Interpreting Shocks to the Relative Price of Investment with a Multi-Sector Model

Luca Guerrieri, Federal Reserve Board, and

Jinill Kim, Korea University

November, 2015

Intro: Motivation and Question

- The assumptions underlying commonly used measures of productivity imply that an aggregate strategy can be used to formulate measures of aggregate inputs feeding an aggregate function that can aggregate production, much in the tradition of Solow (1957), Griliches and Jorgenson (1966).
- These assumptions were at the center of the Cambridge-Cambridge debate of the 1960s.
- What I want to explore today is whether commonly used identification schemes for VARs can distinguish between the alternative properties of sector-specific technology shocks for models in which aggregation of the sectoral production functions is possible vs. properties of other models in which aggregation does not apply (and in which TFP measures would not be sensible).

Intro: The Models

- To facilitate this kind of exploration, we used two specific models.
- The first is a typical RBC model, extended to encompass investment-specific technology (IST) shocks following Greenwood, Hercowitz, and Krusell (1997), but otherwise maintaining an aggregate production structure.
- The second is a model that encompasses multiple sectors and multiple final goods produced with inputs from these various sectors, following the Input-Output tables for the United States.
- For this second class of models, the factor inputs and production functions of the individual sectors cannot be recombined to imply aggregate inputs and an aggregate production function for the entire economy.

Intro: VAR Identification of Technology Shocks

- An alternative to growth accounting exercises that promises to make only "light" assumptions was suggested by Galí (2001) .
- He used a parsimonious VAR framework identified with the assumption that only permanent movements in technology affect aggregate labor productivity permanently.
- However, Galí did not allow for multiple sources of technology shocks with (possibly) different trends.

Intro: The Empirical Strategy

- One of the striking features of post WWII U.S. data is that the price of investment shows a downward trend and varies substantially over the cycle.
- We follow the empirical strategy of Fisher (2006) to identify shocks that move the relative price of investment permanently.
- We extend Fisher's VAR to include measures of aggregate consumption and investment.
- We show that conditional on shocks that move the price of investment permanently, consumption and investment comove.

Intro: Models and VAR Evidence

- To ensure that the VAR evidence is admissible to discriminate between our models, we verify that the identification scheme is consistent with both models.
- Whenever possible we choose parameters to match (unconditional) moments of the same variables included in our VAR.
- In many respects the VAR evidence cannot discriminate between the two models.
- However, focusing on the comovement of consumption and investment, the model with multiple sectors is much closer to the VAR evidence than the aggregate model.

VAR

Let's start by considering some properties of the VAR we estimated.

- A key discriminating factor between a one-sector model with IST shocks and a model with multiple sectoral MFP shocks is the correlation between consumption and investment.
- Fisher's seminal work on identifying IST shocks did not include a measure of consumption in the VAR, making it difficult to investigate this correlation.
- We update Fisher's results and extend them to gauge this correlation by including measures of consumption and investment in the VAR.

VAR: Variables

The VAR that we estimate includes five variables:

1. the growth rate of the relative price of investment, constructed as the implicit price deflator for equipment and software divided by non-farm business output prices (net of equipment and software);
2. labor productivity growth, measured as log-differenced labor productivity in the nonfarm business sector;
3. hours per capita, constructed as the log of hours worked in the nonfarm business sector minus the log of civilian non-institutional population 16 years and over;
4. the growth rate of equipment and software per capita;
5. the growth rate of consumption per capita.

VAR: Estimation Choices

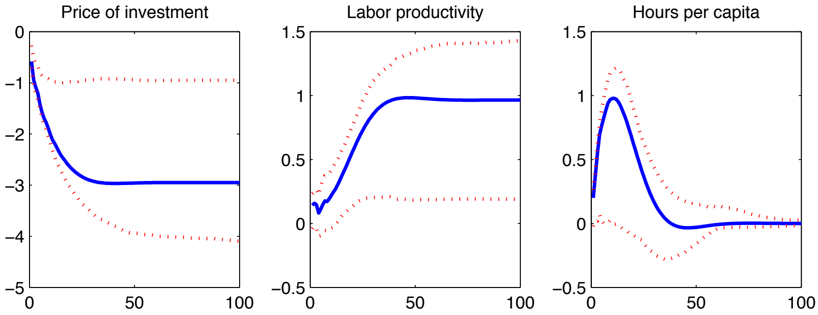
- Beaudry and Lucke (2009), Schmitt-Grohe and Uribe (2011), and Sims (2011) replaced or augmented labor productivity growth in the VAR with the growth of TFP measures obtained from growth accounting exercises.
- That strategy relies on aggregation of sectoral production functions.
- We continue to use labor productivity growth since the conditions for aggregation underlying those TFP measures do not hold in the model that we are interested in examining.
- We estimate a VAR of order 4.
- The start date for the estimation sample is 1982:Q3, avoiding the adjustment from the Volcker disinflation.
- We end the sample in 2008:Q3 to avoid the ZLB period.
- In robustness analysis we also consider a longer sample, spanning the entire length of the dataset.

