Journal of Monetary Economics 101 (2019) 64–81

Contents lists available at ScienceDirect

Journal of Monetary Economics

journal homepage: www.elsevier.com/locate/jmoneco

Macroeconomic policy games[☆]

Martin Bodenstein[a], Luca Guerrieri[a,*], Joe LaBriola[b]

[a] Federal Reserve Board, 20th and C Streets NW, Washington, D.C. 20551, United States
[b] UC Berkeley, 410 Barrows Hall, Berkeley, CA 94720, United States

ARTICLE INFO

Article history:
Received 22 September 2014
Revised 23 July 2018
Accepted 27 July 2018
Available online 29 July 2018

JEL classification:
E44
E61
F42

Keywords:
Optimal policy
Strategic interaction
Welfare analysis
Monetary policy cooperation
Macroprudential regulation

ABSTRACT

We develop a toolbox that characterizes the welfare-maximizing cooperative Ramsey policies under full commitment and open-loop Nash games between policymakers. We adopt the timeless perspective. Two examples for the use of our toolbox offer novel results. The first example revisits the case of monetary policy coordination in a two-country model to highlight sensitivity to the choice of policy instruments. For the second example, a central bank and a macroprudential policymaker are assigned distinct objectives in a model with financial frictions. Lack of cooperation can lead to large welfare losses even if technology shocks are the only source of fluctuations.

1. Introduction

Policymakers face the challenging task of finding the appropriate response to the actions of other policymakers. This task has informed active research on the gains from monetary policy coordination across countries, as described in detail by Canzoneri and Henderson (1991). Strategic interactions also arise within a country when different policymakers are assigned or pursue distinct objectives. For instance, the expansion and reorganization of regulatory responsibilities spurred by the Financial Crisis has been approached differently across countries. In the United States the Dodd-Frank Act substantially increased the macroprudential responsibilities of the Federal Reserve. In the United Kingdom, the Financial Services Act 2012 established an independent Financial Policy Committee as a subsidiary of the Bank of England, with some policymakers participating in both the Monetary and the Financial Policy Committee. By contrast, in the euro area, monetary policy is strictly separated from macroprudential regulation, although both functions involve the European Central Bank. Other examples include the interaction between fiscal and monetary authorities or games between countries about improving global competitiveness by setting tariffs and taxes across countries.

☆ We are grateful to Roberto Chang for useful comments. We are also indebted to Andrew Levin and David López-Salido for encouragement, useful comments, and for sharing their programs that solve the Ramsey problem for a single regulator. Of course, remaining errors are our own. The views expressed in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or of any other person associated with the Federal Reserve System. The toolbox and replication codes for the examples discussed in this paper are available from https://sites.google.com/site/martinbodenstein/ and from http://www.lguerrieri.com/games_code.zip.

* Corresponding author.
E-mail addresses: Martin.R.Bodenstein@frb.gov (M. Bodenstein), Luca.Guerrieri@frb.gov (L. Guerrieri), joelabriola@berkeley.edu (J. LaBriola).

https://doi.org/10.1016/j.jmoneco.2018.07.015
0304-3932/© 2018 Published by Elsevier B.V.

To facilitate the study of strategic interactions between policymakers, we develop a toolbox that characterizes the welfare-maximizing cooperative Ramsey policies under full commitment and open-loop Nash games from the timeless perspective. Our algorithm automates the analytical derivation of the conditions for an equilibrium under cooperative and open-loop Nash games. The algorithm has four main advantages: 1) it is fast; 2) it is widely applicable; 3) it avoids the error-prone manual derivation of the conditions for an equilibrium; and 4) it makes results easy to replicate. These characteristics open up the possibility to consider questions beyond the reach of other approaches to setting up games in a DSGE setting, as we showcase in our examples.

The toolbox is designed to extend Dynare, a convenient and popular modeling environment.[1] Our work augments the single regulator framework of Lopez-Salido and Levin (2004). The general framework for the policy games that we consider distinguishes between two groups of agents: the first group consists of private agents who incorporate the (expected) path of the policy instruments in their decisions; the second group consists of the policymakers, who determine policies taking into account the private sector's response to the implemented policies. Taking as input a set of equilibrium conditions given arbitrary rules for the reactions of the policy instruments, our toolbox replaces those rules with either the welfare-maximizing Ramsey policies or with the policies for the open-loop Nash game under the timeless perspective.

To showcase the wide applicability of our toolbox, we consider two examples and provide new results regarding the gains from cooperative policies. The first example is a two-country monetary model that closely follows Benigno and Benigno (2006) and Corsetti et al. (2010). These authors characterize the optimal monetary policies under cooperation and the open-loop Nash game between two monetary policy authorities in a dynamic general equilibrium model with sticky prices. If we take a linear approximation to the policymakers' first-order conditions around the optimal deterministic steady state of the model, we confirm that our toolbox produces the same results as the linear-quadratic approach in Benigno and Benigno (2006) and Corsetti et al. (2010).

A key advantage of our toolbox is the automation of the analytical derivation of the cooperative and open-loop Nash policies, once the actions of the private agents are characterized. Beyond the replication of existing results, the rapidity and convenience of deploying the toolbox allows us to explore with ease different strategy spaces associated with alternative instruments. We find that the instrument typically selected for this kind of exercise, producer price inflation, would not be selected if policymakers could enter a meta-game on instrument selection prior to the formulation of their optimal strategies. They would instead choose real output, a finding related to the lower spillover effects abroad associated with the strategy space for this instrument. Following the linear-quadratic approach, each of the twenty-five combinations of instruments that we consider would involve a new set of long and tedious algebraic derivations.

As a second example, we consider the workhorse New Keynesian model with financial frictions of Gertler and Karadi (2011). An agency problem on financial intermediaries has two effects. First, the problem inefficiently limits the provision of credit. Second, the agency problem also magnifies the reaction of the economy to shocks through familiar financial accelerator mechanisms. We extend the model of Gertler and Karadi (2011) to include a transfer tax between households and firms. Within that model, we consider a game between a financial policymaker and a central bank – a question not previously explored. The policy instrument of the central bank is the inflation rate; the policy instrument of the financial policymaker is the transfer tax. The objectives of the two policymakers reflect the preferences of households, but in both cases include an extra term. The central bank has an objective biased towards stabilizing inflation. The financial policymaker has an objective biased towards stabilizing the provision of credit. We characterize optimal cooperative Ramsey and open-loop Nash policies. Crucially, we constrain the choice of biases so that the cooperative policies with the skewed objectives come close to replicating the allocations under policies that maximize the welfare of the representative household. Nonetheless, the strategic interaction between policymakers lead to large and persistent deviations from cooperative outcomes and imply substantial welfare losses.

To highlight the wide applicability and rapidity of our toolbox, we also consider how the introduction of altruistic objectives that would (at least partially) internalize the bias of the other policymaker affect the open-loop Nash equilibrium. Intuitively, we confirm that altruistic preferences move the Nash allocations closer to the cooperative allocations, even in the presence of biases.

The optimal control literature that focuses on DSGE models typically derives first-order conditions but does not check second-order conditions. Forward-looking variables in DSGE models complicate substantially the analysis of second-order conditions especially when considering fully nonlinear solutions. An exception is the work of Benigno and Woodford (2012), who derive second order-conditions for an optimal control problem in the case of a single planner under a linear-quadratic solution. Benigno and Woodford do not provide analogous derivations for the more involved case of the open-loop Nash problem considered here.

Our approach to checking second-order conditions relies on taking perturbations of the optimal solution in the direction of arbitrary policy rules. We verify that a convex combination of the optimal rule and an arbitrary policy rule does not improve the objective function of the policymaker. This check applies both under cooperation, and under the open-loop Nash solution. After all, in the open-loop Nash case, we are interested in the best response of a policymaker to the best response of the other policymaker.

[1] See Adjemian et al. (2011).

Our toolbox is not limited to solving the particular examples above. Following the approach in Dixit and Lambertini (2003), differences in objectives are fertile ground to explore the strategic interactions between policymakers. For instance, the solution under coordinated optimal monetary and fiscal policies explored in Schmitt-Grohe and Uribe (2004) could be readily extended for strategic interactions after allowing for differences in the objectives of the monetary and fiscal authorities. More recent examples of stylized models that set the stage for strategic interactions between policymakers include Costinot et al. (2014), who illustrate the use of capital controls to manipulate the terms of trade, and Brunnermeier and Sannikov (2014), who show how capital controls may improve welfare in a model with financial frictions (but who do not consider a non-cooperative solution). Furthermore, our toolbox greatly facilitates the analysis of more fully articulated models. Examples include Bergin and Corsetti (2013), who introduce firm entry into a two-country model to study how the resulting production relocation externality influences monetary policy, and Fujiwara and Teranishi (2013), who allow for nominal rigidities in loan contracts.

The rest of the paper is organized as follows. Section 2 outlines the algorithm for calculating cooperative optimal policy and extends the algorithm to the calculation of optimal policies in open-loop Nash games. Section 3 applies the algorithm to an open-economy model where each country wishes to maximize welfare, and Section 4 considers the application of our algorithm to a model with a monetary authority and a macroprudential policymaker. Section 5 concludes. An online appendix provides instructions for the use of our toolbox as well as a more detailed description of our examples.

2. Equilibrium definitions and solution algorithms

This section defines an equilibrium under cooperative Ramsey policies and under an open-loop Nash game. We discuss computational issues and concepts as appropriate.

In maximizing the policy objectives subject to the structural equations of the private sector our toolbox employs a Lagrangian approach. The exact nonlinear first-order conditions that characterize the optimal policies under cooperation and the open-loop Nash game, respectively, are obtained by symbolic differentiation. Each system of equations is then approximated around its deterministic steady state using higher order perturbation methods. An alternative approach to characterizing optimal policies uses linear-quadratic (LQ) techniques. The LQ approach involves finding a purely quadratic approximation of each policymaker's objective function, which is then optimized subject to a linear approximation of the structural equations of the model. Following Benigno and Woodford (2012); Levine et al. (2008) and Debortoli and Nunes (2006) we show how the LQ approach relates to the approach underlying our numerical procedure and that the LQ approach delivers the same solution if the nonlinear output of our toolbox is approximated to the first order.

