The Effects of Foreign Shocks When Interest Rates Are at Zero*

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Abstract

In a two-country DSGE model, the effects of foreign demand shocks on the home country are greatly amplified if the home economy is constrained by the zero lower bound for policy interest rates. This result applies even to countries that are relatively closed to trade such as the United States. Departing from many of the existing closed-economy models, the duration of the liquidity trap is determined endogenously. Adverse foreign shocks can extend the duration of the trap, implying more contractionary effects for the home country. The home economy is more vulnerable to adverse foreign shocks if the neutral rate is low – consistent with "secular stagnation" – and trade openness is high.

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1 Introduction

For large and relatively closed economies such as the United States, foreign shocks have typically been perceived as having small effects on domestic output. This view is in line with the Mundell-Fleming model, which indicates for an economy in which monetary policy can freely adjust, changes in policy rates should be able to offset the effects of foreign shocks. This view is also consistent with analysis based on open economy DSGE models.[1] Moreover, would seem to be confirmed by actual experience during the Great Moderation, a period in which the U.S. economy typically performed well despite several major recessions in key U.S. trading partners, including during the Mexican crisis in the mid-1990s, the Asian and Russian default crises in the late 1990s, and over long periods in which Japan's growth sputtered.

However, developments during the Global Financial Crisis (GFC) and its aftermath suggest that foreign shocks may have much larger effects on the domestic economy when monetary policy is constrained from adjusting interest rates due to the zero lower bound (ZLB). Indeed, Japan's experience during the GFC seems a telling illustration. As seen in Figure 1, financial developments in Japan during the GFC were much less adverse than in the United States or euro area – with corporate bond spreads rising much less (the top panel). However, the Bank of Japan had little scope to cut policy rates – which were around 50 basis points in mid-2008 (the middle panel) – in response to the massive collapse in global demand that occurred in 2008-2009 and Japan experienced a comparatively larger GDP contraction (the bottom panel). In a similar vein, U.S. policymakers during the past several years have often pointed to foreign developments as playing a key role in affecting the U.S. outlook, and have argued that potential spillovers from abroad are likely to be

[1] Drawing on the two country real business cycle model of Backus, Kehoe, and Kydland (1992), Baxter and Crucini (1995) show that a positive country-specific productivity shock in the foreign economy induces a small *contraction* in domestic output. Similarly, Lubik and Schorfheide (2006) and Adolfson, Laseen, Linde, and Villani (2007) show cross-border spillovers are very small even in models including nominal rigidities.

larger due to the ZLB.[2]

In this paper, we conduct a more formal analysis of how the ZLB constraint on policy rates affects the transmission of foreign shocks to the United States using an open economy DSGE model. Our model incorporates a wide array of empirically-relevant features that we calibrate to the U.S. economy, including sticky wages and prices, endogenous capital accumulation (with adjustment costs), and local currency pricing for traded goods.[3] We show that the effects of a given-sized foreign demand shock are much larger than in normal times if the foreign shock occurs against the backdrop of a severe domestic recession that pins domestic policy rates at zero. In particular, under our baseline calibration, a shock that reduces foreign GDP by 1 percent causes U.S. GDP to fall by roughly 0.6 percent in a ten quarter liquidity trap, a drop about twice as large as would occur if U.S. policy rates could be freely adjusted according to a Taylor rule. The larger effects reflect that in normal times policy rates can be cut to crowd in domestic demand, but that such crowding in is attenuated in a liquidity trap.

We next investigate how the effects of foreign shocks depend on the assumed long-run level of the neutral policy rate, i.e., the policy rate consistent with full-employment after economic shocks have worn off. Ball, DeLong, and Summers (2014) argued that the neutral interest rate may in fact be very low for many industrial economies compared with post-war norms, a phenomenon termed "secular stagnation." As suggested by Japan's experience, a drift toward secular stagnation could heighten the vulnerability of many of these economies to a downturn in foreign demand. In this vein, we show how a foreign demand shock could *of itself* push an economy with a low neutral nominal rate into recession i.e., even assuming that the domestic economy was near full employment prior to the shock. Moreover, we illustrate that the effects of the foreign downturn on the domestic economy can be sharply nonlinear if the shock is large enough to push the economy into a liquidity

[2]For a recent example, see Brainard (2015).

[3]See Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2003).

trap, and/or extend its duration. This is important, because it emphasizes how recessions can be caused purely by foreign shocks operating through trade channels, even in the absence of financial spillovers (from which our model abstracts).

Although the rise in global trade during the past few decades has likely had many beneficial effects – allowing, for instance, countries to specialize production based on their comparative advantage – our analysis shows how greater trade openness may heighten vulnerability to adverse foreign developments when the ZLB is binding. Intuitively, while foreign demand shocks should amplify the contribution of net exports to GDP as the trade share rises, monetary policy should be able to offset these larger trade effects on GDP through appropriate adjustment of policy rates if the ZLB doesnt bind; thus, foreign demand shocks should not be expected to cause substantially greater output volatility simply on account of a higher trade share.[4] However, our model simulations illustrate how a higher trade share can translate into much larger effects of foreign shocks on domestic output if the ZLB binds, and in particular, underscore the potential challenges posed by the interaction of greater trade openness and secular stagnation.

From a broader perspective, our analysis of the effects of foreign shocks has a close parallel with the (largely) closed economy literature emphasizing that domestic shocks have outsized effects in a liquidity trap, reflecting that adverse shocks cause the real interest rate to rise (while favorable shocks, including fiscal stimulus, cause the real rate to fall), e.g., Christiano, Eichenbaum, and Rebelo (2011a), Eggertsson (2011), and Woodford (2011). Even so, most of the literature has highlighted the role of either domestic factors or cross-border financial spillovers as the likely catalyst for pushing an economy to the ZLB. Accordingly, the literature has emphasized the desirability of developing a policy framework that minimizes the risks of a financial meltdown arising from either domestic or foreign sources. Our analysis indicates

[4]Consistent with this implication, Doyle and Faust (2005) found that the correlation between U.S. growth and that of its major trading partners showed little tendency to rise as trade ties deepened.

that the spillovers from foreign shocks – operating purely through trade channels, and absent any financial contagion – could be very large for economies that are highly open with a low nominal rate. Given these large trade spillovers, keeping "one's own house in order" may not be enough to minimize ZLB risks.