VAR: Identification

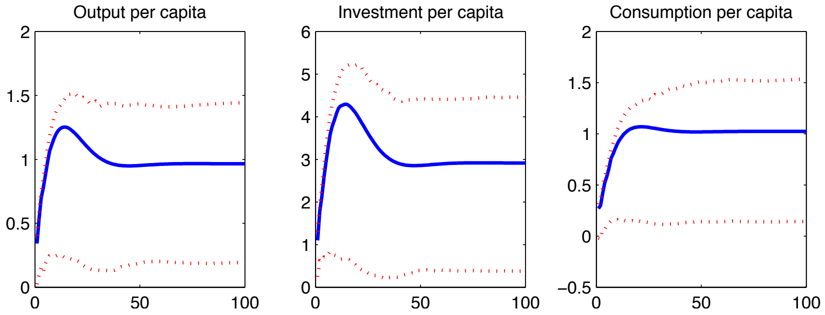
We follow the long-run identification scheme of Fisher (2006):

- Only a shock to the relative price of investment can move that price permanently.
- Moreover, only shocks to the relative price of investment and to labor productivity can move the level of labor productivity permanently.
- All other shocks are left unidentified.

VAR: Response to a Shock that Lowers the Relative Price of Investment Permanently



VAR: Response to a Shock that Lowers the Relative Price of Investment Permanently



VAR: The Importance of the Identified Shocks

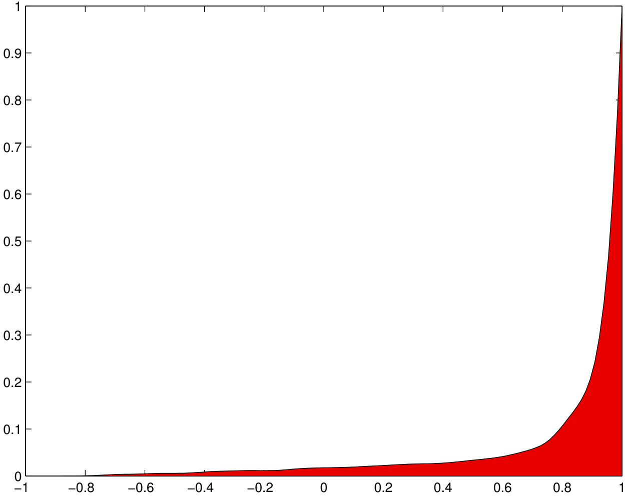
Variance decompositions at business cycle frequencies (population estimates):

Shock	Growth of Price of Investment	Growth of Labor Productivity	Hours
Price of Investment	0.60	0.10	0.71
Neutral MFP	0.10	0.56	0.03

Shock	Growth of Consumption	Growth of Investment
Price of Investment	0.40	0.45
Neutral MFP	0.04	0.19

VAR: Correlation between Consumption and Investment

Cumulative distribution function of the correlation between consumption and investment at business cycle frequencies



Models: Overview

- To interpret his identification scheme, Fisher (2006) wrote down a one-sector model with neutral MFP shocks and IST shocks that enter the capital accumulation equation.
- Under some assumptions—including that of equal factor shares across sectors—the identification scheme would also be consistent with a model with two sectors that produce consumption and investment goods.
- U.S. IO Tables show that factor shares vary substantially across sectors. This variation is at odds with the conditions needed for aggregation.
- We will present a baseline version of our two-sector model one that we use to prove analytically that Fisher's identification scheme applies to a multi-sectoral production structure.

Models: Baseline Two-Sector Model

- There are two sectors of production: machinery and non-machinery.
- There is complete specialization in the production of final goods: investment is assembled with inputs from the machinery sector; consumption is assembled with inputs of the non-machinery sector.
- The model includes only one stock of capital used in both sectors.
- Both capital and labor are perfectly mobile across sectors.

Models: Baseline Two-Sector Model - Household Utility

In period t, the representative household supplies a fixed amount of labor L, and maximizes the intertemporal utility function

$$\max_{C_s, I_s, K_{Ns}, K_{Ms}, B_s} \sum_{s=t}^{\infty} \beta^{s-t} \log C_s,$$

by choosing paths for C, consumption, I, investment, K_N, N goods capital, K_M, M goods capital, and for bonds B that pay the rate of return ρ after one period.