2.1. General framework

Policy games distinguish between two groups of actors. We label the first group "private agents." Private agents take into account the (expected) path of the policy instruments. The second group consists of the policymakers who determine policies taking into account the private sector's response to these policies. With more than one policymaker, strategic interactions between the policymakers can cause the outcomes of the dynamic game to deviate from those of the welfare-maximizing cooperative policy. For simplicity, we restrict the exposition to the case of two policymakers (or players). Furthermore, each policymaker is assumed to have only one instrument.[2]

Let the $N \times 1$ vector of endogenous variables be denoted by x_t, which is partitioned as $x_t = (\tilde{x}_t', i_{1,t}, i_{2,t})'$. The variable $i_{j,\,t}$ is the policy instrument of player $j = [1, 2]$, respectively. The exogenous variables are captured by the vector ζ_t. For given sequences of the policy instruments $\{i_{1,t}, i_{2,t}\}_{t=0}^{\infty}$, the remaining $N - 2$ endogenous variables need to satisfy the $N - 2$ structural conditions that characterize an equilibrium

$$E_t g(x_{t-1}, \tilde{x}_t, x_{t+1}, i_{1,t}, i_{2,t}, \zeta_t) = 0. \tag{1}$$

We assume that the system of equations in g is differentiable up to the desired order of approximation. Without loss of generality and to facilitate changes in the set of policy instruments for our toolbox, the block of structural equations (1) contains two definitions relating the generic instrument variables $i_{1,\,t}$ and $i_{2,\,t}$ to the desired instruments in the model. For example, if player 1 uses the inflation rate $\pi_{1,\,t}$ as instrument, following Woodford (2003), then one of the equations in (1) simply reads $i_{1,t} - \pi_{1,t} = 0$.

To complete our framework, we need to describe how policies are determined. The intertemporal preferences of player j are given by $\mathcal{U}_j = E_0 \sum_{t=0}^{\infty} \beta^t U_j(\tilde{x}_{t-1}, \tilde{x}_t, \zeta_t)$ with the generic utility function $U_j(\tilde{x}_{t-1}, \tilde{x}_t, \zeta_t)$ required to be concave. Under cooperation, the two players maximise the joint welfare function $\omega_1 \mathcal{U}_1 + \omega_2 \mathcal{U}_2$ for given weights ω_1 and ω_2. We normalize the welfare weights to satisfy $\omega_1 + \omega_2$=1. Absent cooperation, each policymaker considers his own preferences only.

[2] Our toolbox is currently restricted to games between two policymakers with one instrument each. However, it should be straightforward to extend the toolbox to handle more than one instrument per policymaker and more than two policymakers.

2.2. Definition of equilibrium under cooperation

The welfare-maximizing Ramsey policy with full commitment is derived from the maximization problem

$$\max_{\{\tilde{x}_t, i_{1,t}, i_{2,t}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t [\omega_1 U_1(\tilde{x}_{t-1}, \tilde{x}_t, \zeta_t) + \omega_2 U_2(\tilde{x}_{t-1}, \tilde{x}_t, \zeta_t)]$$
$$s.t.$$
$$E_t g(x_{t-1}, x_t, x_{t+1}, \zeta_t) = 0. \quad (2)$$

The first-order conditions for this problem can be obtained by differentiating the Lagrangian problem of the form

$$\mathcal{L}_0 = E_0 \sum_{t=0}^{\infty} \beta^t \big[\omega_1 U_1(\tilde{x}_{t-1}, \tilde{x}_t, \zeta_t) + \omega_2 U_2(\tilde{x}_{t-1}, \tilde{x}_t, \zeta_t) + \lambda_t' g(x_{t-1}, x_t, x_{t+1}, \zeta_t)\big]. \quad (3)$$

The $(N-2) \times 1$ Lagrange multipliers associated with the private sector equilibrium conditions in (1) are denoted by λ_t for any $t \geq 0$.

Taking derivatives of $\mathcal{L}_0$ with respect to the N endogenous variables in x_t delivers N first-order conditions. Additionally, taking derivatives with respect to λ_t delivers again the $N-2$ private sector conditions. In total, there are $2N-2$ conditions and $2N-2$ variables. In sum, for $t > 0$ the Ramsey equilibrium process $\{\tilde{x}_t, i_{1,t}, i_{2,t}, \lambda_t\}_{t=0}^{\infty}$ satisfies

$$\sum_{j=1,2} \omega_j \{D_x U_j(\tilde{x}_{t-1}, \tilde{x}_t, \zeta_t) + \beta E_t D_{x^-} U_j(\tilde{x}_t, \tilde{x}_{t+1}, \zeta_{t+1})\}$$
$$+\beta E_t \big\{\lambda_{t+1}' D_{x^-} g(x_t, x_{t+1}, x_{t+2}, \zeta_{t+1})\big\} + E_t \big\{\lambda_t' D_x g(x_{t-1}, x_t, x_{t+1}, \zeta_t)\big\}$$
$$+\beta^{-1} \lambda_{t-1}' D_{x^+} g(x_{t-2}, x_{t-1}, x_t, \zeta_{t-1}) = 0 \quad (4)$$

$$E_t g(x_{t-1}, x_t, x_{t+1}, \zeta_t) = 0, \quad (5)$$

while for $t = 0$ the process satisfies

$$\sum_{j=1,2} \omega_j \{D_x U_j(\tilde{x}_{-1}, \tilde{x}_0, \zeta_t) + \beta E_0 D_{x^-} U_j(\tilde{x}_0, \tilde{x}_1, \zeta_1)\}$$
$$+\beta E_0 \big\{\lambda_1' D_{x^-} g(x_0, x_1, x_2, \zeta_1)\big\} + E_0 \big\{\lambda_0' D_x g(x_{-1}, x_0, x_1, \zeta_t)\big\} = 0 \quad (6)$$

$$E_0 g(x_{-1}, x_0, x_1, \zeta_0) = 0. \quad (7)$$

The notation D_x denotes the vector of partial derivatives of any functions with respect to the elements of x_t; likewise, D_{x^-} and D_{x^+} denote derivatives with respect to x_{t-1} and x_{t+1}, respectively, while λ_t' denotes the transpose of the vector of Lagrange multipliers λ_t. Notice that since the generic instruments $i_{1,t}$ and $i_{2,t}$ are encompassed in $E_t g(x_{t-1}, x_t, x_{t+1}, \zeta_t) = 0$ through definitions of the form $i_{j,t} = \tilde{x}_t^j$, where $\tilde{x}_t^j$ is player j's actual policy instrument, taking derivatives with respect to $i_{1,t}$ and $i_{2,t}$ returns the Lagrange multipliers associated with these definitions.

This formulation of the problem implies that period 0 is different from every other period, because the choice of policies is not restricted by previous commitments. Although this system of equations can in general be solved, the equilibrium functions will not be time-invariant. To avoid this problem, we follow most of the literature in adopting the concept of optimality from a *timeless perspective* which is discussed in great detail in Benigno and Woodford (2012).[3] In short, this concept requires an initial pre-commitment to suitably chosen values λ_{-1} at time 0 so that the first-order conditions (4) to (5) apply to all $t \geq 0$. Thus, the planner solves a modified optimization problem with additional constraints for time 0. Equivalently, the planner's utility function in (2) is modified to reflect the initial commitments directly in the objective function

$$E_0 \sum_{t=0}^{\infty} \beta^t [\omega_1 U_1(\tilde{x}_{t-1}, \tilde{x}_t, \zeta_t) + \omega_2 U_2(\tilde{x}_{t-1}, \tilde{x}_t, \zeta_t)] + \beta^{-1} \lambda_{-1} g(x_{-2}, x_{-1}, x_0). \quad (8)$$

[3] The toolbox generates a Dynare model file and sets the predetermined values for the Lagrange multipliers associated with the recursive planner(s) problem to the steady-state values consistent with the timeless perspective. The model examples that follow are then solved with second-order perturbation methods. However, the initial conditions for the Lagrange multipliers associated with the planner's problem could in principle be reset to any desired value. Setting them to 0, would make them consistent with the original problem (excluding the timeless perspective), which could then be solved, for instance, with a shooting algorithm, including the one available in Dynare.

The timeless perspective implies that the optimal deterministic steady state $(\bar{x}, \bar{\lambda})$ needs to satisfy

$$\sum_{j=1,2} \omega_j \{D_x U_j(\bar{\tilde{x}}, \bar{\tilde{x}}, 0) + \beta D_{x^-} U_j(\bar{\tilde{x}}, \bar{\tilde{x}}, 0)\} \tag{9}$$

$$+\bar{\lambda}' \left(\beta D_{x^-} g(\bar{x}, \bar{x}, \bar{x}, 0) + D_x g(\bar{x}, \bar{x}, \bar{x}, 0) + \beta^{-1} D_{x^+} g(\bar{x}, \bar{x}, \bar{x}, 0)\right) = 0$$

$$E_t g(\bar{x}, \bar{x}, \bar{x}, 0) = 0. \tag{10}$$

The problem stated in Eqs. (9) and (10) is linear in the Lagrange multipliers. This feature can be exploited to obtain a reasonably accurate initial guess for computing the steady-state values of the Lagrange multipliers. For an initial guess of $\bar{x}$ that satisfies the equations for the private sector equilibrium (10), we use linear regressions to obtain initial guesses for the values for the Lagrange multipliers. Re-interpreting Eq. (9), the dependent variables in our regressions are stacked in the vector $-\sum_{j=1,2} \omega_j \{D_x U_j(\bar{\tilde{x}}, \bar{\tilde{x}}, 0) + \beta D_{x^-} U_j(\bar{\tilde{x}}, \bar{\tilde{x}}, 0)\}$, the regression coefficients are the Lagrange multipliers $\bar{\lambda}$, and the explanatory variables are the matrix $\left(\beta D_{x^-} g(\bar{x}, \bar{x}, \bar{x}, 0) + D_x g(\bar{x}, \bar{x}, \bar{x}, 0) + \beta^{-1} D_{x^+} g(\bar{x}, \bar{x}, \bar{x}, 0)\right)$. Our toolbox implements these ideas to solve for the steady state numerically, relying on quasi-Newton methods available in Matlab. As is familiar from the numerical literature, in the presence of multiple solutions, different initial guesses can be used to survey the possibility of multiple steady states. If multiple steady states are identified, the optimal steady state must feature the highest value for the objective of the cooperative planner.

Eqs. (4) and (5) can now be replaced by a local approximation around the optimal steady state $\{\bar{x}, \bar{\lambda}\}$ of desired order. The resulting system of (higher-order) difference equations can be solved by standard perturbation algorithms as further outlined in Section 2.5.