A methodological contribution of our paper is to adopt a modeling framework in which the duration of the ZLB depends endogenously on the foreign demand shock, which allows the effect on GDP to rise nonlinearly with the size of the foreign shock.[5] We also show that the amplified effects of the ZLB is quite particular to demand shocks, and reflects that an adverse foreign demand shock hurts home real net exports both through reducing foreign activity, and by appreciating the domestic currency.[6] By contrast, the ZLB has negligible consequences for the effects of foreign technology shocks, reflecting that adverse foreign supply shocks tend to depreciate the domestic currency, which mitigates the adverse effects on exports.

2 The Model

Apart from the explicit treatment of the zero-lower bound on policy rates, our two-country model is close to Erceg, Guerrieri, and Gust (2006) and Erceg, Guerrieri, and Gust (2008) who themselves build on Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2003). We focus on describing the home country as the setup for the foreign country is analogous. The calibration for the home country reflects key features of the United States.

[5] This approach contrasts with much of the literature, which typically assumes that the shocks considered are too small to affect the duration of the liquidity trap, including in the two-state Markov switching framework often used in heuristic models, e.g., Eggertsson (2011), and Woodford (2011).

[6] Stockman and Tesar (1995) extended the model of Backus, Kehoe, and Kydland (1992) to include consumption preference shocks, and highlighted how these shocks have different implications for cross-country co-movements than technology shocks even in their framework which abstracted from nominal frictions.

2.1 Firms and Price Setting

Production of Domestic Intermediate Goods. There is a continuum of differentiated intermediate goods (indexed by $i \in [0,1]$) in the home country, each of which is produced by a single monopolistically competitive firm. Firms charge different prices at home and abroad, i.e., they practice pricing to market. In the home market, firm i faces a demand function that varies inversely with its output price $P_{Dt}(i)$ and directly with aggregate demand at home Y_{Dt} :

$$Y_{Dt}(i) = \left[\frac{P_{Dt}(i)}{P_{Dt}}\right]^{\frac{-(1+\theta_p)}{\theta_p}} Y_{Dt}, \tag{1}$$

where $\theta_p > 0$, and P_{Dt} is an aggregate price index defined below. Similarly, in the foreign market, firm i faces the demand function:

$$X_t(i) = \left[\frac{P^*_{Mt}(i)}{P^*_{Mt}}\right]^{\frac{-(1+\theta_p)}{\theta_p}} M^*_t, \tag{2}$$

where $X_t(i)$ denotes the foreign quantity demanded of home good i, $P^*_{Mt}(i)$ denotes the price, denominated in foreign currency, that firm i sets in the foreign market, P^*_{Mt} is the foreign import price index, and M^*_t is aggregate foreign imports.

Each producer utilizes capital services $K_t(i)$ and a labor index $L_t(i)$ (defined below) to produce its respective output good. The production function has a constant-elasticity of substitution form:

$$Y_t(i) = \left(\omega_K^{\frac{\rho}{1+\rho}} K_t(i)^{\frac{1}{1+\rho}} + \omega_L^{\frac{\rho}{1+\rho}} (z_t L_t(i))^{\frac{1}{1+\rho}}\right)^{1+\rho}, \tag{3}$$

where z_t is a country-specific shock to the level of technology. Firms face perfectly competitive factor markets for hiring capital and labor.

The prices of intermediate goods are determined by Calvo-style staggered contracts, see Calvo (1983). Each period, a firm faces a constant probability, $1-\xi_p$, to reoptimize its price at home $P_{Dt}(i)$ and probability of $1-\xi_{px}$ to reoptimize the price that it sets in the foreign country of $P^*_{Mt}(i)$. These probabilities are independent across firms, time, and countries. Following Gali and Gertler (1999) we allow for a mass ι_p of backward-looking firms that sets its price based on a lagged information set. These firms set their price equal to the previous period Calvo reset price indexed by lagged inflation.

Production of the Domestic Output Index. A representative aggregator combines the differentiated intermediate products into a composite home-produced good Y_{Dt} according to

$$Y_{Dt} = \left[\int_0^1 Y_{Dt}\left(i\right)^{\frac{1}{1+\theta_p}} di \right]^{1+\theta_p}. \tag{4}$$

The optimal bundle of goods minimizes the cost of producing Y_{Dt} taking the price of each intermediate good as given. A unit of the sectoral output index sells at the price P_{Dt}:

$$P_{Dt} = \left[\int_0^1 P_{Dt}\left(i\right)^{\frac{-1}{\theta_p}} di \right]^{-\theta_p}. \tag{5}$$

Similarly, a representative aggregator in the foreign economy combines the differentiated home products $X_t(i)$ into a single index for foreign imports:

$$M_t^* = \left[\int_0^1 X_t\left(i\right)^{\frac{1}{1+\theta_p}} di \right]^{1+\theta_p}, \tag{6}$$

and sells M_t^* at price P_{Mt}^*:

$$P_{Mt}^* = \left[\int_0^1 P_{Mt}^*\left(i\right)^{\frac{-1}{\theta_p}} di \right]^{-\theta_p}. \tag{7}$$

Production of Consumption and Investment Goods. Assuming equal import content of consumption and investment, there is effectively one final good A_t that is used for consumption or investment, (i.e., $A_t \equiv C_t + I_t$, allowing us to interpret A_t as private absorption). Domestically-produced goods and imported goods are combined to produce final goods A_t according to

$$A_t = \left(\omega_A^{\frac{\rho_A}{1+\rho_A}} A_{Dt}^{\frac{1}{1+\rho_A}} + \left(1-\omega_A\right)^{\frac{\rho_A}{1+\rho_A}} M_t^{\frac{1}{1+\rho_A}} \right)^{1+\rho_A}, \tag{8}$$

where A_{Dt} denotes the distributor's demand for the domestically-produced good and M_t denotes the distributor's demand for imports. The quasi-share parameter ω_A determines the degree of home bias in private absorption, and ρ_A determines the elasticity of substitution between home and foreign goods. Each representative distributor chooses a plan for A_{Dt} and M_t to minimize its costs of producing the final good A_t and sells A_t to households at a price P_t. Accordingly, the prices of consumption and investment are equalized.

