# APPENDIX FOR ONLINE PUBLICATION

# OccBin: A Toolkit for Solving Dynamic Models With Occasionally Binding Constraints Easily

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# A. RBC model with a Constraint on Investment: Nonlinear Solution and Robustness Analysis

We use two alternative approaches to finding a full nonlinear solution, dynamic programming and projection methods. We verified that the differences in the two approaches are negligible relative to the differences highlighted with the piecewise linear solution.[1] We present details of both algorithms before covering robustness analysis relative to alternative parametric assumptions.

## A.1. Dynamic Programming Solution.

The capital stock $K_t$ is the only state variable in the model. We seek a rule that will map the current state variable $K_{t-1}$ and the realization of the stochastic process $A_t$ into a choice of $K_t$. We discretize and put boundaries on the support of the decision rule that we seek. We consider a uniformly spaced set of points for $K_{t-1}$ and $K_t$. We discretize the support of both $K_{t-1}$ and $A_t$. The lower boundary for $K_{t-1}$ is 5 percent below the non-stochastic steady state for capital. The upper boundary is 40 percent above the non-stochastic steady state for capital. We experimented with different grids with little change in the results. We constrain $A$ to lie within three standard deviations of its process, i.e. $|\ln A_t| \leq 3\sqrt{\frac{\sigma^2}{1-\rho^2}}$,. We follow Tauchen (1986) in computing a finite state Markov-chain approximation for $\ln A_t$. The finest discretization we considered involved 75,000 points for capital and 201 points for the stochastic process. Use of shape-preserving splines, allowed us to reduce the number of points in the grid for capital without compromising the quality of the solution.

The dynamic programming algorithm that we use follows closely Chapter 12 of Judd (1998) and Chapter 3 of Ljungqvist and Sargent (2004). The initial choice for the decision rule in the dynamic program is taken to be the linear approximation to the decision rule obtained by standard methods. To accelerate the convergence of the dynamic programming algorithm, we use the Howard improvement algorithm.

## A.2. Projection Solution.

We restate the optimization problem in the model in Lagrangian form as:

$$\begin{aligned} \max_{\{C_t, K_{t+1}, \mu_t, \lambda_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \quad & \beta^t \frac{C_t^{1-\gamma} - 1}{1-\gamma} \\ + \quad & \beta^t \mu_t \left(-C_t - K_t + (1-\delta) K_{t-1} + A_t K_{t-1}^{\alpha}\right) \\ + \quad & \beta^t \lambda_t \left(-K_t + (1-\delta) K_{t-1} - \phi I\right) \end{aligned}$$

[1] For instance, the compensating variation for the use of the dynamic programming solution relative to the projection solution is in the order of \$1 in \$100,000,000 for the baseline calibration for the coarsest grid used.

From standard manipulation of the first-order conditions for the Lagrangian problem, one obtains:

$$C_t^{-\gamma} - \lambda_t - E_t\left[\beta C_{t+1}^{-\gamma}\left((1-\delta) + \alpha A_{t+1} K_t^{\alpha-1}\right) - \beta\left(1-\delta\right)\lambda_{t+1}\right] = 0 \tag{A.1}$$

We seek a solution in the form of a function $g(K_{t-1}, A_t)$ that approximates $C_t^{-\gamma} - \lambda_t$ subject to the complementary slackness condition, $\lambda_t\left(-K_t + (1-\delta)K_{t-1} - \phi I\right) = 0$ following the method of parameterized expectations described by den Haan and Marcet (1990) and Christiano and Fisher (2000). We approximate $g(K_{t-1}, A_t)$ with a Chebyshev polynomial of order 6 and approximate the process for $\ln A$ with a Markov process, following Tauchen (1986). A Markov process with 10 states usually provides an adequate approximation to the underlying process. In an abundance of caution, we use 51 states. We constrain $\ln A$ to lie within three standard deviations of its process, i.e. $|\ln A_t| \leq 3\sqrt{\frac{\sigma^2}{1-\rho^2}}$.

We solve for the parameters of the Chebyshev polynomial function using orthogonal collocation. Given $K_t$ and $A_t$, we guess $\lambda_t = 0$. Consistent with that guess, $C_t = g(K_{t-1}, A_t)^{-\gamma}$ and we can back out $K_t$ from the resource constraint. We then check whether $K_t - (1-\delta)K_{t-1} \geq \phi I$. If so, the original guess for $\lambda_t$ was correct. If not, for the complementary slackness condition to hold, it must be that $K_t = (1-\delta)K_{t-1} + \phi I$. $C_t$ is then given by the resource constraint and $\lambda_t = g(K_{t-1}, A_t) - C_t^{-\gamma}$.