2.3. Definition of open-loop Nash equilibrium

To define an open-loop Nash equilibrium, let $\{i_{j,t,-t^*}\}_{t=0}^{\infty}$ denote the sequence of policy choices by player j before and after, but not including period t^*. An open-loop Nash equilibrium is a sequence $\{i^*_{j,t}\}_{t=0}^{\infty}$ with the property that for all t^*, i^*_{j,t^*} maximises player j's objective function subject to the structural equations of the economy for given sequences $\{i^*_{j,t,-t^*}\}_{t=0}^{\infty}$ and $\{i^*_{-j,t}\}_{t=0}^{\infty}$, where $\{i^*_{-j,t}\}_{t=0}^{\infty}$ denotes the sequence of policy moves by all players other than player j. Each player's action is the best response to the other players' best responses.

With policymakers needing to specify a complete contingent plan at time 0 for their respective instruments $\{i_{j,t}\}_{t=0}^{\infty}$ for $j = [1, 2]$, under the open-loop equilibrium concept, the problem can be reinterpreted as a static game, allowing us to recast each player's optimization problem as an optimal control problem given the policies of the remaining players. As under the static Nash equilibrium concept, player j restricts attention to his own objective function, and the maximisation program is given by

$$\max_{\{\tilde{x}_t, i_{j,t}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t U_j(\tilde{x}_{t-1}, \tilde{x}_t, \zeta_t)$$
$$s.t.$$
$$E_t g(x_{t-1}, x_t, x_{t+1}, \zeta_t) = 0$$
$$\text{for given } \{i_{-j,t}\}_{t=0}^{\infty}. \tag{11}$$

The first-order conditions for each player are obtained from differentiating the Lagrangian of the form

$$\mathcal{L}_{j,0} = E_0 \sum_{t=0}^{\infty} \beta^t \left[U_j(\tilde{x}_{t-1}, \tilde{x}_t, \zeta_t) + \lambda'_{j,t} g(x_{t-1}, x_t, x_{t+1}, \zeta_t)\right] \tag{12}$$

for $j = [1, 2]$. Taking derivates of the $\mathcal{L}_{j,0}$ with respect to the $N-1$ choice variables $\left(\tilde{x}_t, i_{j,t}\right)$, excluding the instrument of the other player, and the $N-2$ Lagrange multipliers $\lambda_{j,\,t}$ associated with the $N-2$ structural relationships yields $2N-3$ conditions for each player.

Notice that the full set of $4N-6$ equations includes the $N-2$ structural equations twice. Since in equilibrium all players face the same values of the non-policy variables $\tilde{x}_t$, an interior Nash equilibrium $\{\tilde{x}^*_t, i^*_{1,t}, i^*_{2,t}, \lambda^*_{1,t}, \lambda^*_{2,t}\}_{t=0}^{\infty}$ satisfies the following $3N-4$ conditions for $t > 0$

$$D_x U_1(\tilde{x}^*_{t-1}, \tilde{x}^*_t, \zeta_t) + \beta E_t D_{x^-} U_1(\tilde{x}^*_t, \tilde{x}^*_{t+1}, \zeta_{t+1})$$
$$+\beta E_t \left\{\lambda^{*'}_{1,t+1} D_{x^-} g(x^*_t, x^*_{t+1}, x^*_{t+2}, \zeta_{t+1})\right\} + E_t \left\{\lambda^{*'}_{1,t} D_x g(x^*_{t-1}, x^*_t, x^*_{t+1}, \zeta_t)\right\}$$
$$+\beta^{-1} \lambda^{*'}_{1,t-1} D_{x^+} g(x^*_{t-2}, x^*_{t-1}, x^*_t, \zeta_{t-1}) = 0 \tag{13}$$

$$
\begin{aligned}
&D_x U_2(\tilde{x}^*_{t-1}, \tilde{x}^*_t, \zeta_t) + \beta E_t D_{x^-} U_2(\tilde{x}^*_t, \tilde{x}^*_{t+1}, \zeta_{t+1}) \\
&\quad + \beta E_t \left\{ \lambda^{*'}_{2,t+1} D_{x^-} E_t g(x^*_t, x^*_{t+1}, x^*_{t+2}, \zeta_{t+1}) \right\} + E_t \left\{ \lambda^{*'}_{2,t} D_x g(x^*_{t-1}, x^*_t, x^*_{t+1}, \zeta_t) \right\} \\
&\quad + \beta^{-1} \lambda^{*'}_{2,t-1} D_{x^+} g(x^*_{t-2}, x^*_{t-1}, x^*_t, \zeta_{t-1}) = 0 \qquad (14)
\end{aligned}
$$

$$
E_t g(x^*_{t-1}, x^*_t, x^*_{t+1}, \zeta_t) = 0. \qquad (15)
$$

In a fashion similar to the case of cooperation, the first-order conditions with respect to $i_{1,\,t}$ and $i_{2,\,t}$ imply the restriction that the Lagrange multipliers associated with the definition of the policy instruments are zero for all $t \geq 0$.

Adopting the timeless perspective is again key to obtaining time-invariant decision rules. The optimal response of each player given the policies of the other player derived from the optimal control problem at time 0 is not necessarily time consistent. Last, the deterministic steady state is found as for the cooperative case by exploiting the linearity of the system (13)–(15) in the $2N - 4$ Lagrange multipliers.

2.4. Relationship to linear-quadratic approach

An alternative approach to solve optimal policy problems uses LQ techniques. In the case of a single decision maker, the LQ approach involves finding a purely quadratic approximation of the policymaker's objective function which is then optimized subject to a linear approximation of the structural equations of the model. Benigno and Woodford (2012) and Levine et al. (2008) and Debortoli and Nunes (2006) discuss necessary and sufficient conditions for a "correct LQ approximation" to the optimization problem stated in Eq. (2) to exist. Adopting the timeless perspective is shown to be one of the necessary conditions. In contrast to the early literature the approach followed here does not require the steady state of the model to be efficient.

Appendix B, available online, shows that, to a first-order approximation, the output of our toolbox is equivalent to that of the LQ approach. The appendix also gives a roadmap for constructing the LQ matrices from the output of our toolbox.

2.5. Solution algorithms

For all the examples demonstrating the use of our toolbox, we apply a perturbation approach to approximating the model solution. When reporting impulse response functions for alternative shocks, we use a first-order approximation. When reporting welfare results, we use a "true" second-order approximation by following the pruning algorithm in Kim et al. (2008). Pruning keeps the approximation constant at the second-order by avoiding the accumulation of higher-order terms. Moreover, pruning ensures that the Blanchard–Kahn conditions for stability and local uniqueness for the first-order of approximation apply to the second-order, too.

To compute the welfare costs of suboptimal policies, we focus on differences in conditional welfare. This procedure is motivated by the fact that the planners/players in our examples have objective functions that are conditional on initial states. This focus on conditional welfare avoids spurious welfare reversals that could otherwise occur.[4]

3. Monetary policy in an open-economy model

We first illustrate our toolbox for the workhorse two-country model of monetary economics laid out in Benigno and Benigno (2006) and Corsetti et al. (2010). The model features two countries, each specialized in the production of one type of goods in different varieties. Each household produces exactly one variety and engages in monopolistic competition with all other households. Time-invariant subsidies offset the monopoly distortions in the steady state. A household chooses its nominal price to maximize its utility; as in Calvo (1983) the household can adjust the price at future dates with a fixed probability. Export prices are set in the currency of the producer. Shocks to technology affect the marginal product of labor, whereas a markup shock influences how much prices exceed the marginal cost of production. Finally, goods trade freely across borders and international financial markets are frictionless and complete.

Benigno and Benigno (2006) and Corsetti et al. (2010) derive the optimal monetary policy under commitment from the timeless perspective using the LQ approach for the case of cooperation. Producer price inflation is the policy instrument in both countries. Under cooperation, the objective is an equally-weighted average of the welfare of the representative agents in the two countries. When policymakers do not cooperate, strategic interaction generally leads to welfare inferior outcomes: the failure to account for the international spillovers of domestic policies causes foreign policymakers to adopt policies that in turn negatively impact the domestic country in the open-loop Nash equilibrium. Thus, there are gains from cooperation.

Online Appendix C covers the problems faced by the various agents in the model and reports the conditions (Eqs. C.12–C.36) that characterize the private-sector equilibrium in the model of Corsetti et al. (2010) which generalizes the one in Benigno and Benigno (2006) by allowing for home bias in consumption. The appendix also shows how to cast the model in a form suitable for the application of our toolbox.

[4] See Kim and Kim (2015) for examples of how conditional or unconditional objectives can lead to different optimal policies.

To bolster confidence in our toolbox, we proceed by showing that it reproduces the results derived by Benigno and Benigno (2006) and Corsetti et al. (2010). We then turn to the novel aspects of our analysis. First, we introduce checks to assess the optimality of the computed equilibria under cooperation and in the open-loop Nash game. Second, we explore the impact of the policy instrument choice for the gains from cooperation. The literature has almost exclusively restricted the policy instrument to be producer price inflation in both countries. Expanding the strategy space to include many more candidate instruments is easily accomplished with our toolbox. Thus, we are in a position to set up an extension of the usual game in the form of a meta-game that lets planners choose their instruments prior to choosing optimal strategies for the selected instrument.

3.1. Optimal policy with and without cooperation

The output of our toolbox matches well-known results in the literature. In the face of technology shocks, the welfare-maximising policy under cooperation replicates the flexible price allocations for the two-country model laid out above. As in closed economy models, the “divine coincidence” applies for “efficient shocks” – see Blanchard and Galí (2007). Accordingly, technology shocks move quantities and prices in the same direction relative to the flexible price economy and policymakers do not face a trade-off between inflation and output gap stabilization.

By contrast, a markup shock, an “inefficient disturbance” poses more complex choices for policymakers. As would be the case in an analogous closed economy model, the cooperating policymakers cannot perfectly stabilize the economy. In response to a positive markup shock, the output gap turns negative, whereas inflation is positive.

If policymakers do not cooperate across borders, prices and quantities will in general differ from those under cooperation. Each country has the ability to influence the terms of trade through its monetary policy stance and the open-loop Nash equilibrium does not replicate the flexible-price allocations even for efficient shocks.[5]

Figs. 1 and 2 confirm key findings of previous papers. They show the responses to a positive technology shock and a markup shock under the welfare-maximizing cooperative policy and under an open-loop Nash game. As in Benigno and Benigno (2006) and Corsetti et al. (2010), Fig. 1 shows that output price inflation is perfectly stabilized under the cooperative policy, and that the output response coincides with its counterpart in a flexible price model (not shown) for both countries after a technology shock. In the open-loop Nash game, inflation and output gaps are not perfectly stabilized. In that case, terms-of-trade movements affect the objectives of the foreign policymaker, and those effects are not fully internalized by the home policymaker.