2.2 Households and Wage Setting

A continuum of monopolistically competitive households (indexed on the unit interval) supplies a differentiated labor service to the intermediate goods-producing sector. A representative labor aggregator combines the households' labor hours in the same proportions as firms would choose. This labor index L_t has the Dixit-Stiglitz form:

$$L_t = \left[\int_0^1 N_t\left(h\right)^{\frac{1}{1+\theta_w}} dh \right]^{1+\theta_w}, \tag{9}$$

where $\theta_w > 0$ and $N_t(h)$ is hours worked by a typical member of household h. The aggregator minimizes the cost of producing a given amount of the aggregate labor index, taking each household's wage rate $W_t\left(h\right)$ as given. One unit of the labor index sells at the unit cost W_t:

$$W_t = \left[\int_0^1 W_t\left(h\right)^{\frac{-1}{\theta_w}} dh \right]^{-\theta_w}. \tag{10}$$

W_t is referred to as the aggregate wage index. The aggregator's demand for the labor services of household h satisfies

$$N_t\left(h\right) = \left[\frac{W_t\left(h\right)}{W_t} \right]^{-\frac{1+\theta_w}{\theta_w}} L_t. \tag{11}$$

The utility functional of a representative household h is:

$$\widetilde{\mathbb{E}}_t \sum_{j=0}^{\infty} \beta^j \left\{ \frac{1}{1-\sigma} \left(C_{t+j}(h) - \varkappa \frac{C_{t+j-1}}{\zeta} - \nu_{ct} \right)^{1-\sigma} \right.$$
$$\left. + \frac{\chi_0}{1-\chi} (1 - N_{t+j}(h))^{1-\chi} + V\left(\frac{MB_{t+j+1}(h)}{P_{t+j}} \right) \right\}, \tag{12}$$

where the discount factor β satisfies $0 < \beta < 1$. As in Smets and Wouters (2003), we allow for the possibility of external habits. At date t household h cares about consumption relative to lagged per capita consumption, C_{t-1}. The preference shock ν_{ct} follows an exogenous first order process with a persistence parameter of ρ_ν. The parameter ζ controls for population size. The household's period utility function depends on current leisure $1 - N_t(h)$, the end-of-period real money balances, $\frac{MB_{t+1}(h)}{P_t}$. The liquidity-service function $V(\cdot)$ is increasing in real money balances at a decreasing rate up to a satiation level. Beyond the satiation level, utility from liquidity services is constant. With this specification of the utility function, the demand for real money balances is always positive regardless of the level of the nominal interest rate.[7]

The budget constraint of each household is given by:

$$P_t C_t(h) + P_t I_t(h) + MB_{t+1}(h) - MB_t(h) + \frac{e_t P^*_{Bt} B_{Ft+1}(h)}{\phi_{bt}} - e_t B_{Ft}(h)$$
$$= W_t(h) N_t(h) + \Gamma_t(h) - T_t(h) + R_{Kt} K_t(h) - P_{Dt} \phi_{It}(h). \tag{13}$$

Final consumption and investment goods are purchased at a price P_t. Investment in physical capital augments the per capita capital stock $K_{t+1}(h)$ according to a linear transition law of the form:

$$K_{t+1}(h) = (1-\delta) K_t(h) + I_t(h), \tag{14}$$

where δ is the depreciation rate of capital. The term $R_{Kt} K_t(h)$ in the budget constraint represents the proceeds to the household from renting capital to firms.

[7]More formally, we follow Jeanne and Svensson (2007) in assuming that $V(MB_{t+1}/P_t) < V_0$, $V'(MB_{t+1}/P_t) > 0$, $V''(MB_{t+1}/P_t) < 0$ for $MB_{t+1} < \bar{m}$, the satiation level of real money. And $V(MB_{t+1}/P_t) = V_0$ for $MB_{t+1} \geq \bar{m}$, and $V'(MB_{t+1}/P_t) \to \infty$ for $MB_{t+1}/P_t \to 0$.

Financial asset accumulation consists of increases in nominal money holdings $MB_{t+1}(h) - MB_t(h)$ and the net acquisition of international bonds. Trade in international assets is restricted to a non-state contingent nominal bond. $B_{Ft+1}(h)$ represents the quantity of the international bond purchased by household h at time t that pays one unit of foreign currency in the subsequent period. P^*_{Bt} is the foreign currency price of the bond, and e_t is the nominal exchange rate expressed in units of home currency per unit of foreign currency. Following Turnovsky (1985) households pay an intermediation fee ϕ_{bt}.[8] The intermediation fee depends on the ratio of economy-wide holdings of net foreign assets to nominal output according to:

$$\phi_{bt} = \exp\left(-\phi_b\left(\frac{e_t B_{Ft+1}}{P_{Dt}Y_t}\right)\right). \tag{15}$$

If the home economy has an overall net lender position, a household will earn a lower return on any holdings of foreign bonds. By contrast, if the economy has a net debtor position, a household will pay a higher return on any foreign debt.