### A.3. Robustness Analysis.

Table A reports the subsidy (as a percent of steady-state consumption) that would compensate the agent for the use of the piecewise linear algorithm over a fully nonlinear method with initial conditions set at the non-stochastic steady state. The welfare cost of using our piecewise solution method is a non-monotonic function of risk aversion; it is "high" when risk aversion is around 1, "low" when risk aversion is at 2, and increasing with risk aversion for values of $\gamma$ around 3 or higher. This happens because – under the full nonlinear solution – the irreversibility constraint has two opposing effects on the equilibrium average level of capital. The first effect – an *illiquid capital effect* – works to reduce average capital: when capital is irreversible, it is less useful in smoothing consumption when technology is low. The second effect – a *precautionary capital effect* – works to increase average capital through a precautionary saving effect: as capital is irreversible, consumption is more volatile, and more capital is held even if it is less useful in bad states of the world. These two opposing effects – captured by the fully nonlinear solution but not by the piecewise linear method – explain the non-monotonicity. Under the baseline calibration ($\gamma = 2$), the illiquid capital and precautionary capital effect almost offset each other, and the stochastic fixed point for capital[2] happens to be close to its non-stochastic steady state, which does not

[2] The stochastic fixed point for capital is the level attained by capital with all innovations to technology set to 0,

depend on uncertainty and irreversibility. The precautionary effect dominates for higher values of $\gamma$, whereas the illiquidity effect dominates for low risk aversion.[3] These differences in the demand for capital under different parameterizations of the model influence the performance of the piecewise linear solution method. In particular, for high levels of risk aversion, the linear component of the solution cannot capture the increase in demand for capital stemming from precautionary motives, and the performance of the solution algorithm deteriorates, with a the welfare cost of \$1 in about \$100,000 when $\gamma = 5$. Similarly, the linear component of the piecewise linear solution is not able to capture the drop in capital demand when risk aversion is low, and the performance of the piecewise linear solution also deteriorates.

Table A also considers sensitivity with respect to the choice of the value for the discount factor $\beta$. The welfare cost of using the piecewise method increases as the discount factor rises. We conjecture that in the plain vanilla RBC model the nonlinearities become more pronounced as the risk free rate becomes lower, thus penalizing linearization in general over a fully nonlinear solution algorithm.

## B. New Keynesian Model Subject to the Zero Lower Bound: Nonlinear Solution and Robustness Analysis

The necessary conditions for an equilibrium are:

$$C_t = \frac{1}{\beta_t E_t \left( \frac{R_t}{C_{t+1}\Pi_{t+1}} \right)} \tag{A.2}$$

$$mc_t = w_t \tag{A.3}$$

$$w_t = \psi L_t^{\vartheta} C_t \tag{A.4}$$

$$\varepsilon x_{1t} = (\varepsilon - 1)\, x_{2t} \tag{A.5}$$

$$x_{1t} = \frac{1}{C_t} mc_t Y_t + \theta E_t \beta_t \Pi_{t+1}^{\epsilon} x_{1t+1} \tag{A.6}$$

$$x_{2t} = \Pi_t^* \left( \frac{Y_t}{C_t} + \theta \beta_t E_t \frac{\Pi_{t+1}^{\varepsilon-1}}{\Pi_{t+1}^*} x_{2t+1} \right) \tag{A.7}$$

$$Z_t = R \left[ \left( \frac{\Pi_t}{\Pi} \right)^{\phi_p} \left( \frac{Y_t}{Y} \right)^{\phi_y} \right] \tag{A.8}$$

$$R_t = \max\left(Z_t, 1\right) \tag{A.9}$$

$$G_t = s_g Y_t \tag{A.10}$$

$$1 = \theta \Pi_t^{\varepsilon-1} + (1-\theta)\left(\Pi_t^*\right)^{1-\varepsilon} \tag{A.11}$$

[3] In a partial equilibrium setting, Abel and Eberly (1999) also find that the introduction of capital irreversibility has an ambiguous effect on the long-run level of the capital stock.

$$v_t = \theta \Pi_t^{\varepsilon} v_{t-1} + (1-\theta)\left(\Pi_t^*\right)^{-\varepsilon} \quad (A.12)$$

$$Y_t = C_t + G_t \quad (A.13)$$

$$Y_t = \frac{L_t}{v_t} \quad (A.14)$$

$$\ln \beta_t = (1-\rho)\log\beta + \rho \log \beta_{t-1} + \sigma\epsilon_t. \quad (A.15)$$

Given the balanced budget, $B_t = 0$. The model variables in the system above are $C$, $R$, $\Pi$, $\Pi^*$, $mc$, $w$, $L$, $x_1$, $x_2$, $Z$, $G$, $Y$, $v$, and $\beta$. We seek a solution in the form of three functions $g_1(v_{t-1}, \beta_t)$, $g_2(v_{t-1}, \beta_t)$ and $g_3(v_{t-1}, \beta_t)$ that approximate, respectively $\frac{1}{C_t \beta_t R_t}$, $\frac{\left(x_{1t} - \frac{1}{C_t} mc_t Y_t\right)}{\theta \beta_t}$ and $\frac{\left(\frac{x_{2t}}{\Pi_t^*} - \frac{Y_t}{C_t}\right)}{\theta\beta_t}$ subject to $R_t = \max(Z_t, 1)$. We approximate $g_1$, $g_2$, and $g_3$ with Chebyshev polynomials of order 6 and approximate the process for $\beta_t$ with a Markov process, following Tauchen (1986) and using 51 states. We constrain $\ln\beta_t - \ln\beta$ to lie within 3.5 standard deviations of its process. We also constrain $v_t$ to lie in the interval bounded by 1 and 1.04. We solve for the parameters of the Chebyshev polynomial functions using orthogonal collocation. Given these choices, we follow the same guess-and-verify approach described in the appendix of Fernández-Villaverde et al. (2012).