Under the markup shock in Fig. 2, neither policy completely stabilizes output price inflation and the output gaps.[6] As shown in Corsetti et al. (2010), the home country's real exchange rate appreciates and its terms of trade improve by more under the open-loop Nash policies than under the cooperative policy, resulting in larger spillover effects.[7]

Previous explorations of the gains from cooperation for monetary policy in an open economy setting restricted the strategy space to certain families of instrument rules. A prominent example is Obstfeld and Rogoff (2002). We confirmed that if the two countries in our model were to use simple interest rate rules responding to the lagged interest rate and to producer price inflation, the resulting cooperative and Nash allocations would be very similar to the optimal policies under cooperation and the open-loop Nash game, respectively.[8] This finding, however, is not general. It is driven by the apt choice of variables that enter the interest rate rule. Relatedly, when the analysis of strategic interactions is dependent on the particular family of interest rate rules considered, any results of this analysis could, in principle, be overturned by including additional terms in the rules. The more general policies automatically set up by our toolbox avoid this shortcoming.

3.2. Assessing the optimality of policy choices

The optimal control literature that focuses on DSGE models typically does not go beyond the derivation of first-order optimality conditions. An exception is the work of Benigno and Woodford (2012), who derive second-order conditions for an optimal control problem in the case of a single planner under the LQ solution. Benigno and Woodford do not provide analogous derivations for the more involved case of the open-loop Nash problem considered here. Furthermore, the approach outlined in Benigno and Woodford (2012) is not directly applicable to the verification of optimality conditions under solutions from higher-order approximations even for the case of a single policymaker.

---

[5] A necessary condition for the gains from cooperation to disappear in response to a technology shock is that the intratemporal and intertemporal elasticities of substitution be equal, which is not a feature of our calibration.

[6] The efficient output level does not move at all in response to a markup shock. Hence, movements in actual output are mirrored by movements in the output gap.

[7] To further assess the reliability of our toolbox, we confirmed that its output under a first-order approximation coincides with the results produced by the LQ approach in Benigno and Benigno (2006) and Corsetti et al. (2010). Online Appendix C.3 reconciles the notation in Corsetti et al. (2010) with ours. The toolbox that accompanies this paper provides codes that line up our results with those in Benigno and Benigno (2006) and Corsetti et al. (2010).

[8] In the case of the Nash game, we identified the solution by alternatively optimizing the parameters for the instrument rule (governing the weights on the lagged interest rate and on inflation) of one policymaker keeping the other rule constant at the previously optimized parameters. We stopped the iteration at a fixed point (consistent with the definition of a Nash equilibrium). In all cases, the optimized simple rules featured a very high weight on interest rate smoothing and responded strongly to inflation.

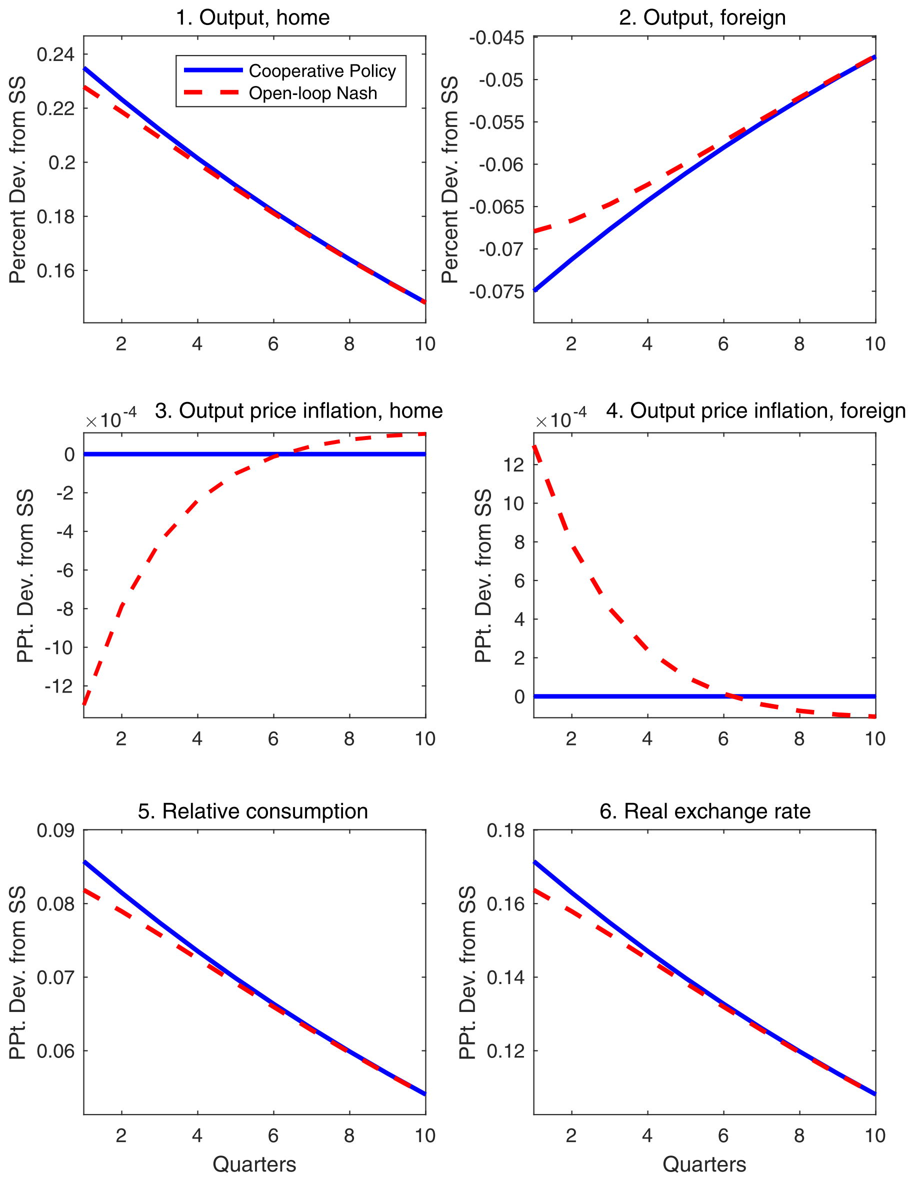

Fig. 1. Cooperative and open-loop Nash policies in the open economy model: Responses to a technology shock.
Notes: The figure plots the transition dynamics of the two economies after a one-standard deviation increase in technology in the home country. The two lines show the responses under full commitment with cooperation (cooperative policy) and without cooperation (open-loop Nash), when policymakers use output price inflation in their respective country as the policy instrument.

Our approach to checking second-order conditions relies on taking perturbations of the optimal solution in the direction of arbitrary policy rules. We verify that a convex combination of the optimal rule and an arbitrary policy rule does not improve on the objective function of the policymaker. This check applies both under cooperation, and under the open-loop Nash solution.

Practically, we stack the necessary conditions for an equilibrium for the optimal control problem (either for the cooperative or the competitive case) with the conditions for an equilibrium for the analogous economy governed by the arbitrary policy rule. All the endogenous variables for the two stacked models remain distinct in order to track numerically the optimal policy for the particular instrument of choice. With this approach we can check the payoffs associated with any convex combination of the optimal policy and the arbitrary instrument rule (as long as the instrument rule does not lead to a violation of the Blanchard–Kahn conditions). Optimality requires intuitively that the value of the objective function of the policymaker be reduced if any non-zero weight is attached to the arbitrary policy rule.

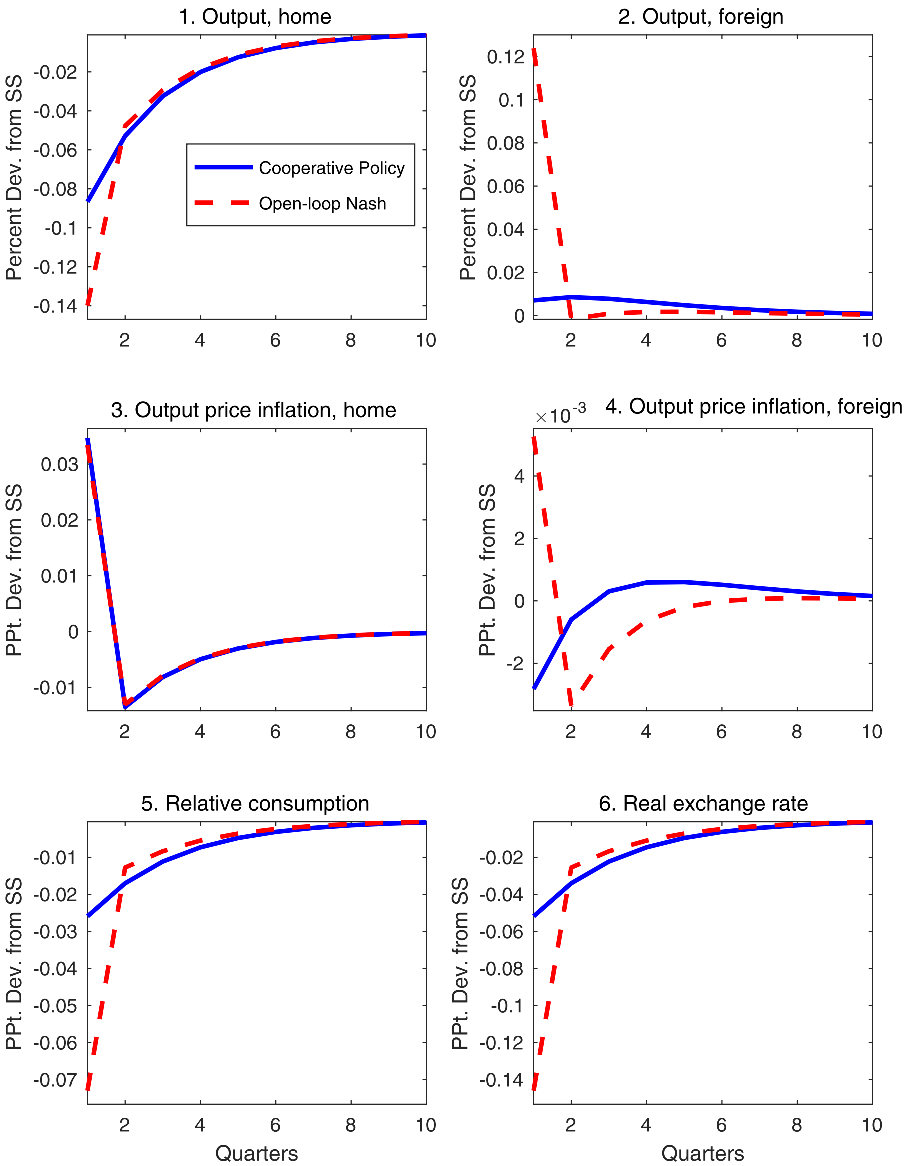

Fig. 2. Cooperative and open-loop Nash policies in the open economy model: Responses to a markup shock.
Notes: The figure plots the transition dynamics of the two economies after a one-standard deviation increase in the price markup in the home country. The two lines show the responses under full commitment with cooperation (cooperative policy) and without cooperation (open-loop Nash), when policymakers use output price inflation in their respective country as the policy instrument.