Households earn labor income, $W_t(h)N_t(h)$, lease capital to firms at the rental rate R_{Kt}, and receive an aliquot share $\Gamma_t(h)$ of the profits of all firms. Furthermore, they pay a lump-sum tax $T_t(h)$. We follow Christiano, Eichenbaum, and Evans (2005) in assuming that households bear a cost of changing the level of gross investment from the previous period, so that the acceleration in the capital stock is penalized:

$$\phi_{It}(h) = \frac{1}{2}\phi_I\frac{(I_t(h) - I_{t-1}(h))^2}{I_{t-1}(h)}. \tag{16}$$

Households maximizes the utility functional (12) with respect to consumption, investment, (end-of-period) capital stock, money balances, and holdings of foreign bonds, subject to the labor demand function (11), budget constraint (13), and transition equation for capital (14). They also set nominal wages in staggered contracts

[8]The assumption of an intermediation fee ensures that given our solution technique the evolution of net foreign assets is stationary. See Schmitt-Grohe and Uribe (2003) and Bodenstein (2006) for a discussion. The intermediation cost is asymmetric, as foreign households do not face these costs. Rather, they collect profits on the monopoly rents associated with these intermediation costs.

that are analogous to the price contracts described above. In particular, each member of a household is allowed to re-optimize its wage contract with probability $1-\xi_w$. We allow for a mass ι_w of backward-looking members within the household that sets its wage using a rule of thumb. These household members set their wages equal to the previous period reset wage indexed by lagged wage inflation.

2.3 Monetary and Fiscal Policy

Monetary policy follows an interest rate reaction function as suggested by Taylor (1993). However, when policy rates reach zero, we assume that no further actions are taken by the central bank. The notional rate that is dictated by the interest rate reaction function is denoted by i_t^{not}, whereas the actual policy rate that is implemented is denoted by i_t. The notional rate set as:

$$i_t^{not} = \bar{i} + \pi_t + \gamma_\pi(\pi_t - \bar{\pi}) + \frac{\gamma_y}{4} y_t^{gap}, \tag{17}$$

where $\bar{i}$ and $\bar{\pi}$ are the steady-state nominal interest rate and inflation rate, respectively. The actual (short-term) policy interest rate satisfies

$$i_t = max(0, i_t^{not}), \tag{18}$$

and accordingly, the actual an notional rates differ only when the notional rate turns negative.

The term $\bar{i}$ is the steady-state value for the nominal interest rate. The inflation rate π_t is expressed as the logarithmic percentage change of the domestic price level, $\pi_t = log(P_{Dt}/P_{Dt-1})$ – it is assumed to be zero in steady state. The term y_t^{gap} denotes the output gap, given by the log difference between actual and potential output, where the latter is the level of output that would prevail in the absence of nominal rigidities. Notice that the coefficient γ_y is divided by four as the rule is expressed in terms of quarterly inflation and interest rates.

Government purchases are a constant fraction of output $\bar{g}$ and they fall exclusively on the domestically-produced good. These purchases make no direct contribution to household utility. To finance its purchases, the government imposes a lump-sum tax on households that is adjusted so that the government's budget is balanced every period.

2.4 Resource Constraints

The home economy's aggregate resource constraint satisfies:

$$Y_{Dt} = C_{Dt} + I_{Dt} + G_t + \phi_{It}. \tag{19}$$

The composite domestically-produced good Y_{Dt}, net of investment adjustment costs ϕ_{It}, is used to produce final consumption and investment goods ($A_{Dt} = C_{Dt} + I_{Dt}$), or directly to satisfy government demand. Moreover, since each individual intermediate goods producer can sell its output either at home or abroad, there are also a continuum of resource constraints that apply at the firm level.

2.5 Calibration of Parameters and Solution Method

The model is calibrated at a quarterly frequency. The values of key parameters are presented in Table 1 and reflect fairly standard calibration choices for the U.S. economy. We choose $\omega_A = 0.15$ to be consistent with an import share of output of 15%. The domestic and foreign population levels, respectively ζ and ζ^*, are set so that the home country constitutes 25 percent of world output. Balanced trade in steady state implies an import (or export) share of output of the foreign country of 5 percent. Because the foreign country is assumed to be identical to the home country except in its size, $\omega_A^* = 0.05$. We set $\rho_A = 4$, so that the price elasticity of import demand is 1.25.

We set the Calvo parameter for prices ξ_p to 0.9 and the parameter for wages ξ_w to 0.85 implying an average contract duration of 10 quarters and about 6 $\frac{1}{2}$, respectively. We choose these parameters to curb the sensitivity of inflation in a way that compensates for the fact that the model abstracts from real rigidities in price and wage setting. Export price rigidities have a shorter duration of 2 quarters, as implied by the parameter $\xi_{px} = 0.5$. We set ι_p, the fraction of backward-looking price setters, to 0.75. Analogously, we set ι_w, the fraction of backward-looking wage setters, to 0.75. Taken together the choices of parameters regulating price and wage setting build a substantial degree of inertia in the price and wage inflation processes. This inertia prevents inflation from dropping precipitously in response to contractionary shocks, in a way consistent with recent U.S. experience. Our results concerning the importance of the zero lower bound for the transmission of foreign shocks would be reinforced if we were to reduce the degree of inflation inertia.

Monetary policy follows the modified Taylor rule (see Taylor 1999), aside from account of the zero lower bound constraint. Thus, the parameter γ_π on the inflation gap is 0.5 and the parameter γ_y on the output gap is 1. The steady state real interest rate is set to 1% per year ($\beta = 0.9975$). Given steady state inflation equal 2%, the implied steady state nominal interest rate is 3%. The values of remaining parameters are also fairly standard in the literature, and are summarized in Table 1.

Following Jung, Teranishi, and Watanabe (2005) and Eggertsson and Woodford (2003), all equilibrium conditions except the non-linear policy rule are linearized around the model's non-stochastic steady state. We solve the model using a piece-wise linear algorithm described in Guerrieri and Iacoviello (2015). Because standard perturbation methods only provide a local approximation, they cannot capture the zero-lower bound constraint without adaptation. The method of Guerrieri and Iacoviello (2015) builds on an insight that has been used extensively in the literature on the effects of attaining the zero-lower bound on nominal interest rates.[9] That insight is that occasionally binding constraints can be handled as different regimes of the same model. Under one regime, the occasionally binding constraint is slack.