Models: Baseline Two-Sector Model - Budget and Capital Constraints

- The utility maximization problem is subject to a budget constraint given by

$$W_s L + R_{Ms} K_{Ms} + R_{Ns} K_{Ns} + \rho_{s-1} B_{s-1} = P_{Cs} C_s + P_{Is} I_s + B_s,$$

- The utility maximization problem is also subject to the following law of motion for the accumulation of capital:

$$K_{Ms+1} + K_{Ns+1} = (1 - \delta)(K_{Ms} + K_{Ns}) + I_s,$$

Models: Baseline Two-Sector Model - Final Goods

- There is complete specialization in the assembly of consumption and investment goods.
- Investment exhausts the output of the M sector, $I_s = Y_{Ms}$ and the price of investment coincides with the price of N goods, $P_{Is} = P_{Ms}$.
- Analogously, consumption exhausts the output of the N sector, $C_s = Y_{Ns}$ and the price of consumption coincides with the price of N goods, $P_{Cs} = P_{Ns}$.

Models: Baseline Two-Sector Model - Firms' Problem

- In each sector, perfectly competitive firms minimize production costs to meet demand subject to the technology constraint as reflected in the following Lagrangian problems:

$$\min_{K_{Ms},L_{Ms},P_{Ms}} R_{Ms}K_{Ms} + W_s L_{Ms} + P_{Ms}(Y_{Ms} - K_{Ms}^{\alpha_M}(A_{Ms}L_{Ms})^{1-\alpha_M}),$$

$$\min_{K_{Ns},L_{Ns},P_{Ns}} R_{Ns}K_{Ns} + W_s L_{Ns} + P_{Ns}(Y_{Ns} - K_{Ns}^{\alpha_N}(A_{Ns}L_{Ns})^{1-\alpha_N}),$$

- Notice that when $\alpha_N = \alpha_M$ the model can be recast as an aggregate model and rules for the state variables of the model can be formulated without reference to sectoral variables.
- In this respect, our two-sector model encompasses the class of models already used to interpret the identification scheme in Fisher (2006).

Reconciling Model and VAR

Theorem

In the long run, equiproportionate shocks to technology in the two production sectors M and N do not affect relative prices, while labor productivity is a log-linear function of these two shocks.

- The proof to this theorem can be constructed using steady-state restrictions for our baseline two-sector model.
- A corollary of this theorem is that the baseline two sector model can be used to interpret the permanent shocks to the relative price of investment and to labor productivity identified from our VAR.

Reconciling Model and VAR: Proof of Theorem, Part 1

$$\frac{P_M}{P_N} = \psi_1 \left(\frac{A_N}{A_M}\right)^{1-\alpha_N}$$

where

$$\psi_1 = \left(\frac{(1-\alpha_N)}{(1-\alpha_M)} \frac{\left(\alpha_N \frac{1}{(1-\beta(1-\delta))}\right)^{\frac{\alpha_N}{1-\alpha_N}}}{\left(\alpha_M \frac{1}{(1-\beta(1-\delta))}\right)^{\frac{\alpha_M}{1-\alpha_M}}} \right)^{1-\alpha_N} .$$

- Thus, equiproportionate changes in technology in the two production sectors will not affect relative prices.
- Notice that variation in relative prices at the sectoral level is a precondition for variation in relative prices at the level of final goods even in models with incomplete specialization.

Reconciling Model and VAR: Proof of Theorem, Part 2

Define aggregate labor productivity (at constant prices) as $\frac{Y_M+Y_N}{L}$. Then:

$$\frac{Y_M+Y_N}{L} =$$

$$A_M\left(\frac{\alpha_M}{(1-\beta(1-\delta))}\right)^{\frac{\alpha_M}{1-\alpha_M}}\frac{(1-\alpha_M)\psi_1}{(1-\alpha_M)\psi_1+(1-\alpha_N)\psi_2}$$

$$+A_M^{\alpha_N}A_N^{(1-\alpha_N)}\left(\frac{\alpha_N}{\psi_1\left(1-\beta(1-\delta)\right)}\right)^{\frac{\alpha_N}{1-\alpha_N}}\frac{(1-\alpha_N)\psi_2}{(1-\alpha_M)\psi_1+(1-\alpha_N)\psi_2}.$$

where $\psi_2=\psi_1\left(\frac{(1-\beta(1-\delta))}{\delta\alpha_N}-\frac{\alpha_M}{\alpha_N}\right)$.

- Thus both types of permanent sectoral MFP shocks affect the level of labor productivity in the long run.