In performing this check, it is necessary to recognize that under the timeless perspective, the objective function of the policymaker is modified relative to the original objective to ensure time-invariant decision rules in equilibrium.[9] Our toolbox provides tools to stack the necessary conditions for an equilibrium for our test, as well as to size the change in the objective function consistent with the timeless perspective – essentially the value of promises made by the policymaker before the initial period.

Fig. 3 provides an example of this check. In the home country, we consider a simple policy rule that sets producer price inflation to its steady-state value. Letting the cooperative policy in each country be denoted by π_t and π_t^*, the alternative combination of policies sets producer price inflation in the home country as $\pi_t^s = \upsilon^s \pi + (1 - \upsilon^s)\pi_t$, where the parameter υ^s governs the convex combination. The foreign country follows $\pi_t^{s*} = \pi_t^*$.

[9] Recall that under the timeless perspective the utility function maximized by the planner is given by Eq. (8). In particular, the switch to an alternative policy could break previous commitments made under the timeless perspective and therefore outperform the optimal policy if the welfare criterion is not taken to be (8).

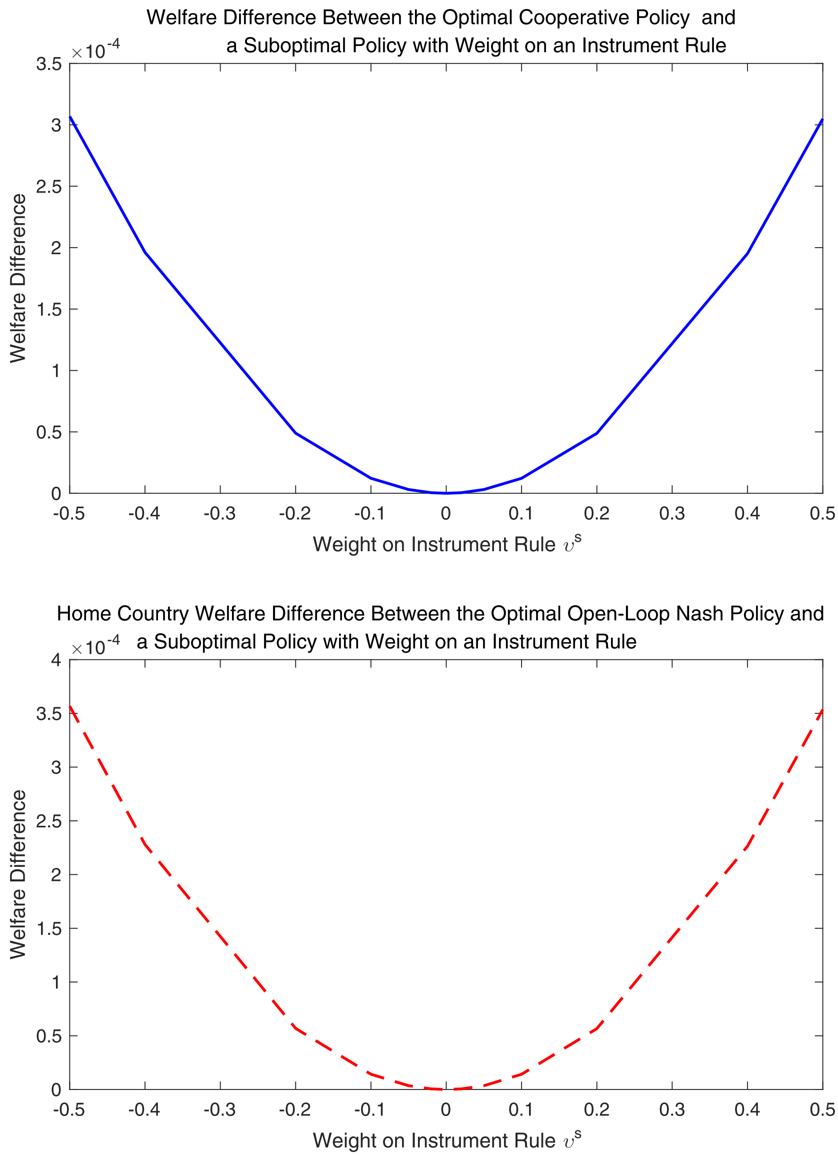

Fig. 3. Assessing optimality of policy choices.
Notes: The top panel plots the difference between conditional welfare under the optimal cooperative policy and a suboptimal policy that assigns weight on both an arbitrary policy rule and the optimal cooperative policy. With the instruments under the cooperative policy chosen as π_t and π_t^*, the suboptimal policy sets producer price inflation in the home country to follow $\pi_t^s = \upsilon^s \pi + (1 - \upsilon^s)\pi_t$ and to follow $\pi_t^{s*} = \pi_t^*$ in the foreign country. The welfare difference being minimized at $\upsilon^s = 0$ implies that the arbitrary rule under consideration cannot improve upon the optimal cooperative policy. The bottom panel reports the results from a similar exercise in the open-loop Nash game by asking whether the home country can improve upon the optimal strategy for the Nash game by assigning weight to the prescription from an arbitrary policy rule. Assuming that the foreign country sets the foreign inflation rate in accordance with its strategy in the open-loop Nash game, the home country prefers the strategy from the open-loop Nash game to any mixture that assigns positive weight to the arbitrary rule under consideration. Similar results obtain when the roles of the home and foreign country are reversed.

The top panel plots the difference between conditional welfare under the optimal cooperative policy and the suboptimal combination policy for different values of υ^s. As indicated by the welfare difference being minimized at $\upsilon^s = 0$, the arbitrary rule considered cannot improve the optimal cooperative policy.

The bottom panel of Fig. 3 reports results analogous to those for the top panel for the open-loop Nash game. In this case, we check whether the home country can improve upon the optimal strategy in the Nash game by also assigning weight to an arbitrary policy rule. With the foreign country setting the foreign inflation rate following the optimal strategy from the open-loop Nash game, the home country prefers the (optimal) strategy from the open-loop Nash game to any convex combination of that strategy and the arbitrary rule. (Given symmetry, identical results obtain when the roles of the home and foreign country are reversed.)

Table 1
Welfare gains from cooperation under alternative instrument choices.

Strategy	π_t^*	$\pi_{C,t}^*$	Y_t^*	$P_t^* Y_t^*$	$\frac{e_t^*}{e_{t-1}^*}$
$\boldsymbol{\pi_t}$	1.00	3.14	0.65	1.39	3.64
$\boldsymbol{\pi_{C,\,t}}$	3.15	21.61	2.79	5.39	27.00
$\boldsymbol{Y_t}$	0.65	2.79	0.25	1.07	3.27
$\boldsymbol{P_t Y_t}$	1.39	5.39	1.07	2.10	6.38
$\frac{e_t}{e_{t-1}}$	3.65	27.00	3.27	6.39	36.78

Note: This table reports the welfare gains from cooperation for each combination of instruments used by the two policymakers in the open-loop Nash game. The welfare gains in the table are expressed relative to the gains under the baseline case of each policymaker using producer price inflation (π_t) as the instrument. Hence, by construction, the entry corresponding to π_t and π_t^* is 1.00. The other instruments considered are: consumption price inflation ($\pi_{C,\,t}$), real output (Y_t), nominal output ($P_t Y_t$), and the change in the nominal exchange rate ($\frac{e_t}{e_{t-1}}$). Strikingly, when the change in the nominal exchange rate is the instrument in each country, the gains from cooperation are 36.78 times the gains from cooperation under the baseline case. (Notice that $e_t = \frac{1}{e_t^*}$.)

Though we fall short of providing a sufficient statistic for optimality that is fully-analytical, our check can go a long way towards ensuring that indeed the solution identified by the analytical first-order conditions has key characteristics of the optimal solution.

3.3. Exploring the strategy space

Exploiting the flexibility of our toolbox, we can easily analyze how the choice of instruments impacts the outcomes of the open-loop Nash game. Suppose that at the first stage policymakers choose the policy instrument from a given set of instruments. At the second stage of the game, each policymaker chooses the optimal strategy given his choice of instrument, taking the strategy of the other policymaker as given. To determine the optimal choice of instruments, we need to recompute and solve the first-order conditions of the open-loop Nash game described in Eq. (11) for all possible combinations of the instruments included in the set of instruments. An exhaustive exploration of the strategy space for the open-loop Nash game has not been undertaken, thus far; the LQ approach followed in Benigno and Benigno (2006) is too complex for this pursuit.

In principle, any variable that enters the model can be taken as instrument in problem (11). For ease of presentation, we restrict attention to the following five instruments: producer price inflation (π_t), consumer price inflation ($\pi_{C,\,t}$), real output (Y_t), nominal output ($P_t Y_t$), or the change in the nominal exchange rate (e_t/e_{t-1}). In total, we allow twenty-five instrument combinations, a number that strikes a balance between comprehensiveness and ease of exposition. In this set of instruments we omitted the nominal interest rate since we found that any combination of instruments involving the nominal interest rate leads to equilibrium indeterminacy in the open-loop Nash game.

Table 1 reports the gains from cooperation for each of the twenty-five combinations of instruments relative to the gains from cooperation under the baseline specification of both countries choosing producer price inflation as the instrument — the specification in Benigno and Benigno (2006) and Corsetti et al. (2010). Notice that, since we translate the gains from cooperation in terms of a consumption subsidy levied in the home country, Table 1 is not symmetric across the diagonal entries.[10] Strikingly, the baseline specification does not imply comparatively large or small gains from cooperation; a finding that stresses how arbitrary this instrument choice is. If both countries adopt real output as the instrument, the outcome of the open-loop Nash game is much closer to the outcomes under cooperation as evidenced by the much reduced welfare gains from cooperation in this case. The largest gains from cooperation obtain if policymakers play the open-loop Nash game using the growth rate of the nominal exchange rate as instrument. In comparison to the best scenario of both countries formulating their strategies in terms of real output, the welfare losses are about 150 times bigger!