[9] Recent examples of the use of this technique include Jung, Teranishi, and Watanabe (2005), Eggertsson and Woodford (2003), Christiano, Eichenbaum, and Rebelo (2011b).

Under the other regime, the same constraint is binding. The piecewise linear solution method involves linking the first-order approximation of the model around the same point under each regime. Importantly, the solution that the algorithm produces is not just linear – with two different sets of coefficients depending on whether the occasionally binding constraint is binding or not – but rather, it can be highly nonlinear. The dynamics in one of the two regimes may crucially depend on how long one expects to be in that regime. In turn, how long one expects to be in that regime depends on the state vector. This interaction produces the high nonlinearity. Further details of the solution algorithm are given in the appendix.

3 Initial Baseline Path

Our principal goal is to compare the impact of foreign shocks on the home country when it faces a liquidity trap with the effects that occur when policy rates can be freely adjusted. In the former case, the impact of a foreign shock depends on the economic conditions that precipitated the liquidity trap. Intuitively, the effects of an adverse foreign shock against the backdrop of a recession-induced liquidity trap in the home country should depend on the expected severity of the recession, and the perceived duration of the liquidity trap. In a shallow recession in which interest rates are only constrained for a short period, the effects of the foreign shock would not differ substantially from the usual case in which rates could be cut immediately.[10] By contrast, the effects of the foreign shock on the home country might be amplified substantially if it occurred against the backdrop of a steep recession in which policy rates were expected to be constrained from falling for a protracted period.

We use the term "initial baseline path" to describe the evolution of the economy that would prevail in the absence of the foreign shock. Given agents' full knowledge of the model, the initial baseline path depends on the underlying shocks that push

[10] In the case of a linear model, the effects of a shock are unrelated to the initial conditions.

the economy into a liquidity trap, including their magnitude and persistence, as these features play an important role in determining agents' perceptions about the duration of the liquidity trap.

Our analysis focuses on the effects of foreign shocks against the backdrop of an initial baseline path that is intended to capture a severe recession in the home country. This "severe recession" baseline is depicted in Figure 2 by the solid lines. It is generated by a preference shock ν_{ct} that follows an autoregressive process with persistence parameter equal to 0.7. The shock reduces the home country's marginal utility of consumption. As the shock occurs exclusively in the home country, the foreign economy has latitude to offset much of the contractionary impact of the shock by reducing its policy rate.

As shown in Figure 2 policy rates immediately fall to 0 and remain frozen at this level for ten quarters.[11] Given that the shock drives inflation persistently below its steady state value of 2% and that nominal interest rates are constrained from falling by the zero bound, real rates increase substantially in the near term. This increase in real interest rates accounts in part for the substantial output decline, which attains a trough of 8 percent below its steady state value. Real interest rates decline in the longer term, helping the economy recover. This longer term decline also causes the home currency to depreciate in real terms, and the ensuing expansion of real net exports mitigates the effects of the shock on domestic output. However, the improvement in real net exports is delayed due to the zero bound constraint, since higher real interest rates limit the size of the depreciation of the home currency in the near-term.

For purposes of comparison, Figure 2 also shows the effects of the same shocks in the case in which the home country's policy rates can be adjusted, i.e., ignoring the zero bound constraint. In this linear simulation, the home nominal interest rate

[11] In Figure 2, most variables are plotted in deviation from their steady-state values, but the policy interest rate, the real interest rate, and inflation are shown in *levels* to highlight the zero bound constraint.

falls more sharply and turns negative, implying a large and front-loaded decline in real interest rates. Hence, the fall in home output is smaller than in the benchmark framework in which the zero bound constraint is binding. The home output contraction is also mitigated by a more substantial improvement in real net exports. Given that real interest rates fall very quickly, the real depreciation is considerably larger, contributing to a faster improvement in real net exports.

4 International Transmission at the Zero Bound

We turn to assessing the impact of a negative foreign demand shock – specifically, a contractionary consumption taste shock ν_{ct}^{*} – when the home country faces a liquidity trap. The foreign shock is scaled to induce a 1 percent reduction in *home* output in the case in which home monetary policy is assumed to be unconstrained by the ZLB. This scaling proves convenient for assessing the effects of the ZLB constraint on the transmission of the foreign shock to the home economy.

Figure 3 shows the "partial" effect of the foreign demand shock both for the case in which home monetary policy is unconstrained by the ZLB (the dashed red lines), and for the case in which it is constrained (the solid black lines). To be precise, the responses in Figure 3 are derived from a simulation that adds both the adverse domestic taste shock from Figure 2 and the foreign taste shock, and then subtracts the impulse response functions associated with the domestic taste shock alone.[12] Thus, all variables are measured as deviations from the baseline path shown in Figure 2.

As shown in Figure 3, the foreign preference shock leads to a contraction in foreign output of $3\frac{1}{2}$ percent (panel 2). Foreign policy rates are cut. As real rates also drop, investment is stimulated. Lower real rates contribute to a real exchange

[12]Because the model we solve is linear when the zero lower bound does not bind, the dashed lines in Figure 3 can alternatively be interpreted as the responses starting from the model's steady state, rather than from the severe recession baseline.

rate depreciation that boosts foreign exports. Perhaps surprisingly, the response of foreign GDP is nearly invariant to whether home monetary policy is constrained by the ZLB (as discussed below).

By contrast, the effects of the foreign demand shock on the home country are strikingly different depending on whether home monetary policy is constrained by the ZLB. While home GDP only falls about 1 percent when monetary policy is unconstrained – about 0.3 as large as the fall in foreign GDP – home GDP declines about twice as much when home monetary policy is constrained by the ZLB. Under either assumption about monetary policy, home real net exports contract because foreign absorption falls and the home real exchange rate appreciates (shown by the fall in panel 7, which reduces exports and boosts imports through standard relative price channels). However, in a liquidity trap, the decline in home export demand causes a fall in the marginal cost of production and inflation that is not accompanied by lower policy rates. The zero bound constraint keeps nominal rates from declining for ten quarters. Real rates rise sharply in the short run, even though they fall at longer horizons. Consequently, domestic absorption does not expand as much as when policy rates can be cut immediately. If the initial recession were more pronounced, private absorption could even fall. With net exports falling and with domestic absorption not filling the gap, output falls by nearly as much in the home country as abroad.