Figure A shows the absolute values of the residuals for the three intertemporal equations, equations (A.2), (A.6), and (A.7). The residuals were normalized respectively by $C_t$, $x_{1t}$, and $x_{2t}$. The maximum residual is in the order of $10^{-5}$. Figure B shows the residuals for the intertemporal equations for the piecewise linear solution and for a linear solution that disregards the zero lower bound. The residuals for the piecewise linear solution for the model with the ZLB enforced stay close to the residuals for the linear solution for a model that disregards the ZLB.

Finally, Table B provides some robustness analysis relative to alternative parametric assumptions.

## C. A Further Example: A Model of Consumption Choice with a Borrowing Constraint

To showcase applicability of our toolbox to a wide array of problems, we provide one further example. Occasionally binding borrowing constraints arise in a wide variety of models where households can "self-insure" by holding and managing an asset, up to some borrowing limit, that can be used to buffer consumption against adverse shocks. In these models, one can distinguish situations when a household is not constrained in the current period, and the traditional Euler equation for consumption holds; and situations when the household is credit constrained, current consumption is too low relative to next period, and the Euler equation for consumption does not hold. This behavioral asymmetry introduced by borrowing constraints, made popular by Zeldes (1989) and

Deaton (1992), can be studied using our solution method.

### C.1. Model Overview.

A consumer maximizes

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma}-1}{1-\gamma}$$

subject to the budget constraint, and to an (occasionally binding) constraint stating that borrowing $B_t$ cannot exceed a fraction $m$ of income $Y_t$ :

$$C_t + RB_{t-1} = Y_t + B_t, \tag{A.16}$$

$$B_t \leq mY_t. \tag{A.17}$$

Above, $R$ denotes the gross interest rate. The discount factor $\beta$ is assumed to satisfy the restriction that $\beta R < 1$, so that in the deterministic steady state the borrowing constraint is binding. Given initial conditions, the impatient household prefers a consumption path that is falling over time, and attains this path by borrowing today. If income is constant, the household will eventually be borrowing constrained and will roll its debt over forever, and consumption will settle at a level given by income less the steady state debt service.

The log of income follows an AR(1) stochastic process of the form

$$\ln Y_t = \rho \ln Y_{t-1} + \sigma \epsilon_t \tag{A.18}$$

where $\epsilon_t$ is an exogenous innovation distributed as standard normal, and $\sigma$ its standard deviation.

Denoting with $\lambda_t$ the Lagrange multiplier on the borrowing constraint given by equation ($A.17$), the set of equations describing the system of necessary conditions for an equilibrium is given by a system of four equations in the four unknowns $\{C_t, B_t, \lambda_t, Y_t\}$ which includes equation ($A.16$), equation ($A.18$) , together with the consumption Euler equation and the Kuhn-Tucker conditions, given respectively by

$$C_t^{-\gamma} = \beta R E_t \left( C_{t+1}^{-\gamma} \right) + \lambda_t \tag{A.19}$$

$$\lambda_t \left( B_t - mY_t \right) = 0. \tag{A.20}$$

The transitional dynamics of this model will depend in an important way on the gap between the discount rate and the interest rate, which can be measured as $g = 1/\beta - R$. In our setup, when the gap is small, the economy can be characterized as switching between two regimes. In the first regime, more likely to apply when income and assets are relatively low, the borrowing constraint binds. Then, borrowing moves in lockstep with income, and consumption is more volatile than income. In the second regime, more likely to apply when income and assets are relatively high, the borrowing constraint is slack, and current consumption can be high relative to future consumption even if borrowing is below the maximum amount allowed. We focus our attention on this case, since it presents an asymmetry that can be studied using our solution method.[4]

In the reference regime, the borrowing constraint binds, and the multiplier is greater than zero. In the alternative regime, the borrowing constraint is slack, and the multiplier is zero. Mapping these conditions into the notation used in Section **??**, $(M1)$ and $(M2)$ differ because of one equation. The optimization problem implies that $B_t = mY_t$ when the borrowing constraint binds. Conversely, when the constraint is slack, the complementary slackness condition implies that $B_t \leq mY_t$ and $\lambda_t = 0$. The conditions in $(M1)$ encompass $B_t = mY_t$, and the function $g$ captures $\lambda_t > 0$. The conditions in $(M2)$ encompass $\lambda_t = 0$, and the function $h$ captures $B_t \leq mY_t$.

### C.2. Calibration and Policy Functions.

Table C summarizes the baseline calibration, which reflects a yearly frequency. We set $\gamma = 1$, so that utility is logarithmic in consumption. We set the maximum borrowing at one year of income, so that $m = 1$. For the income process, we set $\rho = 0.90$ and $\sigma = 0.0131$, so that the standard deviation of $\ln Y$ is 3 percent. Finally, we set $R = 1.05$ and $\beta = 0.945$. Under this calibration, the borrowing constraint, which binds in the reference regime, is slack about 30 percent of the time using the full nonlinear solution.

