Table 1: Welfare Gains from Cooperation under Alternative Instrument Choices

Strategy	π^*_t	$\pi^*_{C,t}$	Y^*_t	$P^*_t Y^*_t$	$\frac{e^*_t}{e^*_{t-1}}$
π_t	1.00	3.14	0.65	1.39	3.64
$\pi_{C,t}$	3.15	21.61	2.79	5.39	27.00
Y_t	0.65	2.79	0.25	1.07	3.27
$P_t Y_t$	1.39	5.39	1.07	2.10	6.38
$\frac{e_t}{e_{t-1}}$	3.65	27.00	3.27	6.39	36.78

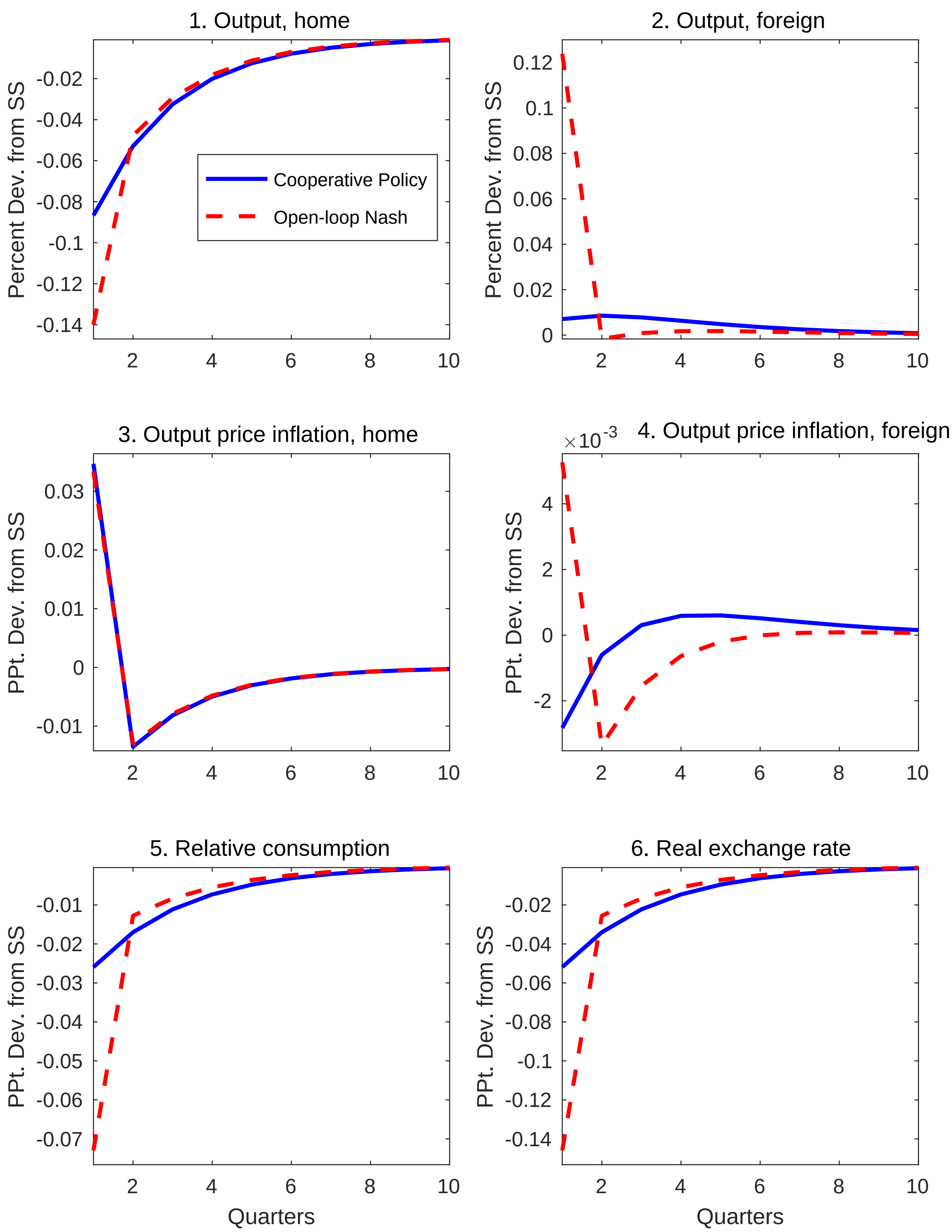
Note: This table reports the welfare gains from cooperation for each combination of instruments by the two policymakers in the open-loop Nash game. The welfare gains are expressed relative to the gains under the baseline case of both central banks using producer price inflation as the instrument. Notice that $e_t = \frac{1}{e^*_t}$.

Figure 1: Cooperative and Open-loop Nash Policies in the Open Economy Model: Responses to a Technology Shock



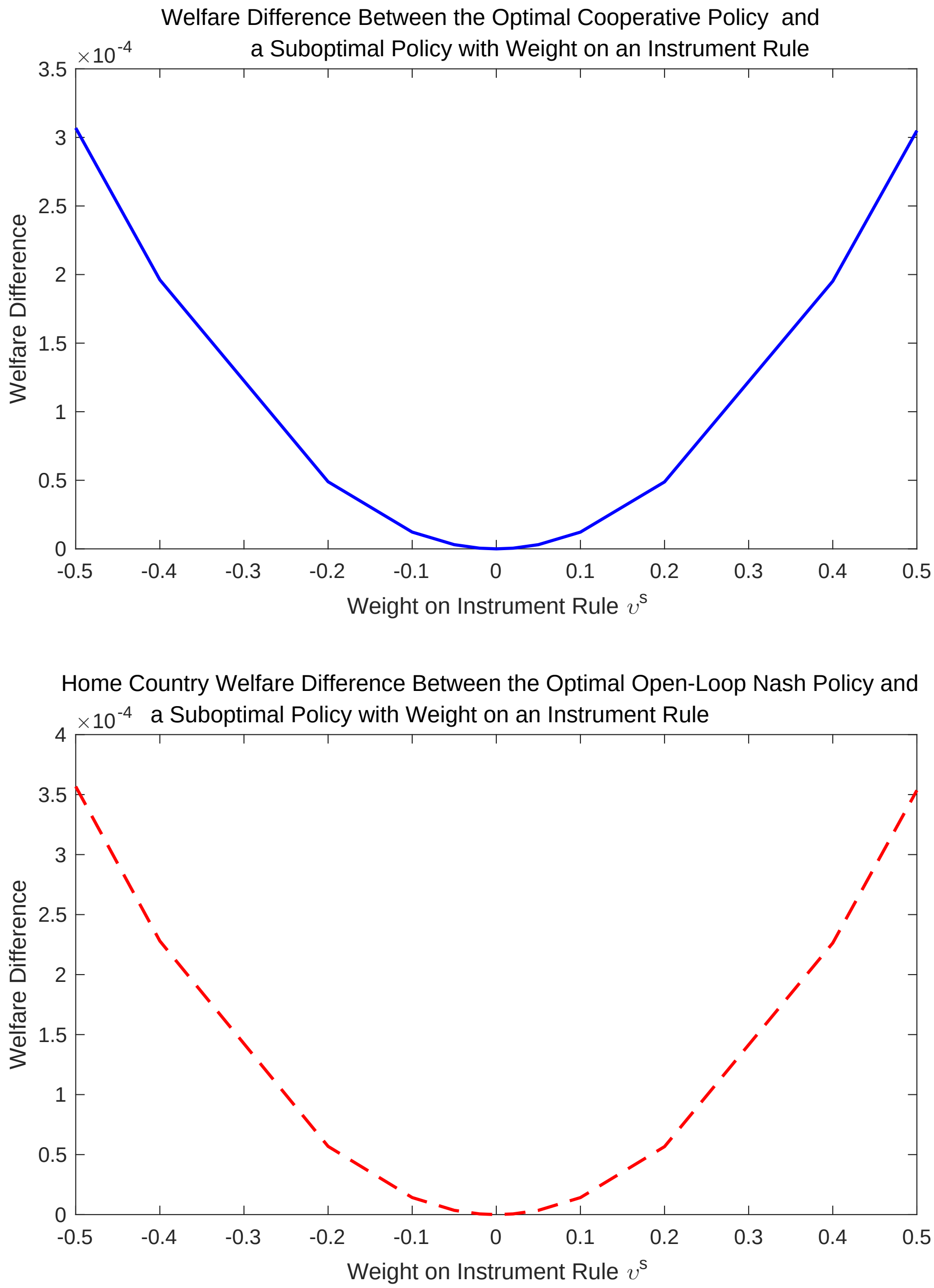
Notes: The figure plots the transition dynamics of the two economies after a one-standard deviation increase in technology in the home country. The two lines show the responses under full commitment with cooperation (Cooperative Policy) and without cooperation (Open-Loop Nash), when policymakers use output price inflation in their respective country as the policy instrument.

Figure 2: Cooperative and Open-loop Nash Policies in the Open Economy Model: Responses to a Markup Shock



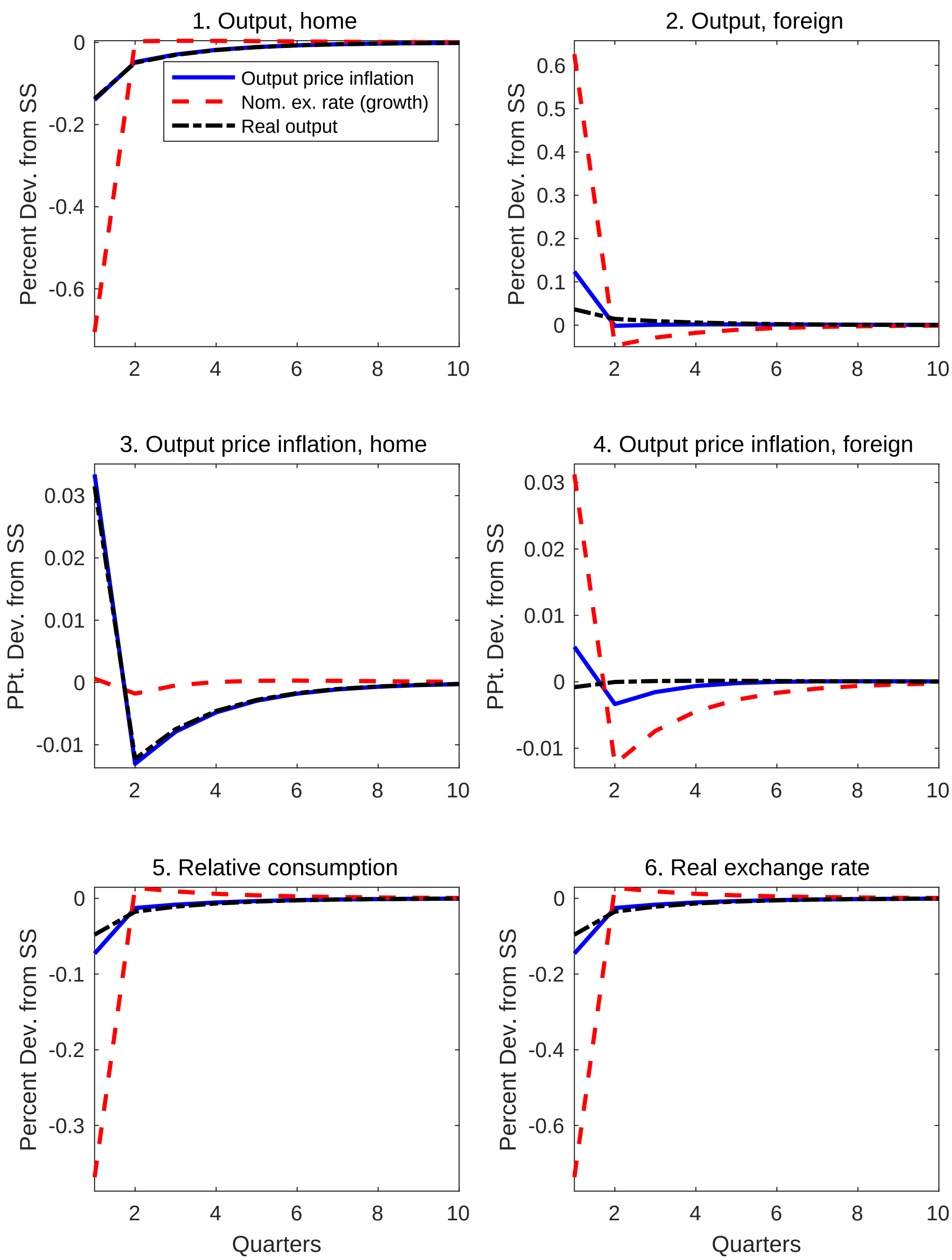
Notes: The figure plots the transition dynamics of the two economies after a one-standard deviation increase in the price markup in the home country. The two lines show the responses under full commitment with cooperation (Cooperative Policy) and without cooperation (Open-Loop Nash), when policymakers use output price inflation in their respective country as the policy instrument.