The implication that the foreign GDP response is nearly invariant to the response of home monetary policy reflects that the home ZLB constraint has offsetting effects on foreign exports. In particular, while the home ZLB constraint reduces home absorption (relative to the unconstrained case) – which hurts foreign exports – it also induces a larger appreciation of the home currency – which benefits foreign exports. With a reasonable calibration of trade price elasticity of around unity, it turns out that these effects on foreign exports (and hence GDP) nearly counterbalance each other.

The magnification of the spillover effects of foreign shocks to the home economy when the ZLB binds is not particular to the consumption shock but also applies, for instance, to shocks to the discount factor, capital tax rates, and government spending.[13] The case of technology shocks is discussed later in this section.

4.1 An Alternative "Secular Stagnation" Baseline

We will next show how the effects of foreign demand shocks vary with the duration of the liquidity trap in the home economy. The liquidity trap in the home economy could be longer either because domestic conditions are comparatively worse, or because the foreign demand shock was sufficiently large and adverse, or both. Either way, an incremental contraction in foreign demand will have a bigger contractionary effect on home GDP when the ZLB binds for a longer duration.

But with this prelude, it is interesting to focus on how foreign demand shocks may potentially push an economy with a low "neutral" policy rate into recession, and exert nonlinear effects on home GDP as the shock increased in size. To do so, we modify our baseline calibration to set the inflation target equal to 0.2 percent, and the discount factor to 0.99925, consistent with a steady state real interest rate of 0.3 percent. The implied steady state nominal interest rate of 0.5 percent is similar to that in Japan prior to the Global Financial Crisis. As we discuss below, this calibration is admittedly quite pessimistic about the long-run neutral rate, it is useful both for showing how the effects of foreign shocks grow with the length of the liquidity trap (the more general point), as well as highlighting some of the risks of a very low neutral rate.

In this vein, Figure 4 shows the effects of three foreign demand shocks of varying size. Importantly, the figure shows responses relative to the steady state baseline – unlike Figure 3, the foreign shocks are the only source of disturbance. The first

[13] The effects of shocks to the discount factor, capital tax rates, and government spending are shown in the working paper version of this article, see Bodenstein, Erceg, and Guerrieri (2009)

shock (solid line) is scaled to reduce foreign GDP by 3 percent. This shock is too small to drive the home economy to the ZLB, and hence reduces home GDP by 0.85 percent (consistent with the partial effect of the foreign demand shock in Figure 3).

The second shock (dashed line) is double in size relative to the first one. As foreign policy rates can be cut aggressively, the effect on foreign GDP is almost linear, with foreign GDP declining just shy of 6 percent. By contrast, home policy rates cannot be cut as much as the policy rule would dictate and the ZLB binds for almost 10 quarters. Accordingly, home GDP declines more sharply and nonlinearly. It drops about 2.2 percent, or 30 percent more than the 1.7 percent drop that a mere doubling of the effects of the first shock would dictate. To put it differently, the "marginal" effect of on home GDP of the additional 3 percent decline in foreign GDP is about 1.35 percent (2.2-0.85 = 1.35), which is much larger than the 0.85 percent home GDP decline implied by the first shock.

The nonlinear effects associated with the zero lower bound in the home country are even more pronounced for the third shock (the dash-dotted line), which doubles in size the second one, or quadruples in size the first one. Again, as the foreign country can adjust the policy rate, foreign GDP declines close to 12 percent, almost double the GDP decline for the second shock. By contrast, as the shock now takes the home policy rate to the lower bound for almost 15 quarters, the magnification of the effects of the shock on the home country is much more than double, and home GDP declines over 6 percent, about three times the size of the GDP decline for the second shock. One simple way of summarizing the nonlinear effects associated with different durations of the liquidity trap in the home country is by the ratio of the GDP decline at home and abroad. When the foreign shocks are small enough to keep the economy away from the zero lower bound, the ratio of home and foreign GDP is 0.35 on impact and declines thereafter. The line shown in the inset box for the first 5 quarters under a linear approximation to the model's solution would be invariant with the size of the shock. By contrast, as the home economy hits that ZLB for the second shock, this ratio increases to 0.42. Finally, as the expected duration of the ZLB is even longer for the third shock, the ratio of home to foreign GDP jumps to 0.62.

The implication of Figure 4 that foreign shocks alone may drive the home economy into recession – even in the absence of financial spillovers from which the model abstracts – may appear to hinge on somewhat extreme assumptions about the neutral policy rate and size of the foreign shocks. Even so, it is useful to underscore that the larger foreign GDP declines of 6-12 percent considered in the second and third scenarios seem reasonably consistent with the experience of many industrial economies during the GFC and its aftermath. For example, an IMF study estimated that GDP in OECD economies fell by 14 percent on average relative to its pre-crisis trend over the 2008-2012 period (Abbas et al. 2014). More generally, it is not essential that the natural rate be as low as the 0.5 percent assumed in Figure 4 – nor for foreign shocks to be as large – in order for the spillovers from foreign shocks to be similar to those shown in Figure 4. For example, the spillovers would be large if the home economy had a considerably lower natural rate than in our baseline (say 1.5 or 2 percent, rather than 3 percent), but was also experiencing a modest recession due to domestic shocks. The more general upshot of our analysis is that a low neutral rate tends to significantly heighten the vulnerability of the domestic economy to foreign shocks.