Extensions of the Baseline Model

In order to bolster the baseline model with empirically relevant features, we extend it along the lines of Guerrieri, Henderson, and Kim 2014.

- We augment the utility function to allow for habit persistence in consumption and for endogenous labor supply.
- We modify the capital accumulation equation so that the capital stocks are distinct and predetermined across sectors, and we introduce investment adjustment costs.
- We allow for the investment and consumption aggregates to be a constant-elasticity functions of machinery and non-machinery inputs.
- We distinguish between two types of capital: equipment and structures.
- This greater degree of flexibility permits the commingling of sectoral inputs and different factor intensities across sectors consistent with the U.S. IO Tables.

Extensions of the Baseline Model – Sectoral Variant

We estimate two variants of this richer model:

1. *Sectoral Model with MFP shocks*
    - With all the extensions that bolster the empirical relevance just described, the resulting model cannot be aggregated.
    - We estimate this richer model capturing the variation in sectoral MFP levels with a neutral shock that varies the levels of MFP in equal ways across sectors and with an MFP shock specific to the machinery sector.

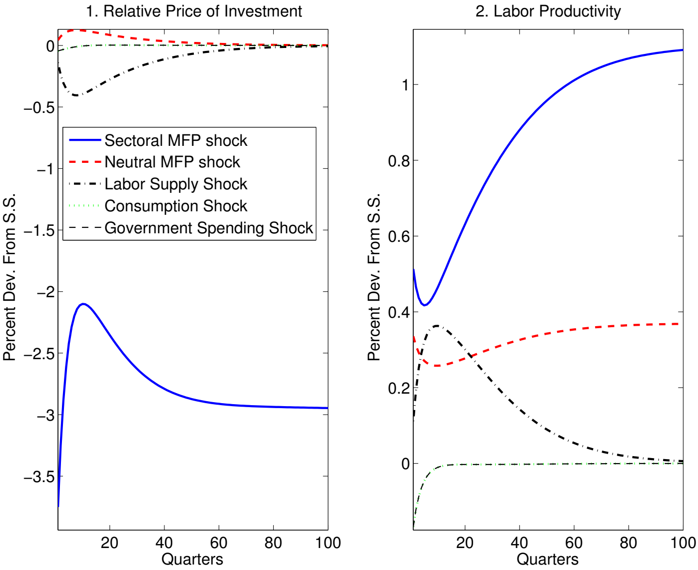
Extensions of the Baseline Model – Aggregate Variant

2. *Aggregate Model with IST shocks.*
    - Under special parametric restrictions that impose, complete sectoral specialization in the production of final goods, equal factor shares across sectors, capital stocks that are predetermined only at the aggregate level, our richer model can still be aggregated to a one sector model.
    - Moreover, under the same restrictions, sectoral variation in multi-factor productivity can be captured with a neutral MFP shock in the aggregate production function and with IST shocks that vary the efficiency of investment to produce installed capital right in the aggregate capital accumulation equation.
    - We estimate the aggregate variant of the model with IST shocks that are in line with Fisher's original interpretation of the shocks that yield a permanent movement in the relative price of investment.

Extensions of the Baseline Model – Estimation

- For estimation purposes, we augment the stochastic structure of the models with non-technology shocks: government spending shocks, consumption preference shocks, and labor supply shocks.
- We focus on matching the variance, the covariance, and the first autocorrelation of the same five variables used in the VAR.
- For each variant, the estimated parameters include the autoregressive coefficients and the standard deviations for all the shock processes. We also estimate the elasticity of substitution between factor inputs in the assembly functions for the aggregate goods (for the sectoral model only), the degree of habit persistence in consumption, and the investment adjustment costs.
- All other parameters are calibrated by determining steady-state ratios with U.S. national accounts data.