Figure 3: Assessing Optimality of Policy Choices



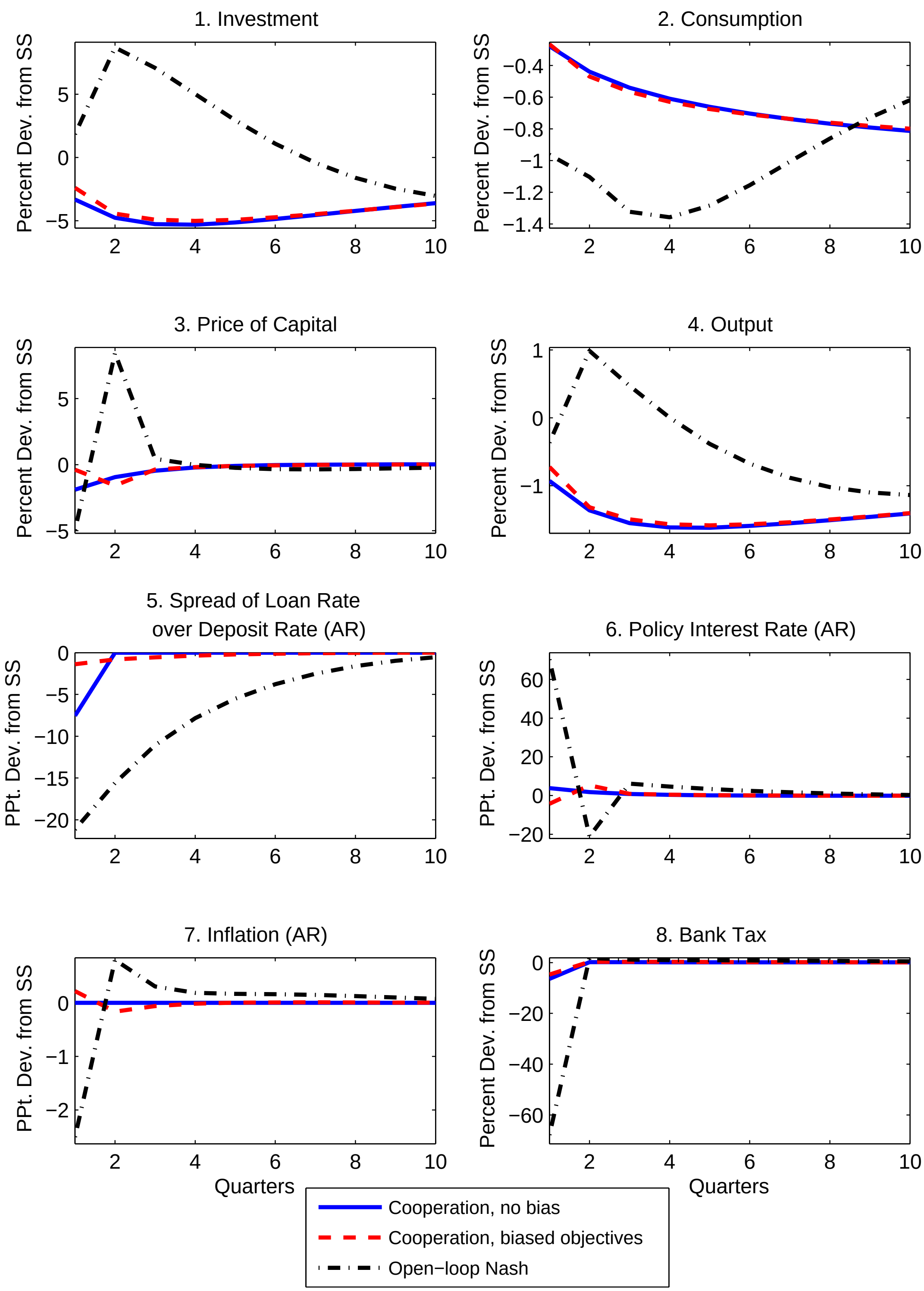
Notes: The top panel plots the difference between conditional welfare under the optimal cooperative policy and a suboptimal policy that assigns weight on both an arbitrary policy rule and the optimal cooperative policy. With the instruments under the cooperative policy chosen as π_t and π_t^*, the suboptimal policy sets producer price inflation in the home country to follow $\pi_t^s = \upsilon^s \pi + (1 - \upsilon^s)\, \pi_t$ and to follow $\pi_t^{s*} = \pi_t^*$ in the foreign country. The welfare difference being minimized at $\upsilon^s = 0$ implies that the arbitrary rule under consideration cannot improve upon the optimal cooperative policy. The bottom panel reports the results from a similar exercise in the open-loop Nash game by asking whether the home country can improve upon the optimal strategy under in the Nash game by assigning weight to the prescription from an arbitrary policy rule. Assuming that the foreign country sets the foreign inflation rate in accordance with its strategy in the open-loop Nash game, the home country prefers the strategy from the open-loop Nash game to any mixture that assigns positive weight to the arbitrary rule under consideration. Similar results obtain when the roles of the home and foreign country are reversed.

Figure 4: Strategy-space under Open-loop Nash policies in the Open Economy Model: Responses to a Markup Shock



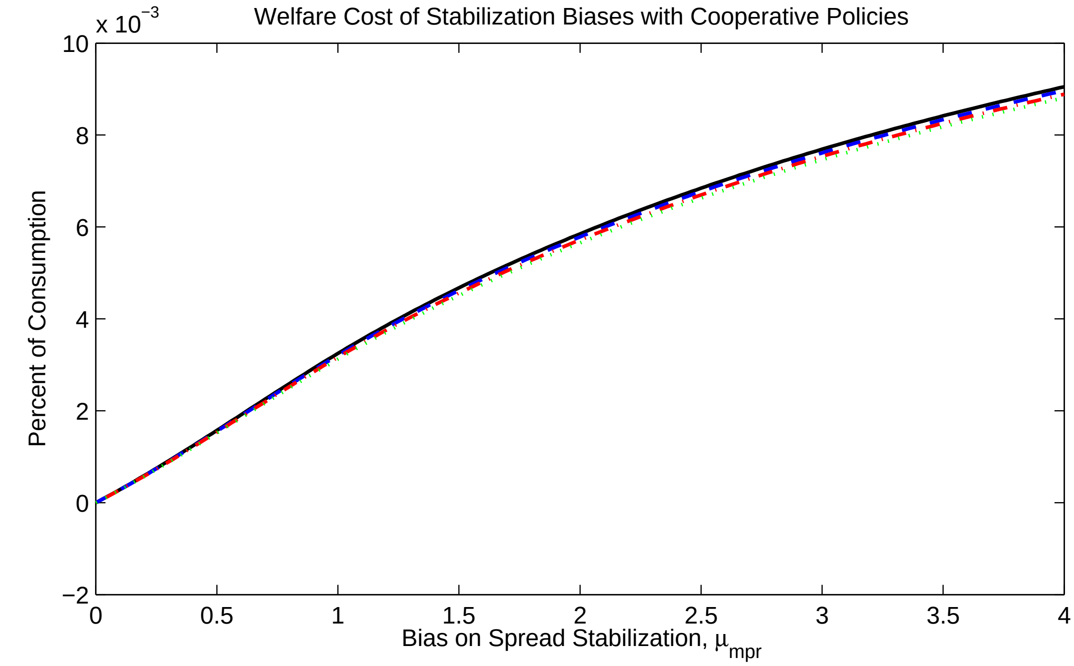
Notes: The figure plots the transition dynamics of the three economies after a one-standard deviation increase in markups in the home country. The three lines show the responses for the open-loop Nash game when policymakers use output price inflation, changes in the nominal exchange rate, and real output as instrument, respectively.

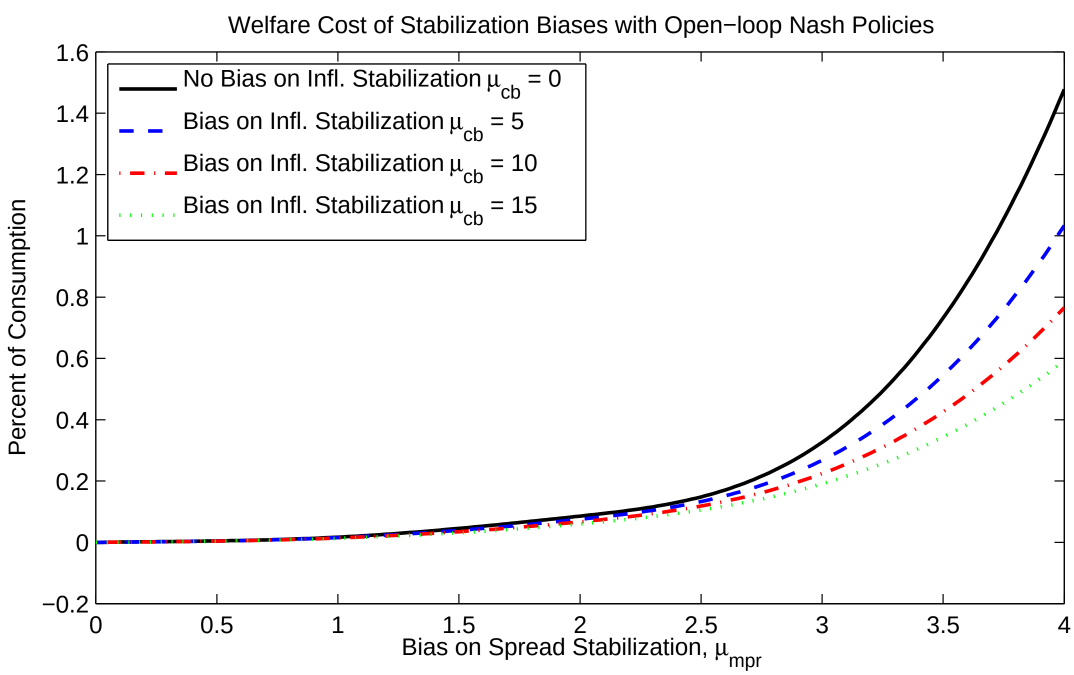
Figure 5: Cooperative and Open-loop Nash Policies in the Macroprudential Regulation Model: Responses to a Technology Shock



Notes: The figure plots the transition dynamics of the economy after a one-standard deviation decline in technology. The central bank uses inflation as its instrument and the macroprudential regulator uses the tax on bank capital as instrument. The three lines show the responses for the cases of cooperation with unbiased policy preferences, cooperation with biased policy preferences, and without cooperation and biased policy preferences, respectively.

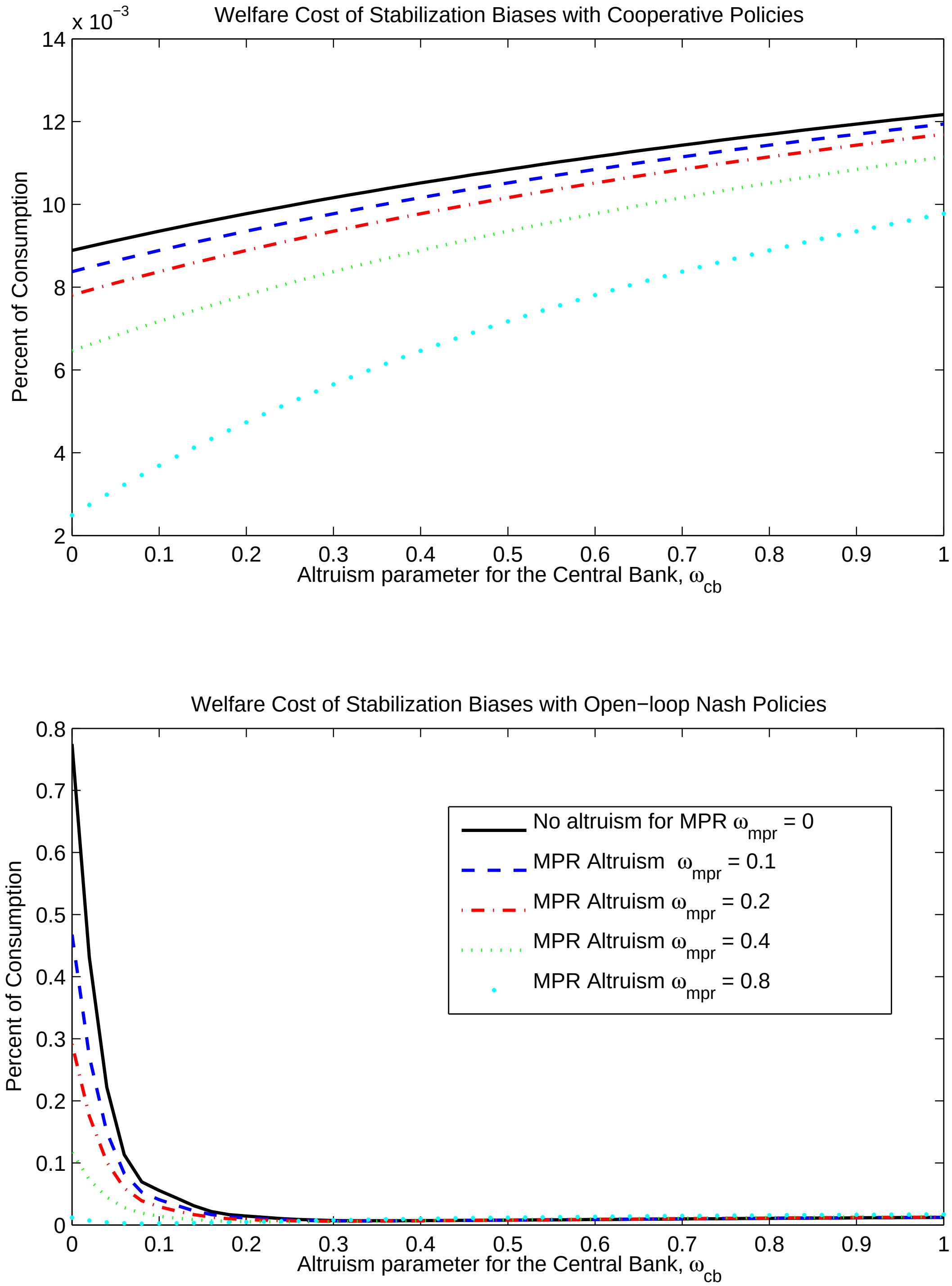
Figure 6: Welfare Implications of Biased Objectives





Notes: The figure plots the welfare costs as a function of the stabilization bias of the macroprudential regulator, μ_{mpr}. The welfare gains of going from a given model to the model without stabilization bias and cooperation is expressed as a consumption equivalent variation. The top panel shows the welfare costs of having biased objectives for the regulators regulators (using the unbiased objectives as welfare metric). The bottom panel plots the welfare costs of open-loop Nash policies if policymakers have biased objectives, relative to cooperative polices from the same biased objectives

Figure 7: Welfare Implications of Biased but Altruistic Objectives



Notes: The figure plots the welfare costs as a function of the altruism parameter for the macroprudential regulator, μ_{mpr}. The welfare gains of going from a given model to the model without stabilization bias and cooperation is expressed as a consumption equivalent variation. The top panel shows the welfare costs of having biased objectives for the regulators regulators (using the unbiased objectives as welfare metric). The bottom panel plots the welfare costs of open-loop Nash policies if policymakers have biased objectives relative to optimal cooperative policies from unbiased objectives.