We use dynamic programming to characterize a high-quality fully-nonlinear solution. We display the policy functions in terms of the optimal consumption chosen by the agent as a function of income and debt, the two state variables of the problem. The top panel of Figure C shows that for lower-than-average realizations of income (and high initial debt) the agent hits the borrowing constraint, the consumption function is relatively steep, and consumption is very sensitive to changes in income. For higher-than-average income, consumption is sufficiently high today relative to the future that it pays off to save for the future: the borrowing constraint becomes temporarily slack, and consumption becomes less sensitive to changes in income. The bottom panel compares the policy function obtained via dynamic programming with that obtained using our piecewise approach. The two policy functions are very similar. The only slightly difference is that – for given level of initial debt – the

[4] Depending on the calibration, other types of solutions may arise. If the discount rate is high and the gap $g$ is large, the borrowing constraint may bind always. Moreover, if the variance of the income process is sufficiently high and the gap approaches zero, consumption may not converge to any finite limit. Even when consumption converges, the stochastic steady state may be drastically different from the deterministic one because the household can accumulate enough assets so that the borrowing constraint is never a concern. Our calibration rules these possibilities out.

anticipation of future shocks leads the agent to save more (consume less) at all income levels.

### C.3. Assessing Performance: Impulse Responses, Moments and Welfare.

Figure D shows the responses to two shocks, starting from a nonstochastic steady state where income is one and the ratio of debt to income is $m = 1$, the maximum limit. The first shock, in period 2, brings up income by 3 percent (a 2 standard deviations shock). The second shock, in period 21, pushes down income by 3 percent. The solid and dashed lines denote the piecewise linear solution and the dynamic programming solution respectively. The dash-dotted lines denote the first-order perturbation solution, which incorrectly assumes that the borrowing constraint always binds. As the figure shows, the piecewise linear algorithm well captures the asymmetric responses of consumption, debt, and debt-to-income ratio following income shocks. A positive income shock makes the borrowing constraint slack; borrowing rises less than income, and consumption rises less than it would were the constraint binding in all states of the world. Conversely, when income drops, the borrowing constraint binds, borrowing falls in proportion with income, and consumption reacts more than under a positive shock.

Table D shows that the moments computed from the piecewise linear and the nonlinear solution method are again strikingly close. The OccBin can capture first, second, and third moments of the distribution of consumption. In particular, it captures the skewness in consumption derived from the occasionally binding constraint, which is missed by the first-order perturbation method.[5] Furthermore, the piecewise linear method comes close to replicating the frequency with which the constraint binds. Under the piecewise linear solution, the borrowing constraint binds 84 percent of the time. Under the fully nonlinear solution, the constraint binds slightly less frequently, 73 times out of 100 periods. The difference reflects the precautionary behavior induced by the anticipation of future shocks which implies higher average saving under the full nonlinear solution.

The differences between the piecewise linear and the full nonlinear solution for this model again highlight aspects of the economic problem that the piecewise method cannot capture. For this particular model, higher income uncertainty, reduced attitudes toward borrowing (caused by higher discount factor or higher risk aversion), and a looser borrowing limit can magnify the differences between the piecewise solution and the global solution. However, in all these cases the piecewise linear solution still performs uniformly better than the solution where the borrowing constraint is assumed to be always binding.

Relative to the full nonlinear solution, the utility cost of using the piecewise linear method is positive, but small as shown in Table E. For the baseline calibration, the household suffers a utility cost of $1 every $150,000 of consumption. The cost would be five times larger using a policy function based on first–order perturbation,

[5] The linearized model exhibits some small amount of skewness in consumption simply because we write the model in linearized form, but the income shocks are log-linear.

assuming that the constraint is always binding. In other experiments reported, we find that the utility cost becomes slightly larger with a higher maximum debt-to-income ratio; with higher risk aversion; with higher uncertainty; and with lower impatience. In all these cases, precautionary considerations become somewhat more important, and ignoring them magnifies the differences between the piecewise method and global nonlinear solution. However, even in these cases the improvements afforded by the piecewise method are substantial: compared to the global solution, the welfare cost of using the piecewise method is between five and six times smaller than the cost of using the linearized solution.

Table A: Utility Cost of the Solution Method: RBC Model with Constraint on Investment.

| Model | Solution Method | | Solution Method | |
|---|---|---|---|---|
| | Piecewise linear | $ 1 every | Constant capital | $ 1 every |
| $\gamma = 2, \beta = 0.96$ (Baseline) | 0.000007% | $ 14,556,184 | 0.04% | $ 2,842 |
| $\gamma = 1$ | 0.000036% | $ 2,793,317 | 0.02% | $ 4,653 |
| $\gamma = 3$ | 0.000113% | $ 881,804 | 0.05% | $ 1,980 |
| $\gamma = 4$ | 0.000413% | $ 242,411 | 0.07% | $ 1,481 |
| $\gamma = 5$ | 0.000958% | $ 104,424 | 0.09% | $ 1,157 |
| $\beta = 0.98$ | 0.000028% | $ 3,557,192 | 0.05% | $ 1,938 |
| $\beta = 0.94$ | 0.000003% | $ 29,844,218 | 0.03% | $ 3,838 |
| $\gamma = 2$, $\phi = 0$ | 0.000014% | $7,194,352 | 0.05% | $ 2,041 |

Note: The "Piecewise Linear" column indicates the subsidy rate (as a percent of steady-state consumption) that would compensate an agent for the use of the piecewise linear algorithm over a fully nonlinear method with initial conditions set at the non-stochastic steady state. The "Constant Capital" column indicates the subsidy when the agent uses a suboptimal decision rule setting the capital stock to its previous value.