Extensions of the Baseline Model: Sectoral Model IRFs

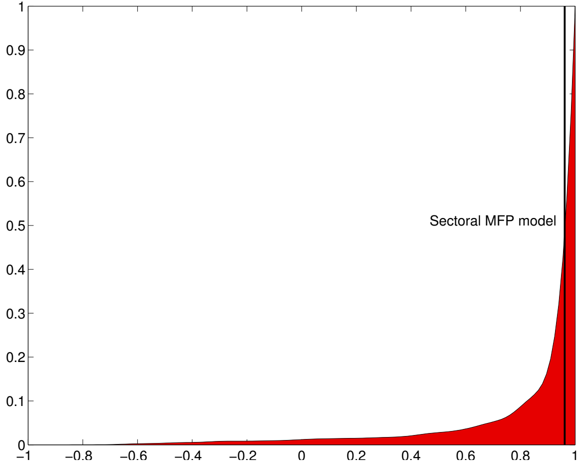


Discriminating across Models based on the VAR

- Having established that the identification scheme for the VAR estimates is consistent with our two-sector model, we proceed by comparing model and the VAR estimates.
- One approach is to check whether the model response to a certain shock is consistent or not with the empirical evidence from the VAR.
- For our purposes, the VAR confidence intervals for standard significance levels are so wide that we cannot tell the models apart.
- A key difference between two models is the correlation between consumption and investment, conditional on shocks to the price of investment.
- In previous work with Dale Henderson, we showed that this correlation is driven by a much stronger substitution effect for IST shocks in the aggregate variant than for investment-sector MFP shocks.

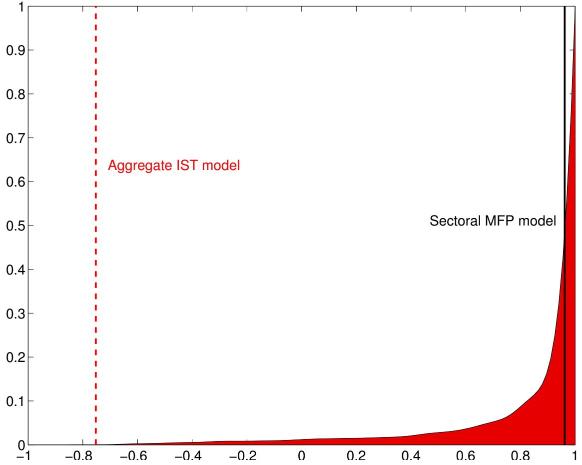
Comparing Model and VAR

Model Estimates and CDFs from VAR of the Correlation Between Consumption and Investment at Business Cycle Frequencies



Comparing Model and VAR

Model Estimates and CDFs from VAR of the Correlation Between Consumption and Investment at Business Cycle Frequencies

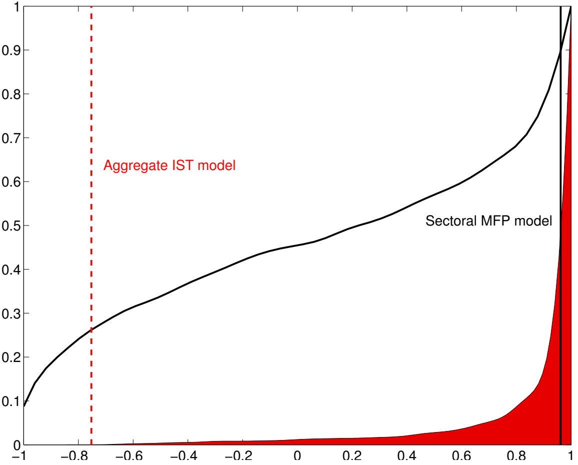


A Monte Carlo Experiment

As a further check we also compared the distribution of the correlation between consumption and investment obtained on artificial data generated from our alternative models.
We estimated the same VAR used on observed data and used the same identification scheme.

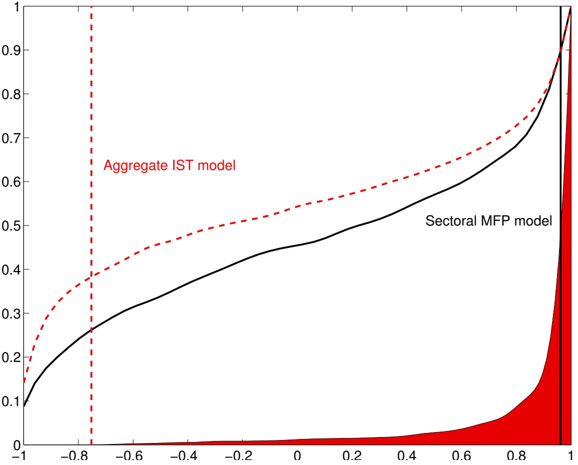
Comparing Model and VAR

Model Estimates and CDFs from VAR of the Correlation Between Consumption and Investment at Business Cycle Frequencies



Comparing Model and VAR

Model Estimates and CDFs from VAR of the Correlation Between Consumption and Investment at Business Cycle Frequencies



Conclusion

- Our estimates show that consumption and investment comove conditional on shocks that change the price of investment permanently.
- We show that this comovement has implications for alternative models of the business cycle.
- We found that the two-sector model that cannot be aggregated matches more closely the evidence of a positive correlation between consumption and investment from our VAR.
- A fruitful direction for further research would be to characterize the general class of DSGE models that is consistent with the assumptions imposed by growth accounting exercises making further contact with the Cambridge-Cambridge debate.