A Description of Codes

The codes underlying this paper can be downloaded from
https://sites.google.com/site/martinbodenstein/ and from
http://www.lguerrieri.com/games_code.zip.
The zipped package includes five folders:

1. `nash_ramsey_toolbox` contains the codes for our toolbox,
2. `plot_support` contains plotting routines,
3. `BBCDL_model` contains the codes for the two-country model,
4. `GK_model` contains the codes for the macroprudential regulation model,
5. `LQ_BBCDL_model` contains the linear quadratic model by Corsetti, Dedola, and Leduc (2010) described in Appendix C.3.

A.1 Toolbox

The toolbox extends the functionality of Dynare (which needs to be installed separately). We have verified that our toolbox is compatible with Dynare 4.4.2 and earlier versions on Mac, Windows, and Linux platforms. Before attempting to run the examples in `BBCDL_model`, `GK_model`, `LQ_BBCDL_model` the paths in `setpathdynare4.m` need to reflect the local setup. The toolbox also requires access to the Matlab Symbolic Math Toolbox. The folder `nash_ramsey_toolbox` contains the codes of our toolbox. In order to generate the first-order conditions that characterize the optimal policies with and without cooperation using our toolbox, the user has to provide a Dynare-formatted model file. In addition to the structural equations derived from optimal behavior of households and firms, the file needs to specify the utility functions of the policymakers and an arbitrary description of the relevant policy rules (e.g., Taylor-style instrument rules in a two-country monetary model).[16] This input file is then used to generate an output file that contains the symbolic derivatives of the Lagrangian functions described in equation (3) for the Ramsey case and equation (14) for the open-loop Nash game. We first describe how to apply the toolbox; then we describe in more detail the key scripts of the toolbox.

A.1.1 Using the Toolbox

Using our toolbox requires the user to follow just a few of conventions. Through the rest of this section, we refer to the original Dynare-formatted model code as `example.mod`.

In `example.mod`:

1. Break the `var` block into two `var` blocks so that the first block contains `Util1`, `Util2`, and all endogenous variables and the second block contains all exogenous variables (the shocks). Insert the line `// Endogenous variables` or `// Exogenous variables` before each block, as appropriate.

[16] A primer on Dynare syntax can be found http://www.dynare.org/wp-repo/dynarewp001.pdf.

2. If parameter values are set directly in `example.mod`, remove them and save them as a separate script with the name `example_paramfile.m`.
3. In the `model` block, before the policy rule for each player, insert the line `// Policy Rule, agent 1` or `// Policy Rule, agent 2`, as appropriate.
4. If the steady-state values for the original N endogenous variables are set in the `initval` block delete the `initval` block and save the steady-state values for endogenous variables as a script in the same folder under the name `example_ss_defs.m`.
5. Collect the equations describing the paths of exogenous variables at the end of the `model` block, **after** all the structural equations.
6. Define the variables `Util1` and `Util2` in the `var` and add the objective functions of the policymakers in the `model` block. The equations defining `Util1` and `Util2` should be declared in the 'model' block as `Util1 = ...;` and `Util2 = ...;` and placed just above the block of the exogenous variables. [N.B.: This is a change from the first version of the toolbox introduced to facilitate the comparison of conditional welfare across models.]

Create a MATLAB function with the name `example_steadystate.m` in the same folder. Dynare will call this program to compute the steady-state of the model. The structure of `example_steadystate.m` should follow this template:

```
function [ys,check] = example_steadystate(junk,ys)
global M_
check = 0;


%% assign parameter values
example_paramfile


%% assign steady-state values
example_ss_defs


%% send parameters and steady states to dynare
nparams = size(M_.param_names,1);
for icount = 1:nparams
eval(['M_.params(icount) = ',M_.param_names(icount,:),';'])
end


nvars = M_.endo_nbr;
ys = zeros(nvars,1);
for i_indx = 1:nvars
eval(['ys(i_indx)=',M_.endo_names(i_indx,:),';'])
end
```

The file `example_steadystate.m` first calls the scripts `example_paramfile.m` to set the parameter values; calling `example_ss_defs.m` assigns the steady-state values of the endogenous variables in the model. The values are saved in the vectors `M_.params` and `ys`, respectively, in order to be passed to Dynare.

Now the model can be processed to create the desired output files by calling the script `convertmodfiles` which is described in the next section.

A.1.2 Description of Toolbox Programs

The first order conditions to the various policy problems associated with the model file `example.mod` are created by executing the script `convertmodfiles.m`. For the open-loop Nash game, calling

```
convertmodfiles('example','nash','instrument1','instrument2')
```

generates the necessary output files `example_nash.mod`, `example_nash_steadystate.m`, `example_nash_ss_defs.m`, and `example_nash_paramfile.m`.[17]

The inputs into `convertmodfiles.m` are:

- `infilename`: a string containing the name of the Dynare file containing the model we want to analyze. Here, we set infilename = `example`, although `example.mod` also works.
- `policy_problem`: a string that must be `ramsey`, `nash`, or `one_agent_ramsey`
  - If `policy_problem = ramsey`, then `convertmodfiles.m` will output the model equations for the cooperative optimal policy (Ramsey).
  - If `policy_problem = nash`, then `convertmodfiles.m` will output the model equations for the open-loop Nash game.
  - If `policy_problem = one_agent_ramsey`, then one of the two players follows the optimal policy given that the other player will follow the arbitrary policy rule that was specified in the original file `example.mod`.
- `instrument1`: a string, giving the name of the instrument variable in the model for the first player. If `policy_problem = one_agent_ramsey`, this is the instrument used by the one player choosing the optimal policy for an arbitrary policy function of the other player.
- `instrument2`: a string, giving the name of the instrument for the second agent. If `policy_problem = one_agent_ramsey`, this should be `'1'` or `'2'`, representing the one player choosing the policy optimally.

Executing the file `convertmodfiles.m` calls the following sequence of scripts:

1. `get_aux.m`
   - replaces lagged endogenous variables in the `model` block with auxiliary variables, which are also inserted under the `var` block as endogenous variables. Given endogenous variables `var_1,...,var_K` entering the structural equations or the utility functions with their lagged values, `get_aux.m` adds `var_1lag,`

[17] The default names of the output files can be changed in to also reflect the names of the instruments.

...,`var_Klag` to the end of the block of endogenous variables in the `var` block, and adds the equations
`var_1lag = var_1(-1);`...`var_nlag = var_n(-1);`
in the 'model' block.

- given `policy_problem`, the script adds appropriate policy variables (`instr1` and `instr2`), parameters (`omega_welf1`, `omega_welf2`, `beta`), and welfare definitions to the Dynare model.The new temporary Dynare file is saved as `example_aux.mod`.
- edits the existing files `example_paramfile.m`, `example_steadystate.m`, and `example_ss_defs.m` to account for the auxiliary and policy variables, parameters, and equations. The new files are named `example_aux_paramfile.m`, `example_aux_steadystate.m`, and `example_aux_ss_defs.m`, respectively.

2. then, depending on the choice of `policy_problem.m`,
   - `get_nash.m` followed by `make_ss_nash` if `policy_problem = nash` to generate the first order conditions of the problem,
   - `get_ramsey.m` followed by `make_ss_ramsey` if `policy_problem = ramsey` to generate the first order conditions of the problem,
   - or, finally, `get_one_agent_ramsey.m` followed by `make_ss_one_agent_ramsey` if `policy_problem = ramsey` to generate the first order conditions of the problem.

We restrict the detailed description to the case of `policy_problem = nash`. The program `get_nash.m`, builds on the program `get_ramsey.m` originally provided by Lopez-Salido and Levin (2004) to find optimal Ramsey policies.[18] Taking the input `example_aux.mod`, `get_nash.m` outputs

1. `example_nash.mod` which contains the first order conditions of the players and removes the arbitrary policy rules from the model.
2. `example_nash_lmss.m` which contains the subset of first order conditions that is linear in the Lagrange multipliers evaluated in the steady state.

Next, the file `make_ss_nash.m` creates four auxiliary files

- `example_nash_steadystate.m`,
- `guess_example_nash_steadystate.m`,
- `example_nash_ss_defs.m`,
- `example_nash_paramfile.m`.

As we have introduced additional endogenous variables, the steady-state values of the existing endogenous variables may have changed and the steady-state values of the new endogenous variables are unspecified. `example_nash_steadystate.m` uses the values provided by `example_nash_ss_defs.m` and `example_nash_lmss.m` to find the new steady-state values via `guess_example_nash_steadystate.m`. To facilitate computation of the new steady state `example_nash_steadystate.m` allows for the choice of

[18] Our version of `get_ramsey.m` extends the version distributed by Lopez-Salido and Levin (2004) by allowing lagged dependent variables in the objective functions.

different algorithms. `example_nash_paramfile.m` sets the same parameter values as `example_paramfile.m`. In addition, the policy parameters are assigned the default values

$$\texttt{omega\_welf1} = 0.5$$
$$\texttt{omega\_welf2} = 0.5$$
$$\texttt{nbeta} = 0.99.$$

The toolbox includes additional programs that may be of use to researchers:

- `add_welfare_vars.m` augments the Dynare model files that have been set up with period utility defined by `Util1` and `Util2` to define the variables `Welf1` and `Welf2` (cumulative welfare variables for each agent) along with `Util` and `Welf` (joint utility and welfare variables using welfare weights `omega_ welf1` and `omega_welf2`).
- `edit_shocks.m` takes in a character matrix of shocks (or the strings `'all'` or `'none'`) and turns on those shocks in all Dynare model files in the current folder. This is helpful when running a program which compares the effects of different groups of shocks in a model.
- `add_shadow_economy.m` takes in four inputs `orig_modfile`, `ramsey_type`, `instrument1`, `instrument2`. `orig_modfile` is the name of the original model file that characterizes the decentralized equilibrium (i.e., `example` in the description above). `ramsey_type` is type of equilibrium desired, either `nash`, `ramsey`, or `one_agent_ramsey`. `instrument1` and `instrument2` govern the instruments used in the implementation of the policies under the equilibrium type chosen. This routine produces a single Dynare .mod file that defines two distinct models. One model is a copy of the model in the original .mod file. The other model is taken from the output of the function `convertmodelfiles` which needs to be called with analogous input arguments prior to invoking `add_shadow_economy`. To keep track of two stacked economies in the same file, the names of the endogenous variables for the original model are changed. The prefix `shadow_` is appended to the names of those variables. Crucially, the innovations to the exogenous variables are imposed to be common across the two models. This setup has multiple purposes:
  1. It facilitates the exploration of instrument rules that respond to variables in both models. Crucially, these rules, could be made to converge to the policy rules to the optimal cooperative policies or to the open-loop Nash policies (or any linear combination of an arbitrary instrument rule and of the optimal rules).
  2. It facilitates the exploration of the costs of suboptimal rules. For this purpose, the cooperative or Nash .mod files created by `convertmodelfiles` need to be augmented with an extra variable capturing the value of commitment to a policy under the timeless perspective. This variable is automatically produced by our routines if `convertmodelfiles` is invoked with an additional argument set to 1.
  3. This routine can also be used to check second-order conditions in line with the method proposed in the main body of the paper.

A.2 Replication Codes

The replication codes for Figures 1, 2, and 4 are stored in the folder `BBCDL_model_excl2ndorder`. The codes for Figure 3 are stored in the folder `BBCDL_model_2ndorder`. The codes for Figures 5 and 6 are provided in the folder `GK_model`. The codes for Figure 7 are stored in the folder `GK_model_with_altruism`.

Finally, the folder `LQ_BBCDL_model` contains the model described in Appendix C.3. The file `call_LQBBCDL` computes the impulse responses to a cost push shock for the linear quadratic model stored in `LQBBCDL.mod` and compares them to those derived from the toolbox output `BBCDLmodelcomp_ramsey_c1pid_c2pid.mod`.

A.2.1 Open Economy Model

`BBDCLmodelcomp.mod` in the folder `BBCDL_model_excl2ndorder` is the Dynare file containing the original model described in equations (36) to (60) with variables to be log-linearized where appropriate, i.e., the variables are surrounded by the expression `exp()`. This model file is ready for being processed by our toolbox. In particular, notice

- the separation of variables into the two blocks of `// Endogenous variables` and `// Exogenous variables`,
- the definition of the period-utility functions of the two policymakers as `Util1` and `Util2`,
- the labelling of the policy rules by `// Policy Rule`,
- the ordering of putting the equations for the exogenous shock processes at the end of the model block.