Table B: Robustness Analysis of Model with ZLB

| | Solution Method | | | | | |
|---|---|---|---|---|---|---|
| | Piecewise linear | | | Nonlinear | | |
| | % at ZLB | log output | | % at ZLB | log output | |
| Model | | St.dev. | Skewness | | St.dev. | Skewness |
| Baseline | 4.2 | 1.44% | -0.22 | 7.13 | 1.54% | -0.49 |
| $\overline{\pi} = 1, \ \beta = 0.9891$ | 6.7 | 1.35% | -0.38 | 9.35 | 1.51% | -0.76 |
| $\phi_\pi = 5$, $\phi_y = 0$ | 6.7 | 0.86% | -0.66 | 9.35 | 0.94% | -1.20 |
| $\phi_\pi = 10$ | 2.91 | 1.69% | -0.13 | 4.41 | 1.72% | -0.14 |

Table C: Baseline Calibration of Model with Borrowing Constraint

| Parameter | Value | Parameter | Value |
|---|---|---|---|
| $\beta$, Discount Factor | 0.945 | $\gamma$, Relative Risk Aversion | 1 |
| $R$, Interest Rate | 1.05 | $m$, Borrowing Limit | 1 |
| $\rho$, Persistence of Income Shock | 0.90 | $\sigma$, St. Dev. Income Shock | 0.0131 |

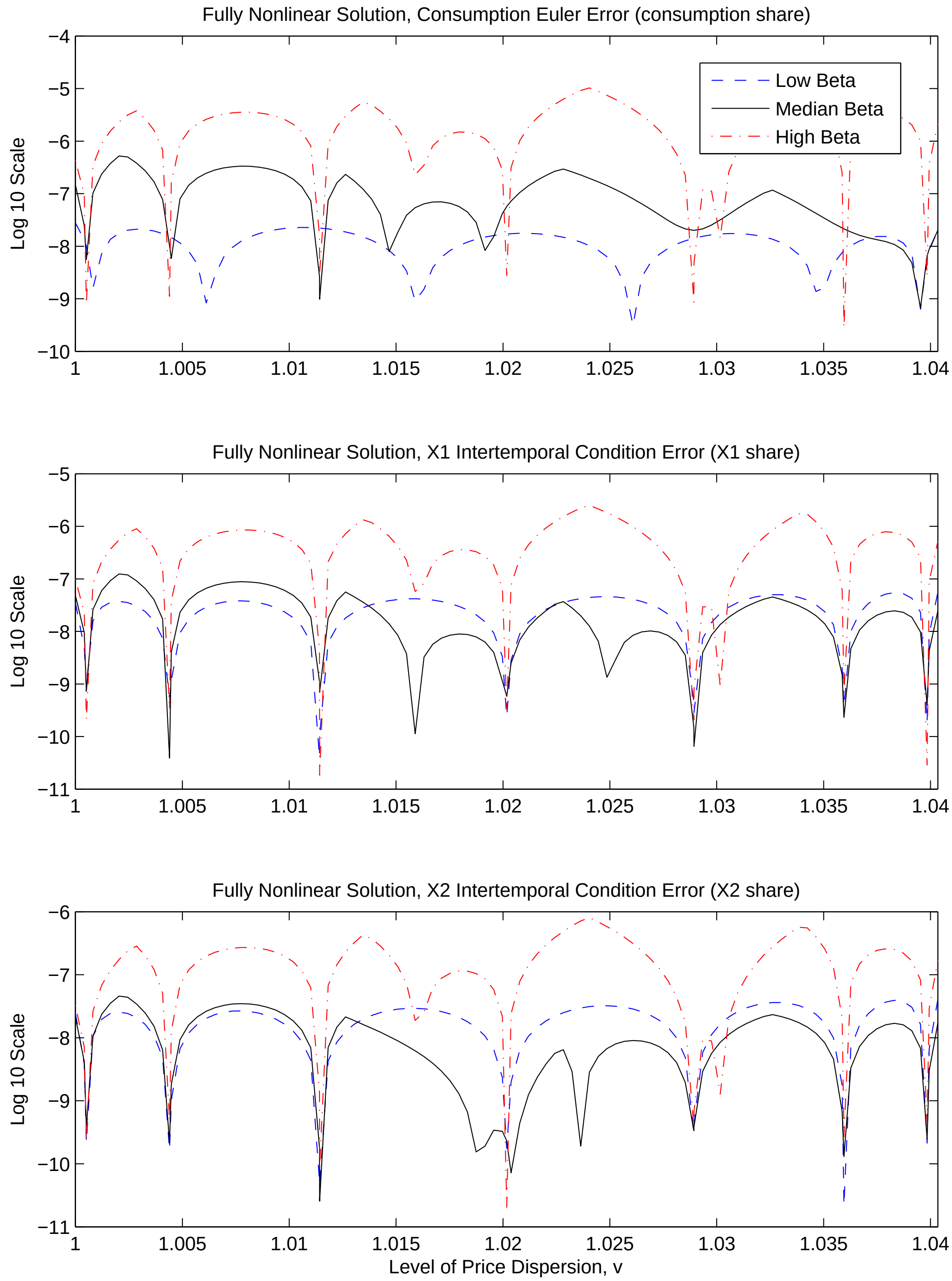
Table D: A Comparison of Key Moments: Model with Borrowing Constraint.