Table 1 focuses on overall welfare implications, but the first stage game in which each country chooses its instrument may not result in the combination of instruments associated with the most desirable outcomes. For the instruments considered here, we confirmed that the home country maximizes its own expected utility by opting for real output as the instrument, irrespective of the foreign country's choice. Likewise, the foreign country maximizes its expected utility by choosing real output as its policy instrument. Thus, real output in both countries is a Nash equilibrium choice at the first stage and leads to outcomes that are closest to those under cooperation.

To shed additional light on the role of the policy instruments, Fig. 4 plots the impulse responses after a markup shock in the home country when both countries adopt 1) producer price inflation, 2) real output, and 3) the change in the nominal exchange rate as the instrument. The response of the real exchange rate can be viewed as a gauge of the international spillover effects — not internalized by each policymaker. The smallest spillover effects, and consequently the smallest gains from cooperation occur when real output is used as the instrument in both countries. Remarkably, when real output is the policy instrument, the foreign country is almost insulated from the shock similar to the case of full cooperation in

[10] Notice also that if we were to assign all the gains to the foreign country, the resulting table would be the mirror image of Table 1.

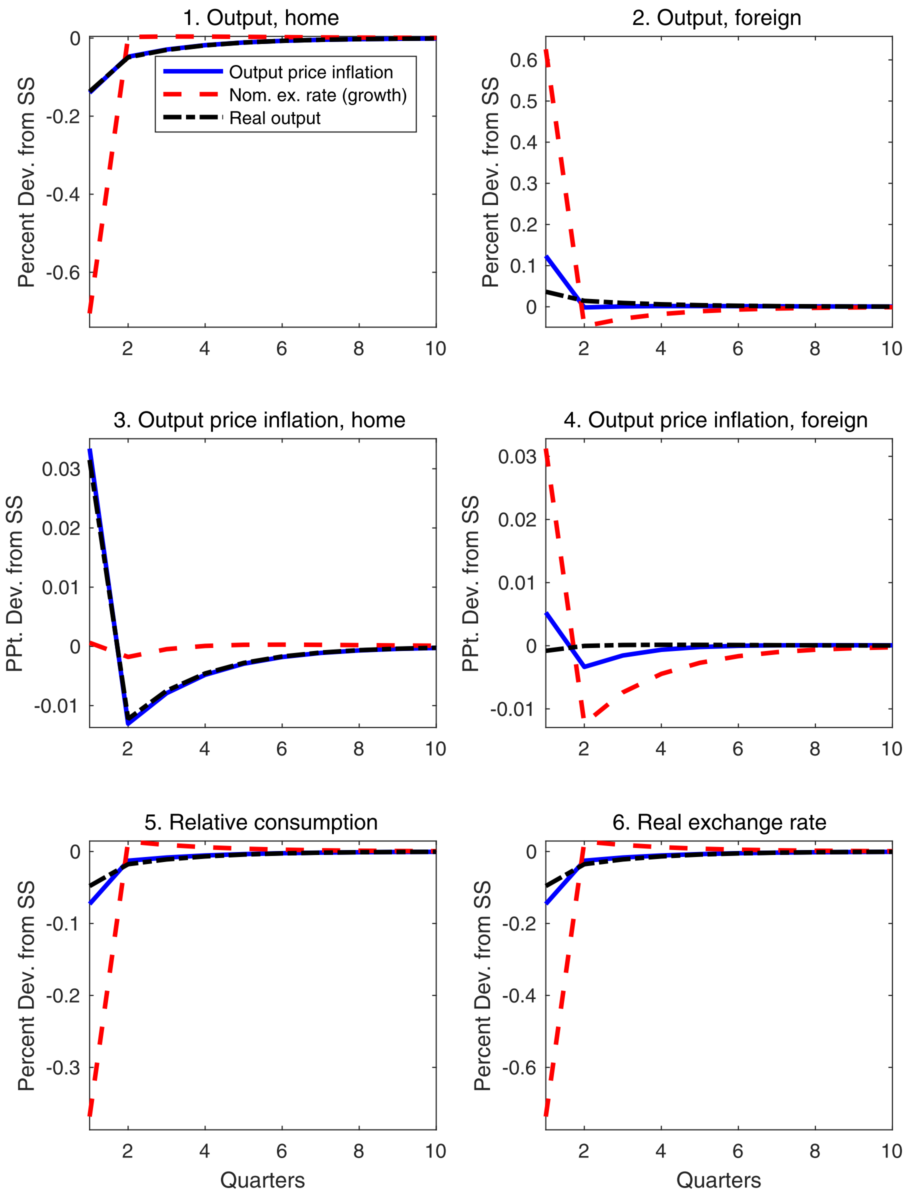

Fig. 4. Strategy-space under open-loop Nash policies in the open economy model: Responses to a markup shock.
Notes: The figure plots the transition dynamics of the three economies after a one-standard deviation increase in markups in the home country. The three lines show the responses for the open-loop Nash game when policymakers use output price inflation, changes in the nominal exchange rate, and real output as instrument, respectively.

Fig. 2. When policymakers use the change in the nominal exchange rate as instrument, they cannot stabilize the economy as effectively as under the other two instruments as exemplified by the larger response of output.

4. Macroprudential regulation model

Our toolbox can also be applied to policy games in a closed economy. As an example, we lay out a policy game between a central bank and a financial policymaker in the model of Gertler and Karadi (2011). That model features two types of rigidities. Allocations are skewed by nominal rigidities as well as by financial frictions. Non-financial firms are prevented from issuing equity to households directly and have to rely on financial intermediaries, referred to as "banks," in order to raise funds. Due to an agency problem, however, banks are limited in their ability to attract deposits and issue credit to non-financial firms. Accordingly, credit is under-supplied, and the reactions to shocks are amplified by a familiar financial-accelerator mechanism.

The only, but crucial, modification that we introduce to the setup of Gertler and Karadi (2011) is a lump-sum tax charged on banks and rebated to households. This is the powerful instrument used by the financial policymaker in our policy game, while inflation is the instrument used by the central bank.[11] Online Appendix D covers the problems faced by the various agents in the model and reports the conditions (Eqs. (D.1)–(D.23)) that characterize the private-sector equilibrium. The appendix also reviews the model calibration. In brief, we stay close the calibration choices in Gertler and Karadi (2011) with two exceptions: 1) for ease of exposition, we simplify the stochastic structure to include technology shocks only; and 2) we impose that the interest rates on deposits and on loans to non-financial firms coincide in the steady state. This second exception implies that the steady-state allocations are efficient and that distortions only open up in response to shocks.

4.1. Analyzing the gains from cooperation

Fig. 5 shows the responses to a contraction in technology under alternative policies. The shock considered brings down technology by 1 percent in the first quarter. Subsequently, technology follows its auto-regressive process.

We first consider the cooperative policy between the two policymakers that maximize the utility of the representative household.[12] The solid lines in Fig. 5 denote the responses for this case. The instruments are so powerful that, for a technology shock, the policymakers replicate the allocations that obtain in the analogous frictionless model. Due to the financial friction, absent intervention from the financial policymaker, banks are undercapitalized after the contractionary technology shock. An infusion of cash into the banks (i.e., a negative bank tax) can prop up their equity position and expand lending next period. At the same time, nominal rigidities call for a slight increase in the policy interest rate to prevent inflation from rising inefficiently. Notice that the welfare-maximizing cooperative policy completely stabilizes the expected spread between the bank return on investment and its cost of funding (the loan rate $E_t R^s_{t+1}$ minus the deposit R_t) in all periods. The same policy also achieves full inflation stabilization.

With identical objectives for the two policymakers, the open-loop Nash and cooperative policies coincide. However, in practice, different policymakers are assigned or pursue different objectives. We assume objectives for the two policymakers that are biased versions of the preferences of the representative agent. Apart from incorporating terms that reflect utility from consumption, C_t, and leisure, L_t, the objective of the central bank also incorporates a term that reflects an inflation stabilization bias (where π_t is inflation and $\bar{\pi}$ is its steady state value):

$$Obj_{cb} = E_0 \sum_{t=0}^{\infty} \beta^t \left[\log(C_t - \gamma C_{t-1}) - \chi_0 \frac{L_t^{1+\chi}}{1+\chi} - \mu_{cb} (\pi_t - \bar{\pi})^2 \right], \tag{16}$$

and where the parameter $\mu_{cb} = 5$ in our benchmark calibration governs the extent of the inflation bias. Analogously, the objective of the macroprudential policymaker is given by

$$Obj_{mpr} = E_0 \sum_{t=0}^{\infty} \beta^t \left[\log(C_t - \gamma C_{t-1}) - \chi_0 \frac{L_t^{1+\chi}}{1+\chi} - \mu_{mpr} \left((R^s_t - \bar{R}^s) - (R_{t-1} - \bar{R}) \right)^2 \right], \tag{17}$$

where the parameter $\mu_{mpr} = 4$ in our benchmark calibration governs the extent of the bias towards stabilizing the interest rate spread for banks, the term $\left((R^s_t - \bar{R}^s) - (R_{t-1} - \bar{R}) \right)^2$. For the baseline calibration, this particular formulation of biased objectives yields minor differences relative to the welfare-maximizing cooperative policies (as quantified below).[13]

As can be seen from Fig. 5, the differences between the cooperative policies with biased and unbiased objectives are relatively minor. The bias implies that the macroprudential policymaker is overzealous in stabilizing the interest rate spread for banks when the shock occurs. Conversely, the central bank accepts small deviations from full stabilization of inflation. Similarly, all other allocations remain close to their counterparts under the welfare-maximizing cooperative policies with biased objectives.

By contrast, an open-loop Nash game with the same biased objectives yields outcomes that are drastically different. To understand the extent of these differences, consider the side effects of a policy that, in reaction to a decline in technology, pushes up the equity positions of banks. Higher equity positions allow banks to expand credit, push up investment, and boost aggregate demand. In the presence of nominal rigidities, this expansion in demand leads to higher resource utilization and higher marginal costs of production, which cause inflation to rise. In reaction to the same decline in technology, the central bank wants to curb the inflationary effects of the shock and increase policy rates. However, higher policy rates bring up the cost of funding for banks, and by reducing profitability ultimately reduce the amount of funds available to support lending.

Accordingly, as the macroprudential policymaker recognizes that the central bank intends to move rates up, he counteracts that action by recapitalizing banks even more (shown as a negative movement of the tax in Fig. 5). In turn, the central

[11] Similar to the case of the two-country model, the open-loop Nash equilibrium is indeterminate when the nominal interest rate is used as policy instrument.

[12] See Eq. (D.1) in the Appendix.