Variables for the home country carry the prefix `c1`; variables for the foreign carry the prefix `c2`.

The model file is accompanied by three user-provided Matlab m-files

- `BBCDLmodelcomp_paramfile` sets the parameter values (via calling the parameter file stored in the folder `parameterfiles` labeled `paramfile_BB` which is common across all model files),
- `BBCDLmodelcomp_ss_defs` assigns the steady-state values to all variables,
- `BBCDLmodelcomp_steadystate` which, after calling the previous two files, sends the parameter and steady-state values to Dynare.

All relevant files for the Ramsey and the open-loop Nash problem are created by calling `convertmodfiles` via `CREATE_RAMSEY_AND_NASH` in the folder `BBCDL_model`. The first line in this script augments the Matlab path to include our toolbox. Output price inflation is denoted by `c1pid` and `c2pid` for countries 1 and 2, respectively. Consumer price inflation is labeled `c1dcore` and `c2dcore`. The files associated with any specific model carry the instrument labels in the file name.

For example, the files needed to compute the solution to the Nash problem using output price inflation as instruments are

- `BBCDLmodelcomp_nash_c1pid_c2pid.mod` containing the final model,

- `BBCDLmodelcomp_nash_c1pid_c2pid_paramfile` setting parameters by calling `paramfile_BB` and assigning values to `omega_welf1`, `omega_welf2`, `nbeta`,
- `BBCDLmodelcomp_nash_c1pid_c2pid_steadystate` generating the new steady state,
- `guess_BBCDLmodelcomp_nash_c1pid_c2pid_steadystate` computing the steady state using the steady state of `BBCDLmodelcomp.mod` as starting guess,
- `BBCDLmodelcomp_nash_c1pid_c2pid_ss_defs` initializing guess for steady-state values of structural variables and via
- `BBCDLmodelcomp_nash_c1pid_c2pid_lmss` initialising the steady-state guess for the Lagrange multipliers.

Notice, that our toolbox assigns the default values

$$\begin{aligned}\texttt{omega\_welf1} &= 0.5\\ \texttt{omega\_welf2} &= 0.5\\ \texttt{nbeta} &= 0.99\end{aligned}$$

to the policy parameters. The steady state of the new model may need to be computed numerically. `BBCDLmodelcomp_nash_c1pid_c2pid_steadystate` allows for different algorithms to be employed by choosing the desired element of `algo` in the `options` variable.

A.2.2 Macroprudential Regulation Model

`rbcb_monprud.mod` is the Dynare file containing the original model with biased objectives described in equations (104) to (129).[19] This model file is ready for being processed by our toolbox. In particular, notice

- the separation of variables into the two blocks of `// Endogenous variables` and `// Exogenous variables`,
- the definition of the period-utility functions of the two policymakers as `Util1` and `Util2`,
- the labelling of the policy rules by `// Policy Rule`,
- the ordering of putting the equations for the exogenous shock processes at the end of the model block.

The model file is accompanied by three user-provided Matlab m-files

- `rbcb_monprud_paramfile` sets the parameter values (via calling the parameter files in the folder `parameterfiles`),
- `rbcb_monprud_ss_defs` assigns the steady-state values to all variables,
- `rbcb_monprud_steadystate` which, after calling the previous two files, sends the parameter and steady-state values to Dynare.

[19] An additional model file with unbiased objectives is provided under the name `rbcb_monprud_nobias.mod`.

All relevant files for the Ramsey and the open-loop Nash problem are created by calling `convertmodfiles` via `CREATE_RAMSEY_AND_NASH` located in the folder `GK_model`. The first line in this script augments the Matlab path to include our toolbox. Inflation is denoted by `infl` and the bank transfer by `bt`. The files associated with any specific model carry the instrument labels in the file name.

For example, the files needed to compute the solution to the Nash problem using output price inflation as instruments are

- `rbcb_monprud_nash_infl_bt.mod` containing the final model,
- `rbcb_monprud_nash_infl_bt_paramfile` setting parameters by calling the parameter files located in the folder `parameterfiles` and assigning values to `omega_welf1`, `omega_welf2`, `nbeta`,
- `rbcb_monprud_nash_infl_bt_steadystate` generating the new steady state,
- `guess_rbcb_monprud_nash_infl_bt_steadystate` recomputing the steady state using the steady state of `rbcb_monprud.mod` as starting guess,
- `rbcb_monprud_nash_infl_bt_ss_defs` initializing guess for steady-state values of structural variables and via
- `rbcb_monprud_nash_infl_bt_lmss` initialising the steady-state guess for the Lagrange multipliers.

Notice, that our toolbox assigns the default values

$$\begin{aligned}
\texttt{omega_welf1} &= 0.5 \\
\texttt{omega_welf2} &= 0.5 \\
\texttt{nbeta} &= 0.99
\end{aligned}$$

to the policy parameters. Furthermore, the steady state of the new model may need to be computed numerically. `rbcb_monprud_nash_infl_bt_steadystate` allows for different algorithms to be employed by choosing the desired element of `algo` in the `options` variable.

B Relationship to Linear-Quadratic Approach

To see the connection between the LQ approach and the approach followed in our toolbox, assume we were interested in the solution to the problem stated in Equation (2) obtained from the linear approximation of the first order conditions (4) to (7) around the optimal steady state. Under the timeless perspective, the first order conditions with respect to the endogenous variables can then be approximated by

$$\begin{aligned}
&\sum_{j=1,2} \omega_j \left\{ D^2_{xx^-}\bar{U}_j \hat{x}_{t-1} + \left[D^2_{xx}\bar{U}_j + \beta D^2_{x^-x^-}\bar{U}_j \right] \hat{x}_t + \beta D^2_{x^-x}\bar{U}_j E_t \hat{x}_{t+1} \right\} \\
&+ \sum_{j=1,2} \omega_j \left\{ D^2_{x\zeta}\bar{U}_j \zeta_t + \beta D^2_{x^-\zeta}\bar{U}_j E_t \zeta_{t+1} \right\} \\
&+ \beta\bar{\lambda} \left\{ D^2_{x^-x^-}\bar{g}\hat{x}_t + D^2_{x^-x}\bar{g} E_t \hat{x}_{t+1} + D^2_{x^-x^+}\bar{g} E_t \hat{x}_{t+2} + D^2_{x^-\zeta}\bar{g} E_t \zeta_{t+1} \right\} \\
&+ \bar{\lambda} \left\{ D^2_{xx^-}\bar{g}\hat{x}_{t-1} + D^2_{xx}\bar{g}\hat{x}_t + D^2_{xx^+}\bar{g} E_t \hat{x}_{t+1} + D^2_{x\zeta}\bar{g}\zeta_t \right\} \\
&+ \beta^{-1}\bar{\lambda} \left\{ D^2_{x^+x^-}\bar{g}\hat{x}_{t-2} + D^2_{x^+x}\bar{g}\hat{x}_{t-1} + D^2_{x^+x^+}\bar{g}\hat{x}_t \ + D^2_{x^+\zeta}\bar{g}\zeta_{t-1} \right\} \\
&+ \beta E_t D_{x^-}\bar{g}' \hat{\lambda}_{t+1} + D_x \bar{g}' \hat{\lambda}_t + \beta^{-1} D_{x^+}\bar{g}' \hat{\lambda}_{t-1} = 0. \qquad (22)
\end{aligned}$$

Note that we have augmented the partial derivatives of the utility functionals to include derivatives with respect to the instrument variables $i_{1,t}$ and $i_{2,t}$ — which are zero — to simplify notation. The notation $D^2_{xx^-}$ marks the matrix of second derivatives of a function with respect to x and x^-. $\bar{U}_j$ and $\bar{g}$ is used as short-hand to indicate that a function (or its partial derivatives) is evaluated at the steady-state values $\{\bar{x}, \bar{\lambda}\}$. 'Hatted' variables refer to the deviation of the original variable from its steady-state value. Regrouping terms delivers

$$\begin{aligned}
&\bar{\lambda}\left[\beta^{-1} D^2_{x^+x^-}\bar{g}\right]\hat{x}_{t-2} + \left\{ \sum_{j=1,2} \omega_j D^2_{xx^-}\bar{U}_j + \bar{\lambda}\left[D^2_{xx^-}\bar{g} + \beta^{-1} D^2_{x^+x}\bar{g}\right] \right\} \hat{x}_{t-1} \\
&+ \left\{ \sum_{j=1,2} \omega_j \left[D^2_{xx}\bar{U}_j + \beta D^2_{x^-x^-}\bar{U}_j \right] + \bar{\lambda}\left[D^2_{xx}\bar{g} + \beta D^2_{x^-x^-}\bar{g} + \beta^{-1} D^2_{x^+x^+}\bar{g} \right] \right\} \hat{x}_t \\
&+ \left\{ \sum_{j=1,2} \omega_j \beta D^2_{xx^-}\bar{U}_j + \beta\bar{\lambda}\left[D^2_{xx^-}\bar{g} + \beta^{-1} D^2_{x^+x}\bar{g}\right] \right\}' E_t \hat{x}_{t+1} \\
&+ \beta^2\bar{\lambda}\left[\beta^{-1} D^2_{x^+x^-}\bar{g}\right]' E_t \hat{x}_{t+2} + \left\{ \sum_{j=1,2} \omega_j \beta D^2_{x^-\zeta}\bar{U}_j + \beta\bar{\lambda} D^2_{x^-\zeta}\bar{g} \right\} E_t \zeta_{t+1} \\
&+ \left\{ \sum_{j=1,2} \omega_j D^2_{x\zeta}\bar{U}_j + \bar{\lambda} D^2_{x\zeta}\bar{g} \right\} \zeta_t + \beta^{-1}\bar{\lambda} D^2_{x^+\zeta}\bar{g}\zeta_{t-1} \\
&+ \beta E_t D_{x^-}\bar{g}' \hat{\lambda}_{t+1} + D_x \bar{g}' \hat{\lambda}_t + \beta^{-1} D_{x^+}\bar{g}' \hat{\lambda}_{t-1} = 0 \qquad (23)
\end{aligned}$$

which coincides with the first order conditions of the following LQ problem

$$\begin{aligned}
&\max_{\{\hat{x}_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2}\hat{x}_t' A(L)\hat{x}_t + \hat{x}_t' B(L)\zeta_{t+1}\right] \\
&s.t. \\
&E_t C(L)\hat{x}_{t+1} + D(L)\zeta_t = 0 \\
&C(L)\hat{x}_0 = d_0
\end{aligned} \tag{24}$$

where

$$\begin{aligned}
A_2 &= \bar{\lambda}\left[\beta^{-1} D^2_{x^+x^-}\bar{g}\right] \\
A_1 &= \sum_{j=1,2} \omega_j D^2_{xx^-}\bar{U}_j + \bar{\lambda}\left[D^2_{xx^-}\bar{g} + \beta^{-1} D^2_{x^+x}\bar{g}\right] \\
A_0 &= \sum_{j=1,2} \omega_j \left[D^2_{xx}\bar{U}_j + \beta D^2_{x^-x^-}\bar{U}_j\right] + \bar{\lambda}\left[D^2_{xx}\bar{g} + \beta D^2_{x^-x^-}\bar{g} + \beta^{-1} D^2_{x^+x^+}\bar{g}\right] \\
A(L) &= A_0 + A_1 L + A_2 L^2 \\
B(L) &= \left\{\sum_{j=1,2} \omega_j \beta D^2_{x^-\zeta}\bar{U}_j + \beta\bar{\lambda} D^2_{x^-\zeta}\bar{g}\right\} + \left\{\sum_{j=1,2} \omega_j D^2_{x\zeta}\bar{U}_j + \bar{\lambda} D^2_{x\zeta}\bar{g}\right\} L \\
&\quad + \beta^{-1}\bar{\lambda} D^2_{x^+\zeta} L^2 \\
C(L) &= D_{x^+}\bar{g} + D_x\bar{g}L + D_{x^-}\bar{g}L^2 \\
D(L) &= D_\zeta \bar{g}.
\end{aligned}$$

The constraint $C(L)\hat{x}_0 = d_0$ is added to implement the timeless perspective by an appropriate choice of d_0. Benigno and Woodford (2012) refer to the program in equation (24) as the "correct LQ approximation" and they show how to derive the correct LQ program directly from the original problem stated in (2) rather than going through the first order conditions associated with (2), which is the approach followed by Levine, Pearlman, and Pierse (2008). Using the above definitions, it is easy to compute the matrices for the LQ problem from our toolbox output numerically. Hence, to a first order approximation the output of our toolbox is equivalent to that of the LQ approach.

C Open Economy Model

We first illustrate our toolbox for a two-country monetary model that closely follows Benigno and Benigno (2006) and Corsetti, Dedola, and Leduc (2010). These authors characterize the optimal monetary policies both with and without cooperation between two central banks in dynamic general equilibrium models with sticky prices. To this end, they derive the true linear quadratic approximation of the model. As discussed in Section 2.4, for given choice of policy instruments and strategies of the players, the linear-quadratic approach delivers the same output as our toolbox if we take a linear approximation of the first-order conditions of the two central banks around the deterministic steady state.

C.1 Model Environment

The two countries are equal in size and symmetric in their economic structure. We only describe the economy of country 1 in detail.