One simple way of summarizing the nonlinear effects associated with different durations of the liquidity trap in the home country is by the ratio of the GDP decline at home and abroad. When the foreign shocks are small enough to keep the economy away from the zero lower bound, the ratio of home and foreign GDP is 0.35 on impact and declines thereafter. The line shown in the inset box for the first 5 quarters under a linear approximation to the model's solution would be invariant with the size of the shock. By contrast, as the home economy hits that ZLB for the second shock, the this ratio increases to 0.42. Finally, as the expected duration of the ZLB is even longer for the third shock, the ratio of home to foreign GDP jumps to 0.62.

To illustrate how a drift towards secular stagnation could interact with a trend towards globalization, the next two sections study how the nonlinear transmission of foreign shocks associated with reaching the ZLB is influenced by the home economy's trade share and by the substitutability of the home and foreign traded goods. To this purpose, we continue to use the same calibration as above, which implies a nominal interest rate of 0.5 percent.

4.2 Changes in Openness

Figure 5 considers the same large foreign consumption shock leading to a decline in foreign GDP of close to 12 percent, as was considered for the third shock in Figure 4 . The dashed-dotted lines denote the effects associated with the benchmark 15% import share. The figure also shows the effects of the same foreign shock for a lower trade share of 10 percent and for a higher trade share of 20 percent. The results shown in the left-hand-side panels (panels 1, 3, 5, and 7) pertain to a configuration of the model for which the ZLB in the home country is not enforced, while the results shown in the right-hand-side panels (panels 2, 4, 6, and 8) pertain to a configuration of the model for which the ZLB is enforced.

When the ZLB is not enforced, the change in the transmission of the foreign shock to the home country is almost proportional to the change in the trade share. In particular the appreciation of the home exchange rate and the decline in foreign activity produce a decline in home exports that is similar in percentage terms relative to steady state, regardless of the trade share. However, when the home economy's trade share is larger, a given percentage decline in exports accounts for a larger share of GDP and lead to an almost proportionately larger decline in GDP. Similarly, with a smaller trade share, the decline in home GDP is buffeted in a way that is proportional to the lower trade share. Accordingly the effects for the benchmark configuration are almost equidistant from those for the alternative trade shares considered. For increases in the trade share that are even larger than the one considered here, monetary policy can crowd in domestic demand so that the drop in GDP is somewhat buffeted.

By contrast, when the ZLB is enforced, the effects of the same foreign consumption shock show greater variation depending on the trade share. For the larger trade share of 20 percent, the home policy rate is expected to remain at the ZLB for even longer, close to 20 quarters. In that case, the direct hit to GDP implied by a more impactful decline in exports cannot be offset by an expansion in domestic demand, as can be engendered by lower policy rates when the ZLB is not enforced. Analogously, when with a lower trade share, the ZLB in the home country is expected to be less protracted, close to 10 quarters instead of 15 under the benchmark calibration. In this case, home absorption expands by more, implying a smaller contraction in GDP relative to the benchmark case.

4.3 Changes in Substitutability of Home and Foreign Goods

The value of the import price elasticity of demand is an important determinant of the duration of a liquidity trap and the spillover effects of country-specific shocks. When the zero bound is not binding, increasing the trade price elasticity of demand magnifies the decline of home real net exports caused by a foreign demand contraction. The spillover effects on home output are partly offset by a more vigorous reaction of domestic monetary policy. However, in a liquidity trap, monetary policy is unable to compensate in such a manner, and the larger effects on real net exports translate into greater effects on home output.

Figure 6 shows how the spillover effects of a foreign consumption shock are affected by a higher elasticity, equal to 2 versus 1.25 in our original calibration, or a lower elasticity, equal to 0.75. The figure considers, again, the spillover effects on the home economy of a foreign consumption shock that brings about a 12 percent reduction in foreign GDP. As in Figure 4, panels on the left-hand-side are from a configuration that disregards the ZLB. Panels on the right-hand-side are from a configuration of the model with the ZLB enforced.

Away from the zero lower bound, the higher elasticity reduces the responsiveness of exchange rates to country-specific shocks. However, the increased sensitivity to movements in relative import prices more than offsets the decreased volatility of exchange rates. Accordingly, with the higher elasticity, home country net exports drop by more in response to a contractionary foreign consumption shock, leading to a larger fall in home GDP. The larger drop in activity leads to a larger drop in policy rates and a longer duration of the ZLB, which then further reinforces the output decline associated with a larger trade elasticity.

In sum, as trade volumes continue to expand and domestic and foreign goods continue to become closer substitutes in line with globalization trends, we can expect that the spillover effects of foreign contractionary shocks will be enhanced even more profoundly should a country reach the ZLB.

4.4 A Foreign Technology Shock

Near unit-root technology shocks are the typical source of fluctuations in open economy models. However, the spillover effects of country-specific technology shocks are quite small and remain so even in a liquidity trap. The basic reason is that lower foreign activity retards the demand for home exports, but this effect is counterbalanced by a depreciation of the home real exchange rate, which boosts home exports. Returning to our benchmark calibration with nominal interest rates at 3 percent, the exchange rate channel initially dominates, as shown in Figure 7, implying a rise in home real net exports, and a small and short-lived *expansion* in home GDP. The effects when the home country is constrained by the zero lower bound are only modestly different.

5 Conclusions

When monetary policy is unconstrained, it can cushion the impact of foreign disturbances. By contrast, in a liquidity trap, monetary policy cannot crowd in domestic demand as effectively, and the spillover effects of foreign shocks can be magnified greatly. The amplification of idiosyncratic foreign shocks depends both on the duration of the liquidity trap and as well as on key structural features such as the trade price elasticity. The size of the foreign shock is of particular importance as it can effect the length of the liquidity trap and thereby decouple the marginal and average spillover effects of the shock. With our autoregressive shock processes, as typically postulated in the empirical validation of DSGE models, the model can generate substantial differences between the marginal and the average effect of a shock. A simplification of the treatment of the zero lower bound that fixed the duration of the liquidity trap exogenously would miss this feature completely.