[13] In analyzing the strategic interaction between fiscal and monetary policy Dixit and Lambertini (2003) assume the central bank to be more aggressive about inflation stabilization than the representative agent (and the fiscal authority) in order to obtain different objective functions for the fiscal and monetary authorities. Our formulation simplifies to the idea captured in Dixit and Lambertini (2003) for $\mu_{mpr} = 0$.

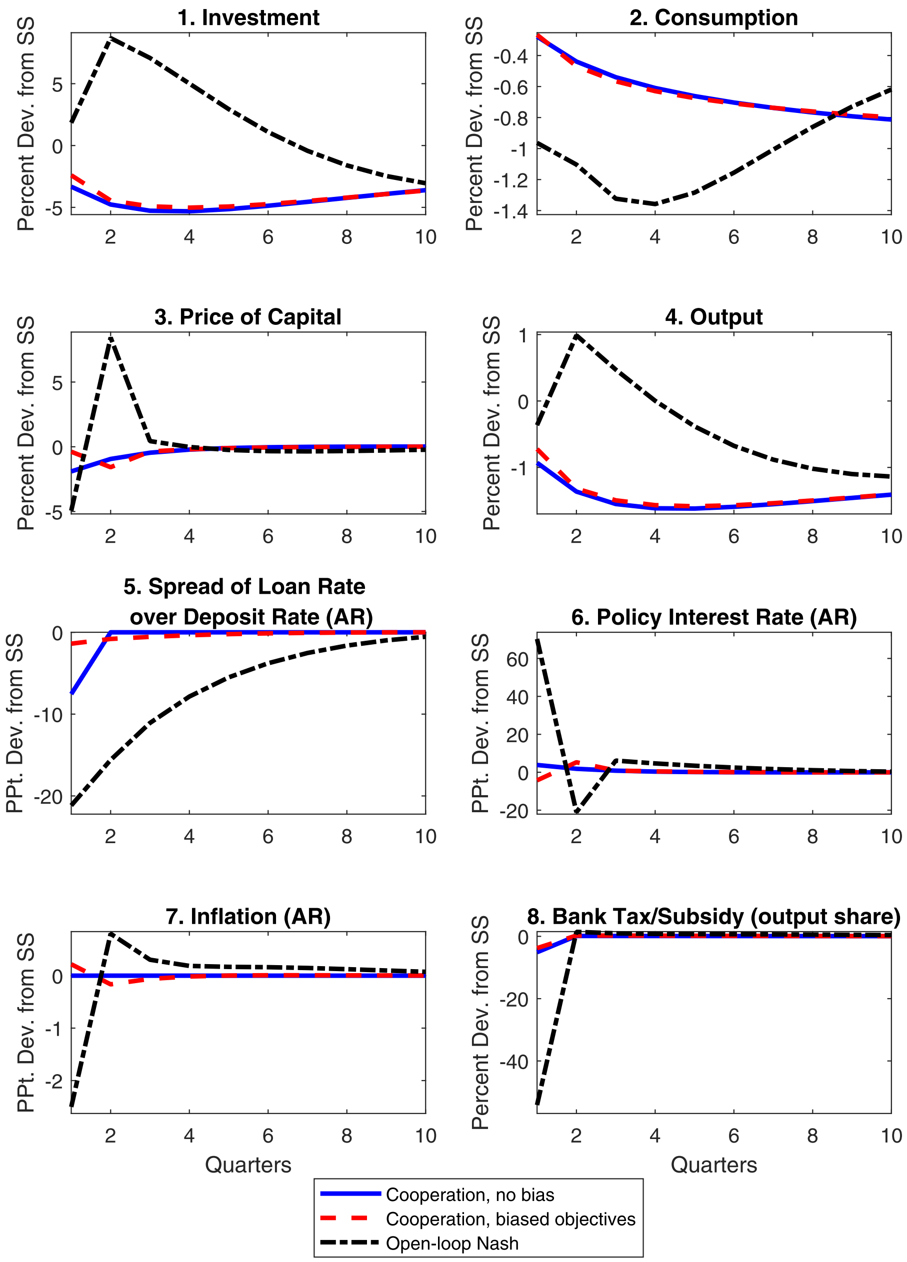

Fig. 5. Cooperative and open-loop Nash policies in the macroprudential regulation model: responses to a technology shock.
Notes: The figure plots the transition dynamics of the economy after a one-standard deviation decline in technology. The central bank uses inflation as its instrument and the macroprudential policymaker uses the tax (if positive, subsidy, if negative) on bank capital as instrument. The three lines show the responses for the cases of cooperation with unbiased policy preferences, cooperation with biased policy preferences, and without cooperation for biased policy preferences, respectively.

bank will have an incentive to increase policy interest rates by more, realizing that the macroprudential policymaker will step up the recapitalization of banks. Effectively, the different biases in the objectives push each policymaker to discount the reverberations of his own actions onto the objectives of the other policymaker. Ultimately, as shown in Fig. 5, the strategic interactions lead to an excessive recapitalization of banks, unnecessarily aggressive tightening in monetary policy, and stark deviations from the allocations under the welfare-maximizing cooperative policies, which imply substantial welfare losses.

The top panel of Fig. 6 confirms that the welfare losses from adopting biased objectives are small for cooperative policies for a broad range of the parameters that govern the biases. The panel's abscissae measure the parameter governing the bias of the macroprudential policymaker towards stabilizing credit spreads. The panel's ordinates measure the welfare loss relative to allocations obtained from cooperative policies with unbiased preferences (expressed in terms of a proportional

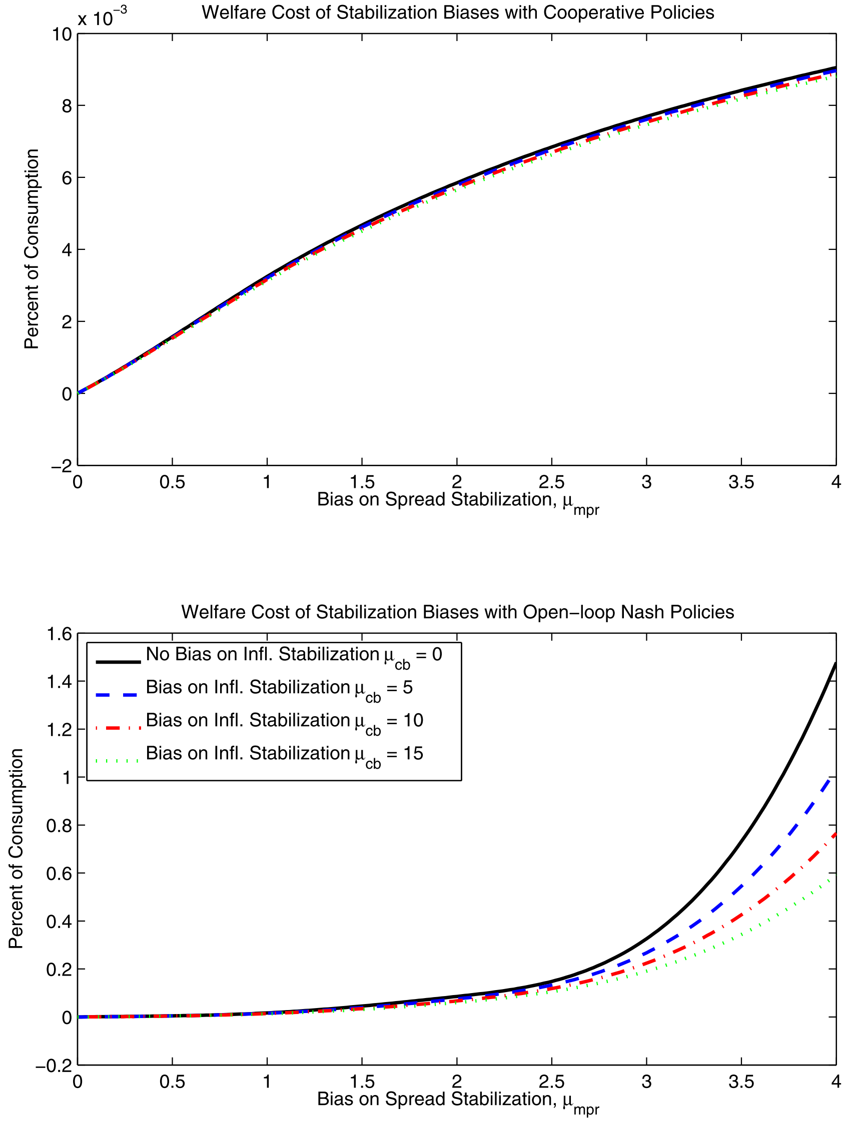

Fig. 6. Welfare implications of biased objectives.
Notes: The figure plots the welfare costs as a function of the stabilization bias of the macroprudential policymaker, μ_{mpr}. The welfare gains of going from a given model to the model without stabilization bias and cooperation is expressed as a consumption equivalent variation. The top panel shows the welfare costs of having biased objectives for the policymakers (using the unbiased objectives as welfare metric). The bottom panel plots the welfare costs of open-loop Nash policies if policymakers have biased objectives, relative to cooperative polices for the same biased objectives.

consumption tax that would leave the households indifferent between cooperative policies with and without biased objectives). The chart shows multiple contours of the tax schedule for different values of the parameter governing the bias of the central bank towards stabilizing inflation.

By contrast, the bottom panel of Fig. 6 shows that the welfare gains from cooperative policies increase substantially with the bias towards spread stabilization. With biased objectives, the welfare cost of open-loop Nash policies relative to the welfare maximizing policies can be orders of magnitude higher than the losses from allowing for biased objectives under cooperative policies (relative to the case of unbiased objectives). Notice also that these welfare costs are orders of magnitudes larger than the welfare costs of business cycles reported in Lucas (2003). Notably, the cost of open-loop Nash policies decreases in the bias of the central bank. This feature is easy to understand. The optimal cooperative policy entails complete inflation stabilization in response to technology shocks. Consequently, a more pronounced bias towards inflation stabilization fosters allocations more closely aligned with those of the cooperative policy.