C.1.1 Households

Following Benigno and Benigno (2006) and Corsetti, Dedola, and Leduc (2010) each country is populated by a continuum of households. Each of them engages in the production of a specific good for which the household uses its own labor as the sole input. The good produced by household h carries the index f. Before describing the production and pricing of goods in detail, we first set up the household's optimization problem for given labor and production choices, $L_t(h)$ and $Y_t(f)$ with financial markets being complete at the domestic and the international level

$$\begin{aligned}
&\max_{\{C_t(h),B_{D,t+1}(h),B_{F,t+1}(h)\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t(h)^{1-\sigma}}{1-\sigma} - \chi_0 \frac{L_t(h)^{1+\chi}}{1+\chi} \right) \\
&s.t. \\
&P_{C,t}C_t(h) + \int_S Q_{D,t}B_{D,t+1}(h) + \int_S e_t Q_{F,t} B_{F,t+1}(h) + T_t(h) \\
&= P_t(f)Y_t(f) + B_{D,t}(h) + e_t B_{F,t}(h) \qquad (25)
\end{aligned}$$

Household f uses its income on consumption, $P_{C,t}C_t(h)$, on the acquisition of domestic bonds in domestic currency, $\int_S Q_{D,t}B_{D,t+1}(h)$, and foreign bonds priced in foreign currency, $\int_S e_t Q_{F,t}B_{F,t+1}(h)$, and on lump-sum taxes, $T_t(h)$. The nominal exchange rate is denoted by e_t. Income is derived from selling its product, $P_t(f)Y_t(f)$, as well as the payoffs from foreign and domestic bonds, $Q_{F,t}B_{F,t}(h) + Q_{D,t}B_{D,t}(h)$.

Consumption utility is derived from consuming a domestic good, $C_{D,t}(h)$, and a foreign good, $C_{M,t}(h)$, according to

$$C_t(h) = \left(\omega_c^{\frac{\rho_c}{1+\rho_c}} C_{D,t}(h)^{\frac{1}{1+\rho_c}} + (1-\omega_c)^{\frac{\rho_c}{1+\rho_c}} C_{M,t}(h)^{\frac{1}{1+\rho_c}} \right)^{1+\rho_c} \qquad (26)$$

with the goods price in domestic currency being denoted by P_t and $P_{M,t}$, respectively. Under the assumption of producer currency pricing, the law of one price holds absent transportation costs and the price of the imported foreign good equals the price of the foreign good in the foreign country adjusted by the nominal exchange rate, $P_{M,t} = e_t P_t^*$. The price of the final consumption good, $P_{C,t}$, is obtained from minimizing the costs of obtaining final consumption, $C_t(h)$, subject to the constraint (26).

C.1.2 Production of Final Goods

Competitive producers of the domestic good, Y_t, aggregate a variety of intermediate goods, $Y_t(f)$, produced by the home country's households using the production technology

$$Y_t = \left[\int_0^1 Y_t(f)^{\frac{1}{1+\nu_p}} df \right]^{1+\nu_p}. \tag{27}$$

Profit maximization delivers the well-known result for the price of the domestic good, P_t,

$$P_t = \left[\int_0^1 P_t(f)^{-\frac{1}{\nu_p}} df \right]^{-\nu_p} \tag{28}$$

and the demand function for each variety $Y_t(f)$

$$Y_t(f) = \left[\frac{P_t(f)}{P_t} \right]^{-\frac{1+\nu_p}{\nu_p}} Y_t. \tag{29}$$

C.1.3 Production by Households

Each household produces exactly one variety $Y_t(f)$ and engages in monopolistic competition with all other households. A household chooses its price so as to maximize its utility. Following Calvo (1983) the probability of adjusting prices in a given period is $1 - \xi_p$.

Assuming household h uses a linear technology to produce good f, it is

$$Y_t(f) = (e^{z_t})^{\frac{\chi}{1+\chi}} L_t(h), \tag{30}$$

where the country-wide technology shock, z_t, evolves according to $z_t = \rho_z z_{t-1} + \sigma_z \varepsilon_{z,t}$. The production and pricing problem of household h can be stated as

$$\begin{aligned}
&\max_{P_t(f),\{Y_{t+i}(f)\}_{i=0}^{\infty}} E_t \sum_{i=0}^{\infty} \left(\xi_p \beta\right)^i \left\{ (1+\tau_{p,t}) \frac{C_{t+i}(h)^{-\sigma}}{P_{C,t+i}} P_t(f)\, Y_{t+i}(f) - \chi_0 \left(e^{z_{t+i}}\right)^{-\chi} \frac{Y_{t+i}(f)^{1+\chi}}{1+\chi} \right\} \\
&s.t. \\
&Y_{t+i}(f) = \left[\frac{P_{t+i}(f)}{P_{t+i}} \right]^{-\frac{1+\nu_p}{\nu_p}} Y_t.
\end{aligned} \tag{31}$$

The variable $\tau_{p,t}$ captures an exogenous time-varying subsidy on sales and is isomorphic to mark-up shocks.

C.1.4 Market Clearing

Aggregating over households, market-clearing for the domestic good requires

$$Y_t = C_{D,t} + C^*_{M,t} + G_t \tag{32}$$

where $C^*_{M,t}$ denotes the foreign country's demand for the domestic good and G_t is the demand for the domestic good due to government spending.

Bonds are in zero net-supply, requiring $B_{D,t+1} = 0$ and $B_{F,t+1} + B^*_{F,t+1} = 0$. Finally, the budget constraint of the government is balanced in every period by adjusting lump-sum taxes, T_t, to the stochastic government purchases, G_t. The share of government consumption in output, $\frac{G_t}{Y_t}$, evolves according to

$$\omega_{gy,t} = \rho_{gy}\omega_{gy,t-1} + \sigma_{gy}\varepsilon_{gy,t} \tag{33}$$

where $\omega_{gy,t}$ measures the deviation of $\frac{G_t}{Y_t}$ from its steady-state value.

C.1.5 Equilibrium Conditions and Calibration

Using the notation introduced in Section 2.1, the endogenous variables are collected in the vector

$$\tilde{x}_t = \begin{pmatrix} C_t, C_{D,t}, C_{M,t}, Y_t, G_t, \frac{P_{C,t}}{P_t}, \pi_t, H_{p,t}, G_{p,t}, \frac{P_t^{opt}}{P_t}, \Delta_t, R_t^n, q_t, \\ C^*_t, C^*_{D,t}, C^*_{M,t}, Y^*_t, G^*_t, \frac{P^*_{C,t}}{P^*_t}, \pi^*_t, H^*_{p,t}, G^*_{p,t}, \frac{P_t^{opt*}}{P^*_t}, \Delta^*_t, R_t^{n*} \end{pmatrix}' \tag{34}$$

where the variables $Q_{D,t}, Q_{F,t}, B_{D,t+1}, B_{F,t+1}, T_t, \Pi_t, e_t$ and their foreign counterparts are omitted from $\tilde{x}_t$, since they assume the value of zero in equilibrium or are substituted out in the following. The vector of endogenous variables includes producer price inflation, defined as $\pi_t = \frac{P_t}{P_{t-1}}$, and the nominal interest rate R_t^n. The exogenous variables are collected in vector

$$\zeta_t = \left(z_t, \tau_{p,t}, G_t, z^*_t, \tau^*_{p,t}, G^*_t\right)'. \tag{35}$$

For illustration, we assume as in Benigno and Benigno (2006) that the policymakers use producer price inflation rates π_t and π^*_t as instruments.[20] Without detailed derivations, we provide a complete list of the conditions characterising the private sector equilibrium for given policies in the model.

The following equations result from the households' optimization problems:

[20] For this class of models, the open-loop Nash equilibrium is not unique if policymakers opt for the nominal interest rate as instrument. See for example Coenen, Lombardo, Smets, and Straub (2007) for a discussion of this issue.

1. derivatives with respect to C_t and C_t^* and $B_{D,t+1}$ and $B_{D,t+1}^*$ to define nominal interest rates

$$\beta E_t \left(\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_{C,t}}{P_t} \frac{P_{t+1}}{P_{C,t+1}} \frac{1}{\pi_{t+1}} \right) = \frac{1}{1+R_t^n} \tag{36}$$

$$\beta E_t \left(\left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \frac{P_{C,t}^*}{P_t^*} \frac{P_{t+1}^*}{P_{C,t+1}^*} \frac{1}{\pi_{t+1}^*} \right) = \frac{1}{1+R_t^{n*}} \tag{37}$$

2. derivatives with respect to B_{Ft}

$$\kappa_0 \left(\frac{C_t^*}{C_t} \right)^{-\sigma} = q_t \tag{38}$$

with q_t denoting the consumption based real exchange rate and $\kappa_0 = q_0 \left(\frac{C_0^*}{C_0} \right)^{-\sigma}$

3. optimal choice of $C_{D,t}$, $C_{D,t}^*$ imply

$$C_{D,t} = \omega_c C_t \left(\frac{P_{C,t}}{P_t} \right)^{\frac{1+\rho_c}{\rho_c}} \tag{39}$$

$$C_{D,t}^* = \omega_c^* C_t^* \left(\frac{P_{C,t}^*}{P_t^*} \right)^{\frac{1+\rho_c}{\rho_c}} \tag{40}$$

4. optimal choice of $C_{M,t}$, $C_{M,t}^*$ imply

$$C_{M,t} = C_t(1-\omega_c) \left(\frac{P_{C,t}^*}{P_t^*} \frac{1}{q_t} \right)^{\frac{1+\rho_c}{\rho_c}} \tag{41}$$

$$C_{M,t}^* = C_t^*(1-\omega_c^*) \left(\frac{P_{C,t}}{P_t} q_t \right)^{\frac{1+\rho_c}{\rho_c}} \tag{42}$$

5. the definition of the consumption goods C_t, and C_t^* impose

$$C_t = \left(\omega_c^{\frac{\rho_c}{1+\rho_c}} C_{D,t}^{\frac{1}{1+\rho_c}} + (1-\omega_c)^{\frac{\rho_c}{1+\rho_c}} C_{M,t}^{\frac{1}{1+\rho_c}} \right)^{1+\rho_c} \tag{43}$$

$$C_t^* = \left(\omega_c^{*\frac{\rho_c}{1+\rho_c}} C_{D,t}^{*\frac{1}{1+\rho_c}} + (1-\omega_c^*)^{\frac{\rho_c}{1+\rho_c}} C_{M,t}^{*\frac{1}{1+\rho_c}} \right)^{1+\rho_c}. \tag{44}$$

Profit maximisation by the intermediaries implies the following set of conditions:

1. the optimal (relative) price set by adjusting firms $\frac{P_t^{opt}}{P_t}$ and $\frac{P_t^{opt*}}{P_t^*}$

$$\left(\frac{P_t^{opt}}{P_t}\right)^{1+\frac{1+\nu_p}{\nu_p}\chi} = \frac{H_{p,t}}{G_{p,t}} \tag{45}$$

$$\left(\frac{P_t^{opt*}}{P_t^*}\right)^{1+\frac{1+\nu_p^*}{\nu_p^*}\chi} = \frac{H_{p,t}^*}{G_{p,t}^*} \tag{46}$$

2. with $H_{p,t}$ and $H_{p,t}^*$ following

$$\begin{aligned} H_{p,t} &= \frac{1+\nu_p}{\nu_p}\chi_0\left(\frac{Y_t}{e^{z_t}}\right)^{\chi}\frac{P_{C,t}}{C_t^{-\sigma}P_t}Y_t \\ &+ \xi_p\beta E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{P_{t+1}}{P_{C,t+1}}\frac{P_{C,t}}{P_t}\left(\frac{\bar{\pi}}{\pi_{t+1}}\right)^{-\frac{1+\nu_p}{\nu_p}(1+\chi)}H_{p,t+1}\right] \end{aligned} \tag{47}$$

$$\begin{aligned} H_{p,t}^* &= \frac{1+\nu_p^*}{\nu_p^*}\chi_0^*\left(\frac{Y_t^*}{e^{z_t^*}}\right)^{\chi}\frac{P_{C,t}^*}{C_t^{*-\sigma}P_t^*}Y_t^* \\ &+ \xi_p^*\beta E_t\left[\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\sigma}\frac{P_{t+1}^*}{P_{C,t+1}^*}\frac{P_{C,t}^*}{P_t^*}\left(\frac{\bar{\pi}^*}{\pi_{t+1}^*}\right)^{-\frac{1+\nu_p^*}{\nu_p^*}(1+\chi)}H_{p,t+1}^*\right] \end{aligned} \tag{48}$$

$\bar{\pi}$ is the steady-state (gross) inflation rate

3. with $G_{p,t}$ and $G_{p,t}^*$ following

$$\begin{aligned} G_{p,t} &= \frac{1+\tau_{p,t}}{\nu_p}Y_t \\ &+ \xi_p\beta E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{P_{t+1}}{P_{C,t+1}}\frac{P_{C,t}}{P_t}\left(\frac{\bar{\pi}}{\pi_{t+1}}\right)^{1-\frac{1+\nu_p}{\nu_p}}G_{p,t+1}\right] \end{aligned} \tag{49}$$

$$\begin{aligned} G_{p,t}^* &= \frac{1+\tau_{p,t}^*}{\nu_p^*}Y_t^* \\ &+ \xi_p^*\beta E_t\left[\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\sigma}\frac{P_{t+1}^*}{P_{C,t+1}^*}\frac{P_{C,t}^*}{P_t^*}\left(\frac{\bar{\pi}^*}{\pi_{t+1}^*}\right)^{1-\frac{1+\nu_p^*}{\nu_p^*}}G_{p,t+1}^*\right] \end{aligned}$$

(50)