| | Log Consumption | | |
|---|---|---|---|
| Solution Method | Mean | St. dev. | Skewness |
| Nonlinear | -0.0512 | 3.40% | -0.24 |
| Piecewise Linear | -0.0513 | 3.46% | -0.22 |
| First-Order Perturbation | -0.0516 | 3.60% | -0.04 |
| | Log Income | | |
| | Mean | St. dev. | Skewness |
| Nonlinear | 0.0000 | 3% | 0.00 |
| Piecewise Linear | 0.0000 | 3% | 0.00 |
| First-Order Perturbation | 0.0000 | 3% | 0.00 |
| | Correlations | | |
| | $\ln Y, \ln C$ | $\ln Y, \ln B$ | |
| Nonlinear | 0.96 | 0.96 | |
| Piecewise Linear | 0.95 | 0.98 | |
| First-Order Perturbation | 0.92 | 1 | |
| | Frequency of Hitting the Borrowing Constraint (%) | | |
| Nonlinear | 73 | | |
| Piecewise Linear | 84 | | |
| First-order Perturbation | 100 | | |

Table E: Utility Cost of the Solution Method: Model with Borrowing Constraint.

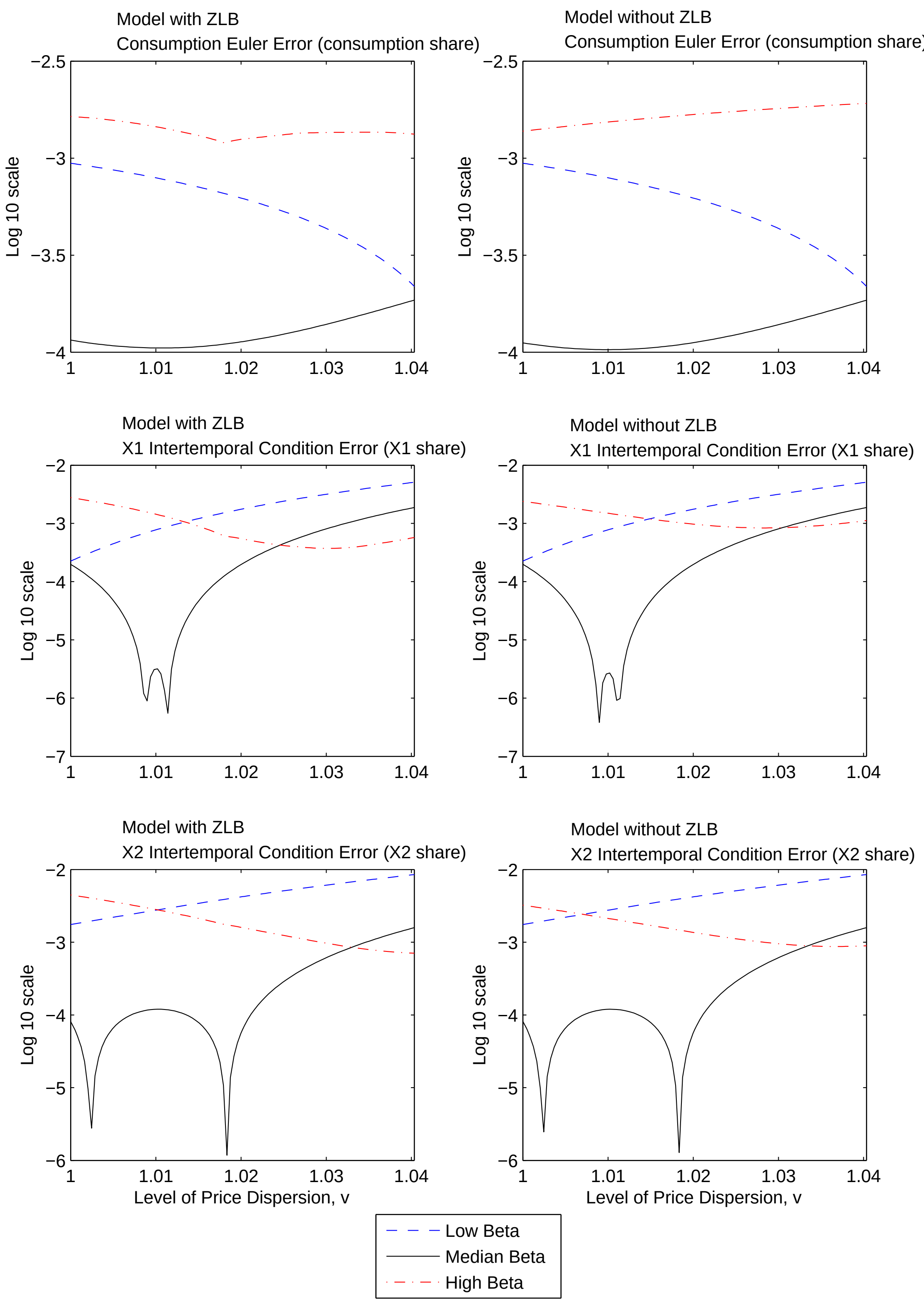
| Model | Solution Method | | Solution Method | |
|---|---|---|---|---|
| | Piecewise linear | $ 1 every | First-order Perturbation (Always Constrained) | $ 1 every |
| Baseline | 0.0007% | $ 149,280 | 0.0033% | $ 30,731 |
| High debt limit, $m = 2$ | 0.0013% | $ 77,444 | 0.0071% | $ 14,082 |
| High risk aversion, $\gamma = 2$ | 0.0023% | $ 44,140 | 0.0131% | $ 7,657 |
| High uncertainty, $\sigma = 0.0196$ | 0.0024% | $ 41,233 | 0.0116% | $ 8,650 |
| Low impatience, $\beta = 0.948$ | 0.001% | $ 102,989 | 0.0056% | $ 17,812 |