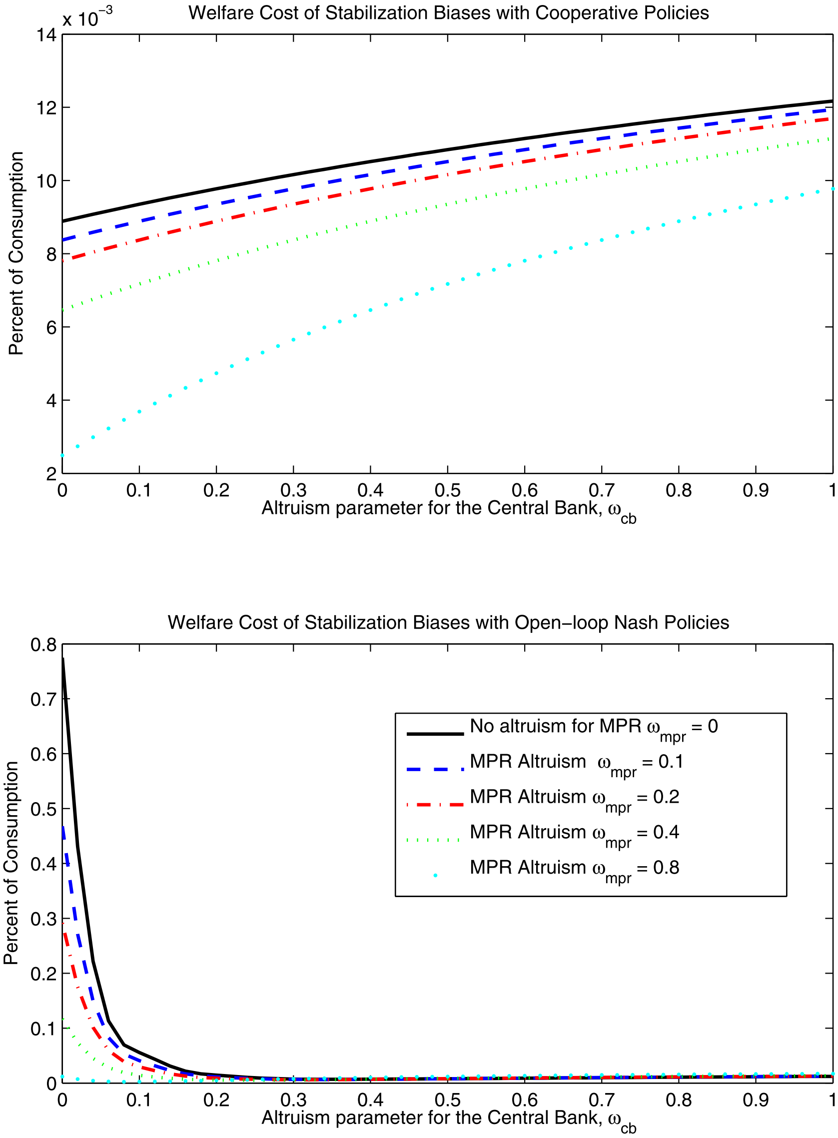

Fig. 7. Welfare implications of biased but altruistic objectives.
Notes: The figure plots the welfare costs as a function of the altruism parameter for the macroprudential policymaker, ω_{mpr}. The welfare gains of going from a given model to the model without stabilization bias and cooperation is expressed as a consumption equivalent variation. The top panel shows the welfare costs of having biased objectives for the policymakers (using the unbiased objectives as welfare metric). The bottom panel plots the welfare costs of open-loop Nash policies if policymakers have biased objectives relative to optimal cooperative policies from unbiased objectives.

4.2. Altruistic objectives

To showcase the flexibility of our toolbox, we also consider how the introduction of altruistic objectives that (at least partially) internalize the bias of the other policymaker affect the open-loop Nash equilibrium. For this exercise, we modify the objective functions of the two policymakers as follows:

$$Obj_{cb} = E_0 \sum_{t=0}^{\infty} \beta^t \Bigg[\log(C_t - \gamma C_{t-1}) - \chi_0 \frac{L_t^{1+\chi}}{1+\chi} - (1-\omega_{cb})\mu_{cb}(\pi_t - \bar{\pi})^2 - \omega_{cb}\mu_{mpr}\Big((R_t^s - \bar{R}^s) - (R_{t-1} - \bar{R})\Big)^2 \Bigg], \tag{18}$$

$$Obj_{mpr} = E_0 \sum_{t=0}^{\infty} \beta^t \left[\log(C_t - \gamma C_{t-1}) - \chi_0 \frac{L_t^{1+\chi}}{1+\chi} \right.$$
$$\left. -(1-\omega_{mpr})\mu_{mpr}\Big((R_t^s - \bar{R}^s) - (R_{t-1} - \bar{R})\Big)^2 - \omega_{mpr}\mu_{cb}(\pi_t - \bar{\pi})^2 \right], \tag{19}$$

where the parameters ω_{cb} and ω_{mpr} govern the extent of the altruism of each policymaker towards the bias of the other policymaker. In Fig. 7, we explore how variation in ω_{cb} and ω_{mpr} affect the welfare costs of open-loop Nash games for an intermediate calibration of the bias parameters that sets $\mu_{cb} = 10$ and $\mu_{mpr} = 4$. The top panel shows that the costs of biased objectives remain small for all the alternative levels of the altruism parameters considered. The bottom panel shows the welfare costs of the open-loop Nash game relative to allocations from an unbiased cooperative policy. Intuitively, we confirm that higher values of the altruism parameters move the Nash allocations closer to the cooperative allocations.

Our results point to two implications for the design of institutional arrangements. Bringing different regulatory functions under the same institution fosters the recognition of alternative objectives and avoids potentially large welfare losses from strategic interactions. When this solution is politically infeasible, our results argue for devising altruistic objectives for each policymaker as a way to minimize the welfare-reducing impact of strategic behavior.

5. Conclusions

A popular approach to study the strategic interactions between policymakers involves the use of linear-quadratic techniques. Purely quadratic objective functions are derived for each policymaker; the first-order conditions of the problem are then obtained by optimizing the quadratic objectives subject to linear approximations of the structural economic relationships. Unfortunately, this approach becomes laborious and potentially error-prone for larger models, limiting the range of analysis.

A more direct approach is to obtain the first-order conditions of the problem by using the nonlinear structural equations of the model and the nonlinear objective functions assigned to the policymakers. Our toolbox fully automates this procedure using symbolic differentiation. The quadratic approximations to the policymakers' objective functions can in principle be retrieved from the output of our toolbox. Changes to an existing model such as allowing for cooperation between policymakers instead of playing out an open-loop Nash game or changing the policy instruments assigned to the policymakers imply a new set of first-order conditions that is easily generated by our toolbox.

We apply the toolbox introduced in this paper to the well-known case of monetary policy coordination in a two-country model. The flexibility of our toolbox allows us to easily replicate the results in the literature and move beyond them. We show that alternative instruments change the strategy space. In particular, if players were allowed a choice of instruments before a choice of strategies, they would favor real output over producer price inflation, the instrument typically considered by studies of monetary policy coordination.

We also apply the toolbox to address strategic interactions between a macroprudential policymaker and a central bank in a model with financial frictions. The analysis points to potentially large welfare losses stemming from the lack of cooperation between policymakers, even if technology shocks are the only source of fluctuations.

Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.jmoneco.2018.07.015.

References

Adjemian, S., Bastani, H., Karam, F., Juillard, M., Maih, J., Mihoubi, F., Perendia, G., Pfeifer, J., Ratto, M., Villemot, S., 2011. Dynare: Reference Manual Version 4. Dynare Working Papers. CEPREMAP.
Benigno, G., Benigno, P., 2006. Designing targeting rules for international monetary policy cooperation. J. Monet. Econ. 53 (3), 473–506.
Benigno, P., Woodford, M., 2012. Linear-quadratic approximation of optimal policy problems. J. Econ. Theory 147 (1), 1–42.
Bergin, P.R., Corsetti, G., 2013. International Competitiveness and Monetary Policy: Strategic Policy and Coordination with a Production Relocation Externality. NBER Working Papers, 19356. National Bureau of Economic Research, Inc.
Blanchard, O., Galí, J., 2007. Real wage rigidities and the new keynesian model. J. Money Credit Banking 39 (s1), 35–65.
Brunnermeier, M.K., Sannikov, Y., 2014. International Credit Flows, Pecuniary Externalities, and Capital Controls. Technical Report.
Calvo, G.A., 1983. Staggered prices in a utility-maximizing framework. J. Monet. Econ. 12 (3), 383–398.
Canzoneri, M.B., Henderson, D.W., 1991. Monetary Policy in Interdependent Economies: A Game-theoretic Approach. MIT Press Books. The MIT Press.
Corsetti, G., Dedola, L., Leduc, S., 2010. Optimal monetary policy in open economies. In: Friedman, B.M., Woodford, M. (Eds.), Handbook of Monetary Economics. In: Handbook of Monetary Economics, 3. Elsevier, pp. 861–933. Chapter 16.
Costinot, A., Lorenzoni, G., Werning, I., 2014. A theory of capital controls as dynamic terms-of-trade manipulation. J. Political Econ. 122 (1), 77–128.
Debortoli, D., Nunes, R., 2006. On Linear Quadratic Approximations. MPRA Paper, 544. University Library of Munich, Germany.
Dixit, A., Lambertini, L., 2003. Interactions of commitment and discretion in monetary and fiscal policies. Am. Econ. Rev. 93 (5), 1522–1542.
Fujiwara, I., Teranishi, Y., 2013. Financial Stability in Open Economies. CAMA Working Papers, 2013-71. Centre for Applied Macroeconomic Analysis, Crawford School of Public Policy, The Australian National University.
Gertler, M., Karadi, P., 2011. A model of unconventional monetary policy. J. Monet. Econ. 58 (1), 17–34.
Kim, J., Kim, S., 2015. Conditional Versus Unconditional Utility as Welfare Criterion: Two Examples. Mimeo. Korea University.
Kim, J., Kim, S., Schaumburg, E., Sims, C.A., 2008. Calculating and using second-order accurate solutions of discrete time dynamic equilibrium models. J. Econ. Dyn. Control 32, 3397–3414.

Levine, P., Pearlman, J., Pierse, R., 2008. Linear-quadratic approximation, external habit and targeting rules. J. Econ. Dyn. Control 32 (10), 3315–3349.
Lopez-Salido, D., Levin, A.T., 2004. Optimal Monetary Policy with Endogenous Capital Accumulation. 2004 Meeting Papers, 826. Society for Economic Dynamics.
Lucas, R.E., 2003. Macroeconomic priorities. Am. Econ. Rev. 93 (1), 1–14.
Obstfeld, M., Rogoff, K., 2002. Global implications of self-oriented national monetary rules. Q. J. Econ. 117 (2), 503–535.
Schmitt-Grohe, S., Uribe, M., 2004. Optimal fiscal and monetary policy under sticky prices. J. Econ. Theory 114 (2), 198–230.
Woodford, M., 2003. Interest and Prices: Foundations of a Theory of Monetary Policy. Princeton University Press.