4. the evolution of prices

$$(1-\xi_p)\left(\frac{P_t^{opt}}{P_t}\right)^{-\frac{1}{\nu_p}}+\xi_p\left(\frac{\bar{\pi}}{\pi_t}\right)^{-\frac{1}{\nu_p}}=1 \tag{51}$$

$$(1-\xi_p^*)\left(\frac{P_t^{opt*}}{P_t^*}\right)^{-\frac{1}{\nu_p^*}}+\xi_p^*\left(\frac{\bar{\pi}^*}{\pi_t^*}\right)^{-\frac{1}{\nu_p^*}}=1 \tag{52}$$

5. evolution of price dispersion

$$\Delta_t=(1-\xi_p)\left(\frac{P_t^{opt}}{P_t}\right)^{-\frac{1+\nu_p}{\nu_p}(1+\chi)}+\xi_p\left(\frac{\bar{\pi}}{\pi_t}\right)^{-\frac{1+\nu_p}{\nu_p}(1+\chi)}\Delta_{t-1} \tag{53}$$

$$\Delta_t^*=(1-\xi_p^*)\left(\frac{P_t^{opt*}}{P_t^*}\right)^{-\frac{1+\nu_p^*}{\nu_p^*}(1+\chi)}+\xi_p^*\left(\frac{\bar{\pi}^*}{\pi_t^*}\right)^{-\frac{1+\nu_p^*}{\nu_p^*}(1+\chi)}\Delta_{t-1}^* \tag{54}$$

The goods market clearing conditions are:

$$Y_t=C_{Dt}+C_{Mt}^*+G_t \tag{55}$$

$$Y_t^*=C_{Dt}^*+C_{Mt}+G_t^*. \tag{56}$$

Government spending is a fixed stochastic share of output:

$$G_t=\omega_{gy,t}Y_t \tag{57}$$

$$G_t^*=\omega_{gy,t}^*Y_t^*. \tag{58}$$

The period utility functions are:

$$U_t=\frac{C_t^{1-\sigma}}{1-\sigma}-\chi_0\left(e^{z_t}\right)^{-\chi}\frac{Y_t^{1+\chi}}{1+\chi}\Delta_t \tag{59}$$

$$U_t^*=\frac{C_t^{*1-\sigma}}{1-\sigma}-\chi_0^*\left(e^{z_t^*}\right)^{-\chi}\frac{Y_t^{*1+\chi}}{1+\chi}\Delta_t^*. \tag{60}$$

The policy rules, which will be replaced by the first order conditions of the policy-makers, are

$$R_t^n=(1+\bar{R}^n)\left(\frac{1+R_{t-1}^n}{1+\bar{R}^n}\right)^{\gamma_{R^n}}\left(\frac{\pi_t}{\bar{\pi}}\right)^{(1-\gamma_{R^n})\gamma_\pi}-1 \tag{61}$$

$$R_t^{n*} = (1+\bar{R}^{n*}) \left(\frac{1+R_{t-1}^{n*}}{1+\bar{R}^{n*}} \right)^{\gamma^*_{R^n}} \left(\frac{\pi_t^*}{\bar{\pi}^*} \right)^{(1-\gamma^*_{R^n})\gamma^*_\pi} - 1 \tag{62}$$

Augmenting the set of conditions (36)-(60) with the two definitions

$$i_t = \pi_t \tag{63}$$
$$i_t^* = \pi_t^* \tag{64}$$

we have cast the structural equations of the model into the form of (1)

$$E_t g(\tilde{x}_{t-1}, \tilde{x}_t, \tilde{x}_{t+1}, i_{1,t}, i_{2,t}, \zeta_t) = 0.$$

The step of adding equations (63) and (64) and removing the policy rules is automated by our toolbox.

The parameterization of the model is provided in Table 2. The choices are comparable to those in Benigno and Benigno (2006) and Corsetti, Dedola, and Leduc (2010). Most notably, by setting the coefficient governing the intertemporal elasticity of substitution σ equal to 2 and fixing the elasticity of substitution between traded goods at 2, the home and foreign good are substitutes in the utility function the household. Steady-state imports are about 15% of GDP, which reflects home-biased preferences, given that the two countries are equal in size and symmetric. Accordingly, the countries are equally weighted in the global welfare function.

Table 2: Parameters for the Open Economy Model

Parameter	**Used to Determine**	**Parameter**	**Used to Determine**
$\beta = 1/1.01$	discount factor	$\sigma = 2$	intertemporal consumption elasticity
$\chi = 0.5$	labor supply elasticity	$\bar{L} = 1$	steady-state labor supply to fix χ_0
$\frac{1+\rho^c}{\rho^c} = 2$	trade subst. elasticity	$\omega_c = 0.85$	home bias in consumption
$\xi_p = 0.75$	Calvo price parameter	$\frac{1+\nu_p}{\nu_p} = 10$	subst. elasticity of varieties
$\bar{\tau} = 1/9$	steady-state subsidy to producers	$\bar{\pi} = 1$	steady-state inflation
$\rho_z = 0.95$	persistence of tech. shock	$\sigma_z = 0.008$	std. of tech. shock
$\rho_\tau = 0$	persistence of cost push shock	$\sigma_\tau = 0.1$	std. of cost push shock
$\rho_{gy} = 0.99$	persistence of gov. spending shock	$\sigma_{gy} = 0.01$	std. of gov. spending shock
$\omega_{gy} = 0$	share of gov. spending	$\kappa_0 = 1$	
$\omega = 0.5$	weight on home country in Ramsey	$\omega^* = 0.5$	weight on foreign country in Ramsey

Note: This table summarizes the parameterization of the open economy model described in Section 3 at quarterly frequency.

C.2 Extensions

We briefly describe the additional equations if consumer price inflation is used as instruments. Using consumer price inflation, $\pi_{C,t} = \frac{P_{C,t}}{P_{C,t-1}}$ as the policy instrument, we need to define consumer price inflation by relating the relative price of consumption $\frac{P_{C,t}}{P_t}$ to producer price inflation:

$$\pi_{C,t} = \left(\frac{P_{C,t}}{P_t}\right)\left(\frac{P_{t-1}}{P_{C,t-1}}\right)\pi_t \tag{65}$$

$$\pi^*_{C,t} = \left(\frac{P^*_{C,t}}{P^*_t}\right)\left(\frac{P^*_{t-1}}{P^*_{C,t-1}}\right)\pi^*_t. \tag{66}$$

Furthermore, the vector of endogenous variables is modified to include $\pi_{C,t}$ and $\pi^*_{C,t}$, i.e.,

$$\tilde{x}_t = \begin{pmatrix} C_t, C_{D,t}, C_{M,t}, Y_t, G_t, \frac{P_{C,t}}{P_t}, \pi_t, H_{p,t}, G_{p,t}, \frac{P_t^{opt}}{P_t}, \Delta_t, R_t^n, q_t, \pi_{C,t}, \\ C_t^*, C_{D,t}^*, C_{M,t}^*, Y_t^*, G_t^*, \frac{P^*_{C,t}}{P^*_t}, \pi_t^*, H_{p,t}^*, G_{p,t}^*, \frac{P_t^{opt*}}{P_t^*}, \Delta_t^*, R_t^{n*}, \pi^*_{C,t} \end{pmatrix}'. \tag{67}$$

C.3 Relationship with Linear-Quadratic Solution

Corsetti, Dedola, and Leduc (2010) deviate from the setup in Benigno and Benigno (2006) by allowing for home bias, but by eliminating government spending. In the following, we allow for home bias, abstract form government spending, and focus on the case of the efficient steady state in order to restate the model presented in Corsetti, Dedola, and Leduc (2010) using our notation. Absent home bias ($\omega_c = \omega_c^* = 0.5$), this model coincides with the one in Benigno and Benigno (2006) for equally-sized countries.

The set of relevant structural relationships of the economy can be reduced to the following set of equations if the model is (log-)linearised around its deterministic steady state

$$\pi_t = \kappa\left(\widetilde{y}_t + \frac{\tau}{\chi+\sigma}\widetilde{\delta}_t + u_t\right) + \beta E_t \pi_{t+1} \tag{68}$$

$$\pi^*_t = \kappa^*\left(\widetilde{y}^*_t - \frac{\tau}{\chi+\sigma}\widetilde{\delta}_t + u^*_t\right) + \beta E_t \pi^*_{t+1} \tag{69}$$

$$\widetilde{y}_t - \widetilde{y}^*_t = \frac{1-2\tau}{\sigma}\widetilde{\delta}_t \tag{70}$$

where

$$\lambda = \frac{\left(1-\beta\xi_p\right)\left(1-\xi_p\right)}{\xi_p\left(1+\frac{1+\nu_p}{\nu_p}\chi\right)}$$

$$\lambda^* = \frac{\left(1-\beta\xi^*_p\right)\left(1-\xi^*_p\right)}{\xi^*_p\left(1+\frac{1+\theta^*_p}{\theta^*_p}\chi\right)}$$

$$\begin{aligned}
\kappa &= \lambda\left(\chi+\sigma\right)\\
\kappa^* &= \lambda^*\left(\chi+\sigma\right)\\
\tau &= -2\omega_c(1-\omega_c)\left(\sigma\frac{1+\rho_c}{\rho_c}-1\right).
\end{aligned}$$

Following Corsetti, Dedola, and Leduc (2010) we assume symmetry, i.e., $\omega_c = \omega_c^*$. As before, the remaining parameters governing preferences over types and timing of consumption and leisure are identical across countries. For the home country π_t denotes the producer price inflation rate in deviation from its steady state, $\widetilde{y}_t$ is the output gap, and $\widetilde{\delta}_t$ stands for the terms of trade gap. The terms of trade are denoted as the price of imports divided by the price of exports. π_t^* and $\widetilde{y}_t^*$ are defined analogously. Notice that for $\sigma\frac{1+\rho_c}{\rho_c} = 1$, the terms of trade interaction is shut down as discussed in Benigno and Benigno (2006).

Relative consumption and the real exchange rate gaps are determined as

$$\begin{aligned}
\widetilde{q}_t &= \sigma\left(\widetilde{c}_t-\widetilde{c}_t^*\right)\\
\widetilde{q}_t &= (\omega_c+\omega_c^*-1)\widetilde{\delta}_t.
\end{aligned}$$

By taking the true linear-quadratic approximation to the utility function, Corsetti, Dedola, and Leduc (2010) show that the loss function under symmetry is given by

$$L_t = -\frac{1}{2}\left(\lambda_y\left(\widetilde{y}_t\right)^2+\lambda_y^*\left(\widetilde{y}_t^*\right)^2+\lambda_\pi\left(\pi_t\right)^2+\lambda_\pi^*\left(\pi_t^*\right)^2+\lambda_\delta\left(\widetilde{\delta}_t\right)^2\right) \tag{71}$$

where

$$\begin{aligned}
\lambda_y &= \chi+\sigma && (72)\\
\lambda_y^* &= \chi+\sigma && (73)\\
\lambda_\pi &= \frac{1}{\lambda}\frac{1+\nu_p}{\nu_p} && (74)\\
\lambda_\pi^* &= \frac{1}{\lambda^*}\frac{1+\nu_p^*}{\nu_p^*} && (75)\\
\lambda_\delta &= \frac{1-2\tau}{\sigma}\tau. && (76)
\end{aligned}$$

D Macroprudential Regulation Model

Our toolbox can also be applied to policy games in a closed economy. We lay out a policy game between a central bank and a financial regulator in a model following Gertler and Karadi (2011). In addition to nominal rigidities, the economy features financial frictions. Non-financial firms are prevented from issuing equity to households directly, but have to go through financial intermediaries, referred to as "banks," in order to raise funds. Due to an agency problem, however, banks are limited in their ability to attract deposits and issue credit to non-financial firms. Accordingly, credit is undersupplied, and the reactions to shocks are amplified by the familiar financial-accelerator mechanism.

D.1 Model Environment

D.1.1 Households

The representative household consists of a continuum of members. A fraction $1-f$ of its members supplies labor to firms and returns the wage earned to the household. The remaining fraction f works as bankers. The household utility function is

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\log(C_t - \gamma C_{t-1}) - \chi_0 \frac{L_t^{1+\chi}}{1+\chi} \right]. \tag{77}$$

The importance of internal habits in consumption is governed by the parameter γ. The budget constraint takes the form

$$P_t C_t = P_t W_t L_t + P_t \Pi_t - P_t T_t - P_t D_t + (1 + R_{t-1}) P_t D_{t-1} \tag{78}$$

Households use their income to consume, C_t, make tax transfers to the government, T_t, and to save in terms of deposits with banks, D_t. Income is derived from returns on deposits, wages, and profits of banks, Π_t.

Financially constrained bankers have an incentive to retain earnings. To prevent the financial constraint from becoming irrelevant by the retention of bank earnings, a banker ceases operations next period with the i.i.d. probability $1-\theta$. Upon exiting, bankers transfer retained earnings to the households and become workers. Each period $(1-\theta) f$ workers are selected to become bankers. These new bankers receive a startup transfer from the family. By construction, the fraction of household members in each group is constant over time. Π_t is net funds transferred to the household from its banker members; that is, funds transferred from existing bankers minus the funds transferred to new bankers (measured by $\bar{\omega}$).