Figure A: New Keynesian Model Subject to the Zero Lower Bound: Intertemporal Errors of the Collocation Solution




Note: "Median Beta" corresponds to $\beta_t = 0.994$. "Low Beta" corresponds to $\beta_t = 0.965$. "High Beta" corresponds to $\beta_t = 1.023$. An open circle indicates that the zero lower bound on the nominal interest rate is binding.

Figure B: New Keynesian Model: Intertemporal Errors for the Piecewise Linear Solution (with ZLB enforced) and for the Linear Solution (with ZLB disregarded)




Note: "Median Beta" corresponds to $\beta_t = 0.994$. "Low Beta" corresponds to $\beta_t = 0.965$. "High Beta" corresponds to $\beta_t = 1.023$. An open circle indicates that the zero lower bound on the nominal interest rate is binding.

Figure C: Consumption Function, Model with Borrowing Constraint.

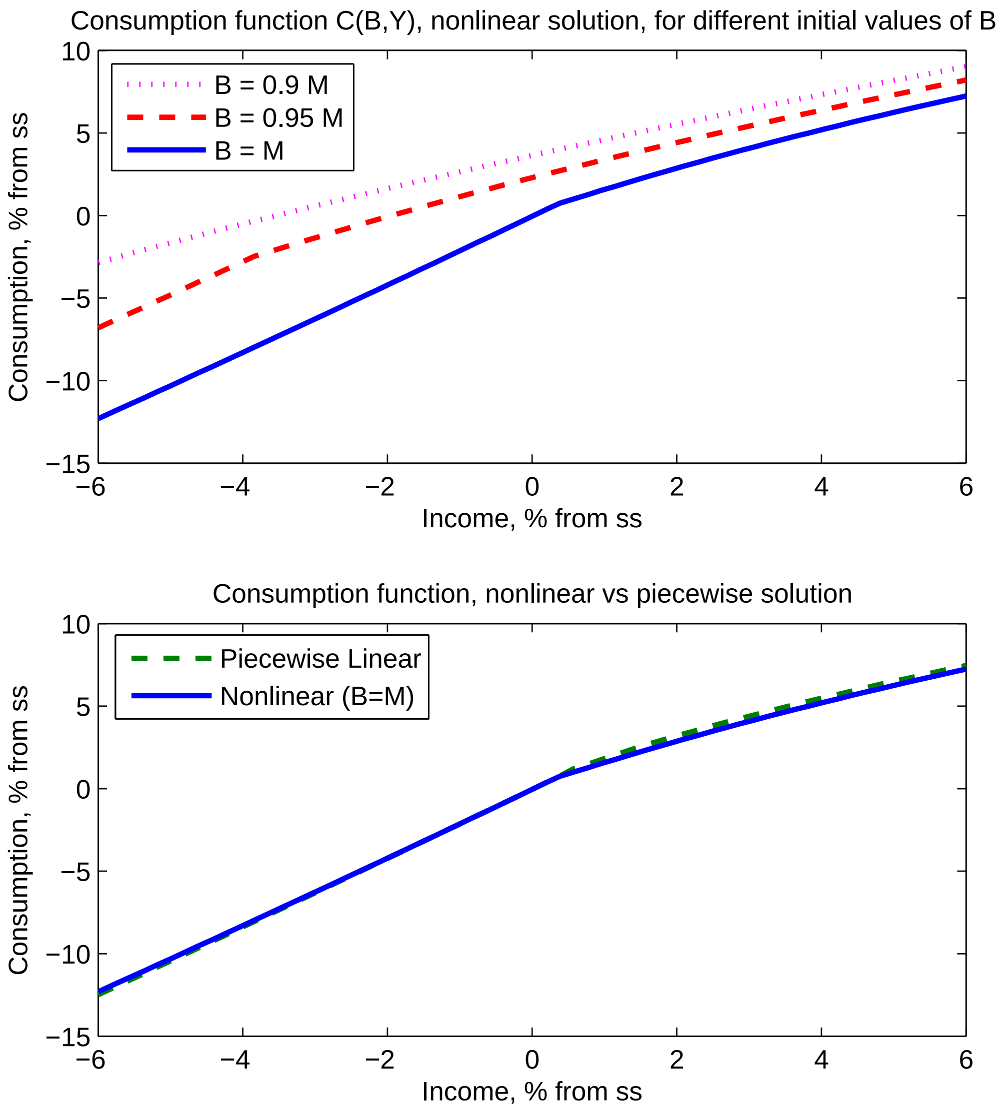
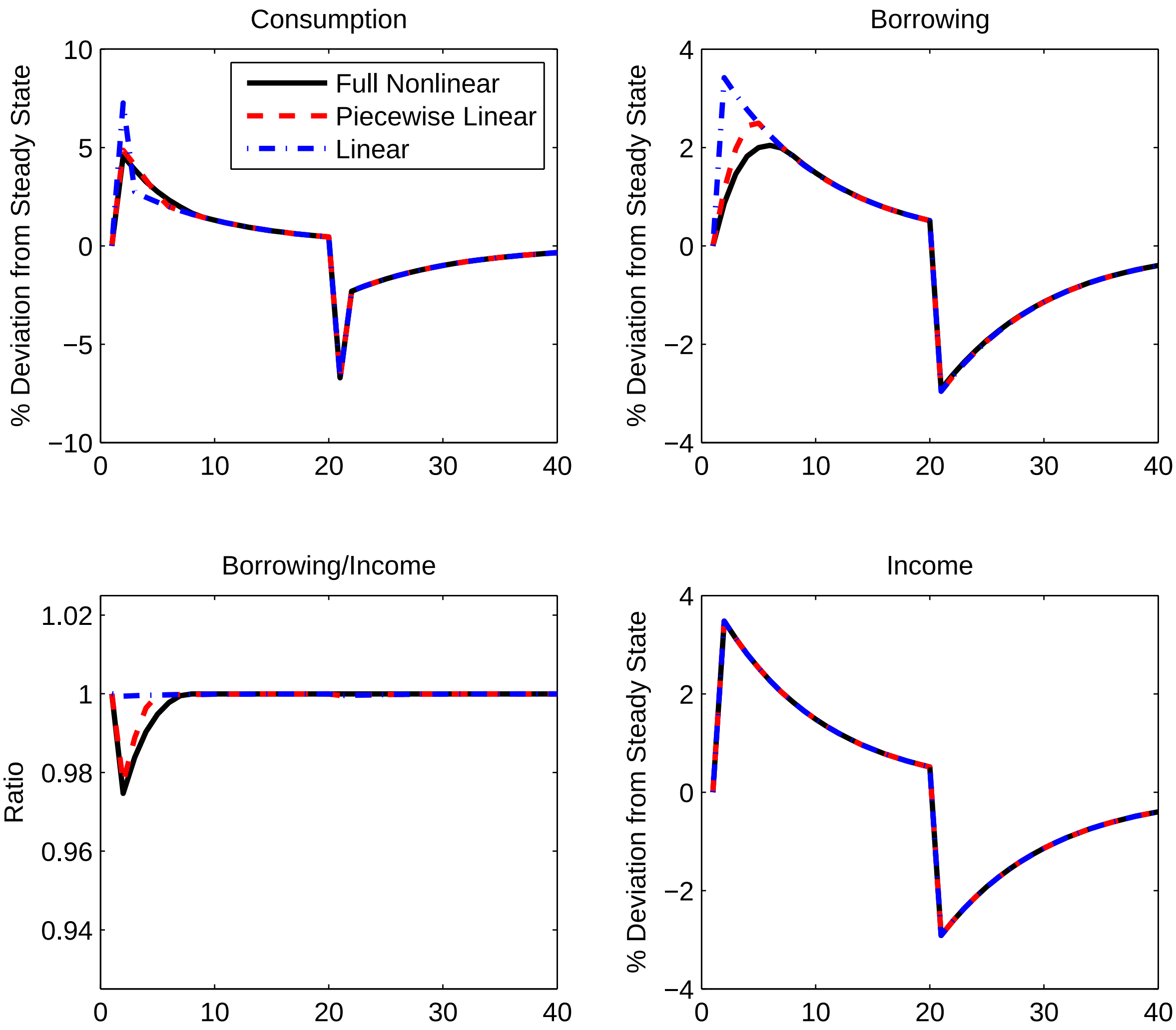


Figure D: Model with Borrowing Constraint: An Unexpected Increase in Income, Followed by a Decrease




Units on the abscissae denote years.

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