D.1.2 Banks

Bank j takes in deposits, $D_t(j)$, from households and invests into non-financial firms through an equity contract. Continuing banks do not consume but accumulate all earnings. Due to taxes/subsidies on equity, the bank operates with the amount $(1 - BT_t)N_t(j)$, where BT_t is the tax rate and $N_t(j)$ is the equity of bank j. Since assets equal liabilities on the bank balance sheet

$$Q_t S_t(j) = (1 - BT_t) N_t(j) + D_t(j). \tag{79}$$

Let deposits $D_t(j)$ pay the non-state-contingent (real) return $(1+R_t)$ and let shares $S_t(j)$ pay the stochastic return $(1+R^s_{t+1})$ at time $t+1$. Net worth in $t+1$ is then determined as the difference between earnings on assets and interest payments on liabilities

$$N_{t+1}(j) = (1+R^s_{t+1})Q_tS_t(j) - (1+R_t)D_t(j) \tag{80}$$

or combing (79) and (80)

$$N_{t+1}(j) = \left(R^s_{t+1} - R_t\right)Q_tS_t(j) + (1+R_t)(1-BT_t)N_t(j). \tag{81}$$

The expected terminal wealth of a bank is then given by

$$\max_{\{S_{t+i}(j)\}} V_t(j) = E_t \sum_{i=0}^{\infty} (1-\theta)\,\theta^i \Lambda_{t,t+1+i} N_{t+1+i}(j) \tag{82}$$

with the stochastic discount factor $\Lambda_{t,t+j} = \beta^j \frac{\lambda_{ct+j}}{\lambda_{ct}}$.

Absent financial frictions, the bank expands its balance sheet when the expected discounted excess return on loans, $E_t\Lambda_{t,t+1+i}\left(R^s_{t+1+i} - R_{t+i}\right)$, is positive. To limit the ability of banks to attract deposits, Gertler and Karadi (2011) introduce the following agency problem. At the beginning of each period, a banker can choose to transfer a fraction λ of assets to his household. If the banker makes this transfer, depositors will force the bank into bankruptcy and recover the remaining fraction $1-\lambda$ of assets. Thus, households will deposit funds with bank j only if the expected terminal wealth, $V_t(j)$ exceeds the fraction of assets that can be diverted, $\lambda Q_tS_t(j)$, in period t

$$V_t(j) \geq \lambda Q_tS_t(j). \tag{83}$$

If equation (83) binds a bank's ability to raise deposits is limited and expected positive excess returns can persist in equilibrium.

As shown in Section D.3 of this Appendix a bank's ability to attract deposits is directly related to its net worth. At the aggregate level this relationship is shown to obey

$$Q_tS_t = \frac{\eta_t}{\lambda - \upsilon_t}(1-BT_t)N_t. \tag{84}$$

The term $\frac{\eta_t}{\lambda-\upsilon_t}$ is the ratio of assets to equity. Condition (84) limits the aggregate leverage ratio to the point where the incentives to cheat are balanced by the costs for each bank. The marginal values of loans, υ_t, and of equity, η_t, are defined recursively as

$$\begin{aligned} \upsilon_t \;=\; & E_t\left\{(1-\theta)\,\Lambda_{t,t+1}\left(R^s_{t+1} - R_t\right)\right\} \\ & + E_t\left\{\theta\Lambda_{t,t+1}\frac{\frac{\eta_{t+1}}{(\lambda-\upsilon_{t+1})}}{\frac{\eta_t}{(\lambda-\upsilon_t)}}\left[\left(R^s_{t+1}-R_t\right)\frac{\eta_t}{(\lambda-\upsilon_t)} + (1+R_t)\right](1-BT_{t+1})\upsilon_{t+1}\right\} \end{aligned} \tag{85}$$

$$\eta_t = (1-\theta) + \theta E_t \left\{ \Lambda_{t,t+1} \left[\left(R^s_{t+1} - R_t \right) \frac{\eta_t}{(\lambda - \upsilon_t)} + (1+R_t) \right] (1 - BT_{t+1}) \eta_{t+1} \right\}. \tag{86}$$

Finally, aggregate net worth evolves according to

$$N_t = \theta \left[(R^s_t - R_{t-1}) \frac{\eta_{t-1}}{(\lambda - \upsilon_{t-1})} + (1 + R_{t-1}) \right] (1 - BT_{t-1}) N_{t-1} + \bar{\omega} Q_t S_{t-1}. \tag{87}$$

D.1.3 Production of Goods

The representative firm uses capital and labor to produce its output

$$Y_t = e^{z_t} K_t^{\alpha} L_t^{1-\alpha}, \tag{88}$$

where technology evolves according to $z_t = \rho_z z_{t-1} + \sigma_z \varepsilon_{z,t}$. Each firm operates for only one period, but it must purchase the capital used in period $t+1$ one period in advance. To do so, the firm issues one share for each unit of capital purchased in period t to be used in period $t+1$. Absent arbitrage opportunities, the value of capital equals the value of shares

$$P_t Q_t K_{t+1} = P_t Q_t S_t. \tag{89}$$

The firm's revenues consist of output sales (priced at marginal costs) and the value of undepreciated capital. Payments for servicing the shares and for labor services enter the accounting as expenses. Hence, profits in period $t+1$ are given by

$$\Pi^f_{t+1} = MC_{t+1} Y_{t+1} + P_{t+1} Q_{t+1} (1-\delta) K_{t+1} - P_{t+1} W_{t+1} L_{t+1} - (1 + R^s_{t+1}) P_t Q_t S_t. \tag{90}$$

With the decision on the capital stock made in period t and labor hired in the $t+1$ spot market, the firm's maximization problem taking prices as given satisfies

$$\begin{aligned} &\max_{S_t, K_{t+1}} E_t \left[\Lambda_{t,t+1} \max_{L_{t+1}} \Pi^f_{t+1} \right] \\ &s.t. \\ &Y_t = e^{z_t} K_t^{\alpha} L_t^{1-\alpha} \\ &Q_t P_t K_{t+1} = Q_t P_t S_t. \end{aligned} \tag{91}$$

The zero profit condition implies that the return on shares is given by

$$(1 + R^s_{t+1}) = \frac{1}{Q_t} \frac{\alpha MC_{t+1} Y_{t+1}}{P_{t+1} K_{t+1}} + \frac{(1-\delta)}{Q_t} Q_{t+1} \tag{92}$$

where

$$(1 + R^s_t) = \frac{(1 + r^s_t)}{\frac{P_t}{P_{t-1}}}. \tag{93}$$

The optimal choice of labor satisfies

$$L_t = (1-\alpha)\frac{Y_t}{W_t}\frac{MC_t}{P_t}. \tag{94}$$

To support an environment with nominal price rigidities, we introduce an intermediate layer of firms between producing-firms and firms that assemble the final goods. Each intermediate firm acquires the product of a producing firm and applies a stamp to it that differentiates it from those of others. In choosing the optimal resale price $P_t(f)$ an intermediate firm faces adjustment costs as in Rotemberg (1982)

$$\max_{P_{t+i}(f)} E_t \sum_{i=0}^{\infty} \Lambda_{t,t+i} \left\{(1+\tau_p) P_{t+i}(f) - MC_{t+i}\right\} Y_{t+i} \left(\frac{P_{t+i}(f)}{P_{t+i}}\right)^{-\frac{1+\nu_p}{\nu_p}} - \phi_{P,t+i}(f) P_{t+i} Y_{t+i} \tag{95}$$

where $Y_{t+i}\left(\frac{P_{t+i}(f)}{P_{t+i}}\right)^{-\frac{1+\nu_p}{\nu_p}}$ is the demand schedule for good f. The adjustment cost for prices follows

$$\phi_{P,t}(f) = \frac{\phi_p}{2}\left(\frac{P_t(f)}{(\iota^p \pi_{t-1} + (1-\iota^p)\bar{\pi}) P_{t-1}(f)} - 1\right)^2. \tag{96}$$

D.1.4 Production of Capital

Physical capital accumulates according to

$$K_{t+1} = I_t^n + (1-\delta) K_t. \tag{97}$$

The capital stock is augmented by net investment, I_t^n, and requires gross investment in the amount, I_t^g

$$I_t^n = \left[1 - \frac{\psi}{2}\left(\frac{I_t^g}{I_{t-1}^g} - 1\right)^2\right] I_{t.}^g \tag{98}$$

Taking the price of capital, Q_t, as given, capital producing firms solve

$$\max_{I_{t+i}^g} E_t \sum_{i=0}^{\infty} \Lambda_{t,t+i} \left[Q_{t+i}\left[1 - \frac{\psi}{2}\left(\frac{I_{t+i}^g}{I_{t+i-1}^g} - 1\right)^2\right] I_{t+i}^g - I_{t+i}^g\right]. \tag{99}$$

D.1.5 Market Clearing

The aggregate resource constraint requires

$$Y_t = C_t + I_t^g + G_t \tag{100}$$

where government spending is set to be

$$G_t = \omega_{gy} Y_t. \tag{101}$$

D.2 Equilibrium conditions in the Macroprudential Regulation Model

Using the notation introduced in Section 2.1, the endogenous variables are collected in the vector

$$\tilde{x}_t = \begin{pmatrix} Y_t, L_t, K_{t-1}, W_t, R_t^s, \frac{MC_t}{P_t}, \lambda_t^c, C_t, R_t, S_t, N_t, v_t, \eta_t, \\ I_t^n, I_t^g, G_t, \pi_t, \phi_t, \frac{\partial \phi_t}{\partial P_t} P_t, \frac{\partial \phi_t}{\partial P_{t-1}} P_t, R_t^n, \Delta R_t^s, \left[\frac{QS}{N}\right]_t, \left[\frac{N}{Y}\right]_t \end{pmatrix}' \tag{102}$$

where the nominal interest rate, R_t^n, the interest rate spread, ΔR_t^s, the loan to net worth ratio, $\left[\frac{QS}{N}\right]_t$, and the net worth to output ratio, $\left[\frac{N}{Y}\right]_t$, are defined below. The exogenous vector ζ_t contains the technology shock

$$\zeta_t = z_t. \tag{103}$$

We provide a complete list of the conditions characterising the private sector equilibrium for given policies for the model described above.

The following equations result from the households' optimization problem:

1. choice of optimal consumption

$$\lambda_t^c = \frac{1}{C_t - \gamma C_{t-1}} - E_t \beta \frac{\gamma}{C_{t+1} - \gamma C_t} \tag{104}$$

2. choice of optimal labor supply

$$\chi_0 L_t^{\chi} = \lambda_t^c W_t \tag{105}$$

3. choice of optimal deposit holdings

$$E_t \frac{\lambda_{t+1}^c}{\lambda_t^c} = \frac{1}{\beta(1 + R_t)}. \tag{106}$$

The following equations result from the banks:

1. leverage constraint

$$Q_t S_t = \frac{\eta_t}{(\lambda - v_t)} (1 - BT_t) N_t \tag{107}$$

2. bank capital evolves according to

$$N_t = \theta \left[(R_t^s - R_{t-1}) \frac{\eta_{t-1}}{(\lambda - v_{t-1})} + (1 + R_{t-1}) \right] (1 - BT_{t-1}) N_{t-1} + \bar{\omega} Q_t S_{t-1} \quad (108)$$

3. the marginal value of loans

$$\begin{aligned} v_t \;\; &= \;\; E_t \left(1 - \theta\right) \Lambda_{t,t+1} \left(R_{t+1}^s - R_t\right) \\ &\quad + \theta \Lambda_{t,t+1} \frac{\frac{\eta_{t+1}}{(\lambda - v_{t+1})}}{\frac{\eta_t}{(\lambda - v_t)}} \left[\left(R_{t+1}^s - R_t\right) \frac{\eta_t}{(\lambda - v_t)} + (1 + R_t) \right] (1 - BT_{t+1}) v_{t+1} \end{aligned} \quad (109)$$

4. the marginal value of equity

$$\eta_t \;\; = \;\; E_t \left(1 - \theta\right) + \theta \Lambda_{t,t+1} \left[\left(R_{t+1}^s - R_t\right) \frac{\eta_t}{(\lambda - v_t)} + (1 + R_t) \right] (1 - BT_{t+1}) \eta_{t+1}. \quad (110)$$

The following equations result from the basic producers:

1. equity financing for capital

$$K_{t+1} = S_{t.} \quad (111)$$

2. production function

$$Y_t = e^{z_t} K_t{}^{\alpha} L_t^{1-\alpha}. \quad (112)$$

3. choice of optimal labor input

$$L_t = (1 - \alpha) \frac{Y_t}{W_t} \frac{MC_t}{P_t} \quad (113)$$

4. zero profit condition

$$(1 + R_t^s) = \frac{\alpha Y_t}{Q_{t-1} K_t} \frac{MC_t}{P_t} + \frac{(1 - \delta)}{Q_{t-1}} Q_t. \quad (114)$$

The following equations result from the variety producers:

1. first order condition with respect to prices

$$E_t \left[\left(-\tfrac{1}{\nu_p} (1 + \tau_p) + \tfrac{1 + \nu_p}{\nu_p} \tfrac{MC_t}{P_t} \right) Y_t - Y_t P_t \tfrac{\partial \phi_t}{\partial P_t} - \Lambda_{t,t+1} Y_{t+1} P_{t+1} \tfrac{\partial \phi_{t+1}}{\partial P_t} \right] = 0 \quad (115)$$

with the price adjustment cost and its derivatives satisfying

$$\phi_t = \frac{\phi_p}{2}\left(\frac{\pi_t}{\iota^p \pi_{t-1} + (1-\iota^p)\bar{\pi}} - 1\right)^2 \tag{116}$$

$$\frac{\partial \phi_t}{\partial P_t} P_t = \phi_p \left(\frac{\pi_t}{\iota^p \pi_{t-1} + (1-\iota^p)\bar{\pi}} - 1\right) \frac{\pi_t}{\iota^p \pi_{t-1} + (1-\iota^p)\bar{\pi}} \tag{117}$$

$$\frac{\partial \phi_t}{\partial P_{t-1}} P_t = -\phi_p \left(\frac{\pi_t}{\iota^p \pi_{t-1} + (1-\iota^p)\bar{\pi}} - 1\right) \frac{\pi_t}{\iota^p \pi_{t-1} + (1-\iota^p)\bar{\pi}} \pi_t. \tag{118}$$

The following equations result from the physical capital producers:

1. evolution of physical capital

$$K_{t+1} = I_t^n + (1-\delta) K_t \tag{119}$$

2. investment adjustment costs

$$I_t^n = \left[1 - \frac{\psi}{2}\left(\frac{I_t^g}{I_{t-1}^g} - 1\right)^2\right] I_t^g. \tag{120}$$

3. price of capital from optimal investment choice

$$Q_t \left[1 - \frac{\psi}{2}\left(\frac{I_t^g}{I_{t-1}^g} - 1\right)^2 - \psi\left(\frac{I_t^g}{I_{t-1}^g} - 1\right)\frac{I_t^g}{I_{t-1}^g}\right] + \Lambda_{t,t+1} Q_{t+1} \psi \left(\frac{I_{t+1}^g}{I_t^g} - 1\right)\left(\frac{I_{t+1}^g}{I_t^g}\right)^2 = 1 \tag{121}$$

The aggregate resource constraint requires

$$Y_t = C_t + I_t^g + G_t \tag{122}$$

where government spending is set to be

$$G_t = \omega_{gy} Y_t. \tag{123}$$

In addition, we define:

1. the loan rate spread

$$\Delta R_t^s = R_t^s - R_{t-1} \tag{124}$$

2. the ratio of loans to net worth

$$\left[\frac{QS}{N}\right]_t = \frac{\eta_t}{\lambda - v_t} \tag{125}$$

3. the nominal interest rate

$$\frac{1}{(1+R_t^n)} = \beta \frac{\lambda_{t+1}^c}{\lambda_t^c} \frac{1}{\pi_{t+1}} \tag{126}$$

4. the net worth to output ratio

$$\left[\frac{N}{Y}\right]_t = \frac{N_t}{Y_t} \tag{127}$$

The period utility functions are

$$U_t^{cb} = \log(C_t - \gamma C_{t-1}) - \chi_0 \frac{L_t^{1+\chi}}{1+\chi} - \mu_{cb}(\pi_t - \bar{\pi})^2 \tag{128}$$

and

$$U_t^{mpr} = \log(C_t - \gamma C_{t-1}) - \chi_0 \frac{L_t^{1+\chi}}{1+\chi} - \mu_{mpr}\left((R_t^s - \bar{R}^s) - (R_{t-1} - \bar{R})\right)^2. \tag{129}$$

The policy rules followed by the central bank and the macroprudential regulator that will subsequently be replaced by the first order conditions of the policymakers are:

$$R_t^n = \bar{R}^n + \gamma_{R^n}\left(R_{t-1}^n - \left(\frac{\bar{\pi}}{\beta} - 1\right)\right) + (1 - \gamma_{R^n})\gamma_\pi(\pi_t - \bar{\pi}) \tag{130}$$

and

$$BT_t = \gamma_{BT} BT_{t-1} + \gamma_S (S_t - S_{t-1}) \tag{131}$$

In the optimal policy exercises, the central bank uses inflation, π_t, as instrument whereas the financial regulator uses the tax on bank capital, BT_t.[21] By augmenting the set of conditions (36)-(60) with the two definitions

$$i_t^{cb} = \pi_t \tag{132}$$
$$i_t^{mpr} = BT_t \tag{133}$$

[21] Similar to the case of the two-country model, the open-loop Nash equilibrium is indeterminate when the nominal interest is used as policy instrument.

we have cast the structural equations of the model into the form of (1)

$$E_t g(\tilde{x}_{t-1}, \tilde{x}_t, \tilde{x}_{t+1}, i_{1,t}, i_{2,t}, \zeta_t) = 0.$$

Table 3 summarises the parameter choices for the subsequent experiments. Most parameters are set at values commonly found in the literature. The parameter ϕ_p in the adjustment cost function for prices is set at 1281. With this value in place the (linearized) Phillips curve features the same slope as that of a model with Calvo contracts and an expected contract duration of one year. Inflation is set to zero in the steady state and the subsidy to the intermediate goods producers is set to remove monopolistic distortions in the steady state. The parameters governing the banking sector mimic those in Gertler and Karadi (2011). The survival probability for banks is set at 0.95 implying an average horizon of bankers of ten years. The steady-state ratio of loans to equity is set equal to 4. For ease of exposition, we abstract from steady-state distortions by setting the interest rate spread between loans and deposits ($R^s - R$) equal to zero.[22] These choices imply that the resource transfer to new banks as a fraction of total loans, $\bar{\omega}$, is 0.0101 and the portion of net worth that the bank management can divert, λ, is 0.25.

When setting up the policy problem under cooperation, the objectives of the individual policymakers receive equal weight in the joint objective function, i.e., $\omega_{cb} = \omega_{mpr} = 0.5$. Positive values of the parameters μ_{cb} and μ_{mpr} introduce biases into the objective functions of the central bank and the macroprudential regulator as described below.

[22] The financial frictions in the model will still imply inefficient allocations away from the state. At the expense of rendering the steady state inefficient, the steady-state interest rate spread can of course be set at the value of one hundred basis points as in Gertler and Karadi (2011) (or any other value).

Table 3: Parameters for the Macroprudential Regulation Model

Free Parameters			
Parameter	**Used to Determine**	**Parameter**	**Used to Determine**
$\beta = 0.99$	discount factor	$\gamma = 0.6$	consumption habits
$\chi = 1$	labor supply elasticity	$\bar{L} = 0.5$	steady-state labor supply to fix χ_0
$\alpha = 0.3$	share of capital in production	$\delta = 0.025$	capital depreciation rate
$\frac{1+\nu_p}{\nu_p} = 11$	subst. elasticity of varieties	$\tau_p = 0.1$	subsidy to producers
$\phi_p = 1281$	price adjustment cost	$\bar{\pi} = 1$	steady-state inflation
$\psi = 1$	investment adjustment cost	$\omega_{gy} = 0$	share of gov. spending
$\rho_a = 0.95$	persistence of tech. shock	$\sigma_a = 0.01$	std. of tech. shock
$\omega_{mpr} = 0.5$	weight of fin. reg. in Ramsey	$\omega_{cb} = 5$	weight of non. pol. in Ramsey
$\mu_{mpr} = 4$	add. term in fin. reg. utility	$\mu_{cb} = 1$	add. term in mon. pol. utility
$\left[\frac{QS}{N}\right] = 4$	steady-state ratio loans to net worth	$\bar{R}^s - \bar{R} = 0$	steady-state interest rate spread
$\theta = 0.95$	probability of bank survival	$\iota^p = 0.5$	inflation persistence
Implied Parameters			
$\lambda = 0.25$	diversion parameter	$\bar{\omega} = 0.0101$	resource transfer to new banks
$\chi_0 = 3.6143$	shift parameter in utility function		

Note: This table summarizes the parameterization of the macroprudential regulation model described in Section 4 at quarterly frequency.

D.3 Details on Conditions (84) and (87)

We begin by restating the expected terminal wealth of a bank as

$$\max_{\{S_{t+i}(j)\}} V_t(j) = E_t \sum_{i=0}^{\infty} (1-\theta)\,\theta^i \Lambda_{t,t+1+i} N_{t+1+i}(j) \tag{134}$$

where

$$N_{t+1}(j) = \left(R^s_{t+1} - R_t\right) Q_t S_t(j) + (1+R_t)(1-BT_t)N_t(j). \tag{135}$$

$V_t(j)$ can be split into two parts

$$\begin{aligned} V_t(j) \quad = \quad & E_t\left(\sum_{i=0}^{\infty} (1-\theta)\,\theta^i \Lambda_{t,t+1+i}\left(R^s_{t+1+i} - R_{t+i}\right) Q_{t+i} S_{t+i}(j)\right) \\ & + E_t\left(\sum_{i=0}^{\infty} (1-\theta)\,\theta^i \Lambda_{t,t+1+i}(1+R_{t+i})N_{t+i}(j)\right). \end{aligned} \tag{136}$$

Defining $v_t(j)$ and $\eta_t(j)$

$$v_t(j) = E_t\left(\sum_{i=0}^{\infty}(1-\theta)\,\theta^i\Lambda_{t,t+1+i}\left(R^s_{t+1+i}-R_{t+i}\right)\frac{Q_{t+i}S_{t+i}(j)}{Q_tS_t(j)}\right) \tag{137}$$

$$= E_t\left((1-\theta)\,\Lambda_{t,t+1}\left(R^s_{t+1}-R_t\right)+\Lambda_{t,t+1}\theta\frac{Q_{t+i}S_{t+i}(j)}{Q_tS_t(j)}v_{t+1}(j)\right) \tag{138}$$

$$\eta_t(j) = E_t\left(\sum_{i=0}^{\infty}(1-\theta)\,\theta^i\Lambda_{t,t+1+i}(1+R_{t+i})\frac{N_{t+i}(j)}{N_t(j)}\right)$$

$$= E_t\left((1-\theta)+\Lambda_{t,t+1}\theta\frac{N_{t+1}(j)}{N_t(j)}\eta_{t+1}(j)\right). \tag{139}$$

we arrive at

$$V_t(j) = v_t(j)Q_tS_t(j)+\eta_t(j)N_t(j). \tag{140}$$

In order to aggregate over banks, we make use of the fact that all banks have access to the same investment opportunities as we will show now. $\frac{Q_{t+1}S_{t+1}(j)}{Q_tS_t(j)}$ will be equalized across surviving firms, and similarly for $\frac{N_{t+1}(j)}{N_t(j)}$. Substitute

$$V_t(j) = v_tQ_tS_t(j)+\eta_tN_t(j) \tag{141}$$

into the incentive-compatibility constraint

$$V_t(j) \geq \lambda Q_tS_t(j) \tag{142}$$

to obtain

$$v_t(j)Q_tS_t(j)+\eta_t(j)N_t(j) \geq \lambda Q_tS_t(j). \tag{143}$$

Assuming this constraint binds with equality and substituting $Q_tS_t(j) = \frac{\eta_t}{(\lambda-v_t)}N_t(j)$ into the evolution of net worth $N_{t+1}(j) = \left(R^s_{t+1}-R_t\right)Q_tS_t(j)+(1+R_t)N_t(j)$ we arrive at

$$\frac{N_{t+1}(j)}{N_t(j)} = \left(R^s_{t+1}-R_t\right)\frac{\eta_t}{(\lambda-v_t)}+(1+R_t). \tag{144}$$

In turn, $\frac{Q_{t+1}S_{t+1}(j)}{Q_tS_t(j)}$ is given by

$$\frac{Q_{t+1}S_{t+1}(j)}{Q_tS_t(j)} = \frac{\frac{\eta_{t+1}}{(\lambda-v_{t+1})}}{\frac{\eta_t}{(\lambda-v_t)}}\frac{N_{t+1}(j)}{N_t(j)}$$

$$= \frac{\frac{\eta_{t+1}}{(\lambda-v_{t+1})}}{\frac{\eta_t}{(\lambda-v_t)}}\left[\left(R^s_{t+1}-R_t\right)\frac{\eta_t}{(\lambda-v_t)}+(1+R_t)\right]. \tag{145}$$

Consequently, v_t and η_t are identical for each bank and evolve according to

$$\begin{aligned} v_t &= E_t\left\{(1-\theta)\,\Lambda_{t,t+1}\left(R^s_{t+1}-R_t\right)\right\} \\ &\quad +E_t\left\{\theta\Lambda_{t,t+1}\frac{\frac{\eta_{t+1}}{(\lambda-v_{t+1})}}{\frac{\eta_t}{(\lambda-v_t)}}\left[\left(R^s_{t+1}-R_t\right)\frac{\eta_t}{(\lambda-v_t)}+(1+R_t)\right]v_{t+1}\right\} \qquad (146) \\ \eta_t &= (1-\theta)+\theta E_t\left\{\Lambda_{t,t+1}\left[\left(R^s_{t+1}-R_t\right)\frac{\eta_t}{(\lambda-v_t)}+(1+R_t)\right]\eta_{t+1}\right\}. \qquad (147) \end{aligned}$$

Finally, aggregate net worth is the sum of the net worth of two groups: old and new bankers. Bankers that survive from period $t-1$ to period t will have aggregate net worth equal to

$$\theta\left[\left(R^s_t-R_{t-1}\right)\frac{\eta_{t-1}}{(\lambda-v_{t-1})}+(1+R_{t-1})\right]N_{t-1}. \qquad (148)$$

Assume that new bankers receive as endowment a fixed fraction of the current value of the assets intermediated by exiting bankers in the previous period, i.e., $(1-\ \theta)\,Q_tS_{t-1}$. Let this fraction be $\frac{\bar{\omega}}{(1-\ \theta)}$. Thus,

$$N^n_t=\frac{\bar{\omega}}{(1-\ \theta)}\,(1-\ \theta)\,Q_tS_{t-1}=\bar{\omega}Q_tS_{t-1}. \qquad (149)$$

Current aggregate net worth is then the sum of net worth carried from the previous period by surviving firms plus the net worth of new entrants, or

$$N_t=\theta\left[\left(R^s_t-R_{t-1}\right)\frac{\eta_{t-1}}{(\lambda-v_{t-1})}+(1+R_{t-1})\right]N_{t-1}+\bar{\omega}Q_tS_{t-1} \qquad (150)$$

with v_t and η_t as defined in equations (146) and (147).