Developments during the Global Financial Crisis and its aftermath suggest that foreign shocks may have much larger effects on the domestic economy when monetary policy is constrained from adjusting interest rates. Should neutral policy rates drift downwards, foreign contractionary shocks could have outsize effects even for relatively closed economies such as the United States, much as they did for Japan during the Global Financial Crisis.

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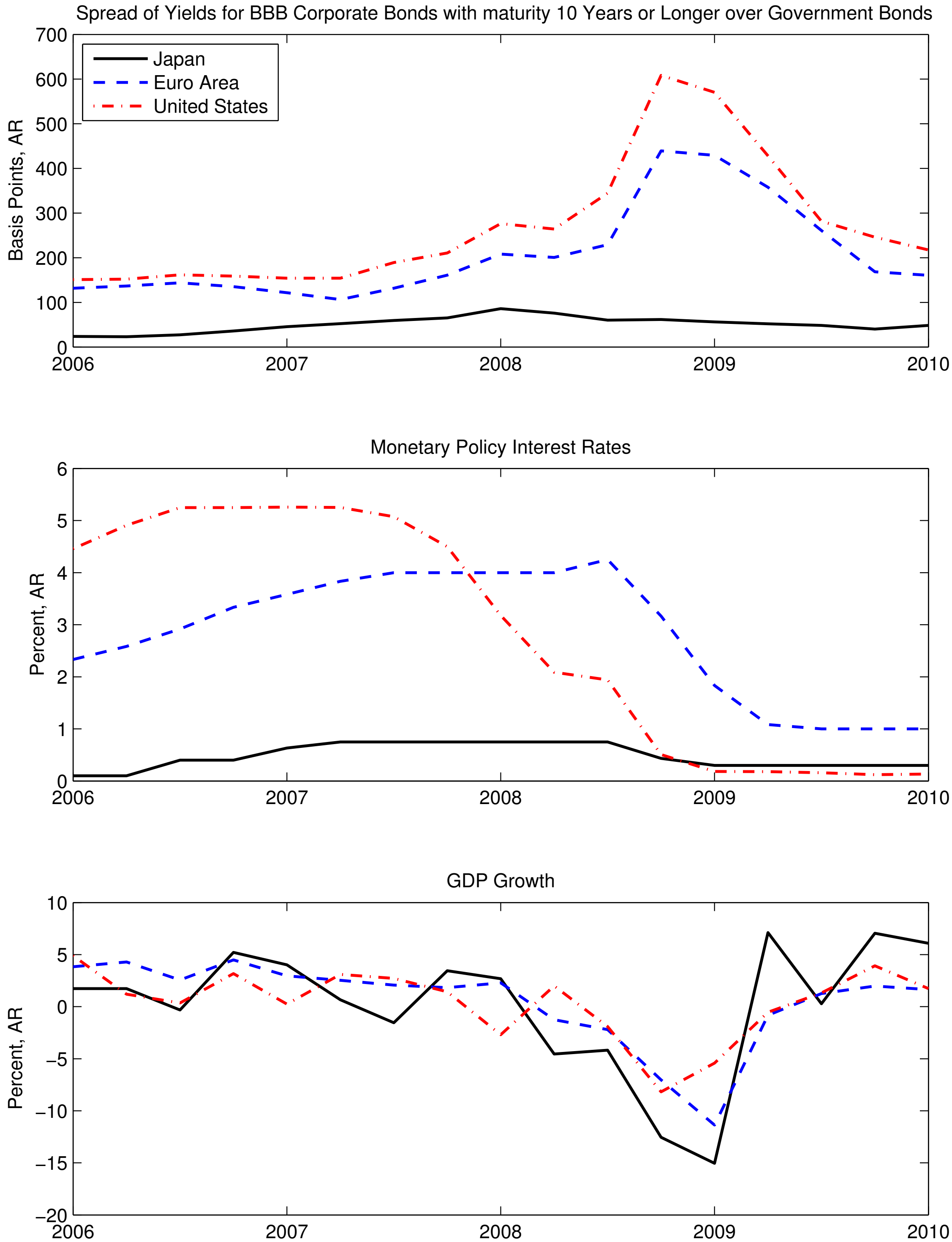
Table 1: Calibration*

Parameter	Determines:	Parameter	Determines:
$\beta = 0.9975$	s.s. real interest rate = 1% per year	$\delta = 0.025$	depreciation rate = 10% per year
χ_0	leisure's share of time = 1/2	$\chi = 2.5$	labor supply elasticity = 1/2.5
$\sigma = 1$	intertemporal substitution elast. 1	$\phi_b = 0.001$	interest elasticity of foreign assets
$\rho = -1$	capital-labor substitution elast. = 1	$\rho_A = 4$	import price elasticity = 1.25
$\omega_A = 0.15$	import share of output = 15%	$\omega_A^* = 0.05$	foreign import share of output = 5%
$\zeta = 1$	population size	$\zeta^* = 3$	foreign population size
$\kappa = 0.85$	consumption habits	$\phi_I = 6$	investment adjustment costs
$\theta_w = 1/3$	wage markup = 33%	$\theta_p = 0.2$	domestic/export price markup = 20%
$\xi_p = 0.90$	price contract expected duration = 10 quarters	$\xi_w = 0.85$	wage contract expected duration = 6.5 quarters
$\xi_{px} = 0.5$	export price contract expected duration = 2 quarters		
$\iota_p = 0.75$	inertia in inflation	$\iota_w = 0.75$	inertia in wage inflation
$\gamma_y = 1$	monetary policy's weight on the output gap	$\gamma_\pi = 0.5$	monetary policy's weight on inflation

* Parameter values for the foreign country are chosen identical to their home country counterparts except for the population size ζ^* and the import share ω_A^*.

Figure 1: Policy Rates, Corporate Spreads, and GDP around the Global Financial Crisis



Sources: BBB corporates spreads are from Bank of America Merrill Lynch via Bloomberg. Monetary policy interest rates are from central banks via Haver. GDP are from national accounts via Haver.

Figure 2: Severe Domestic Recession Scenario (Initial Baseline Path)

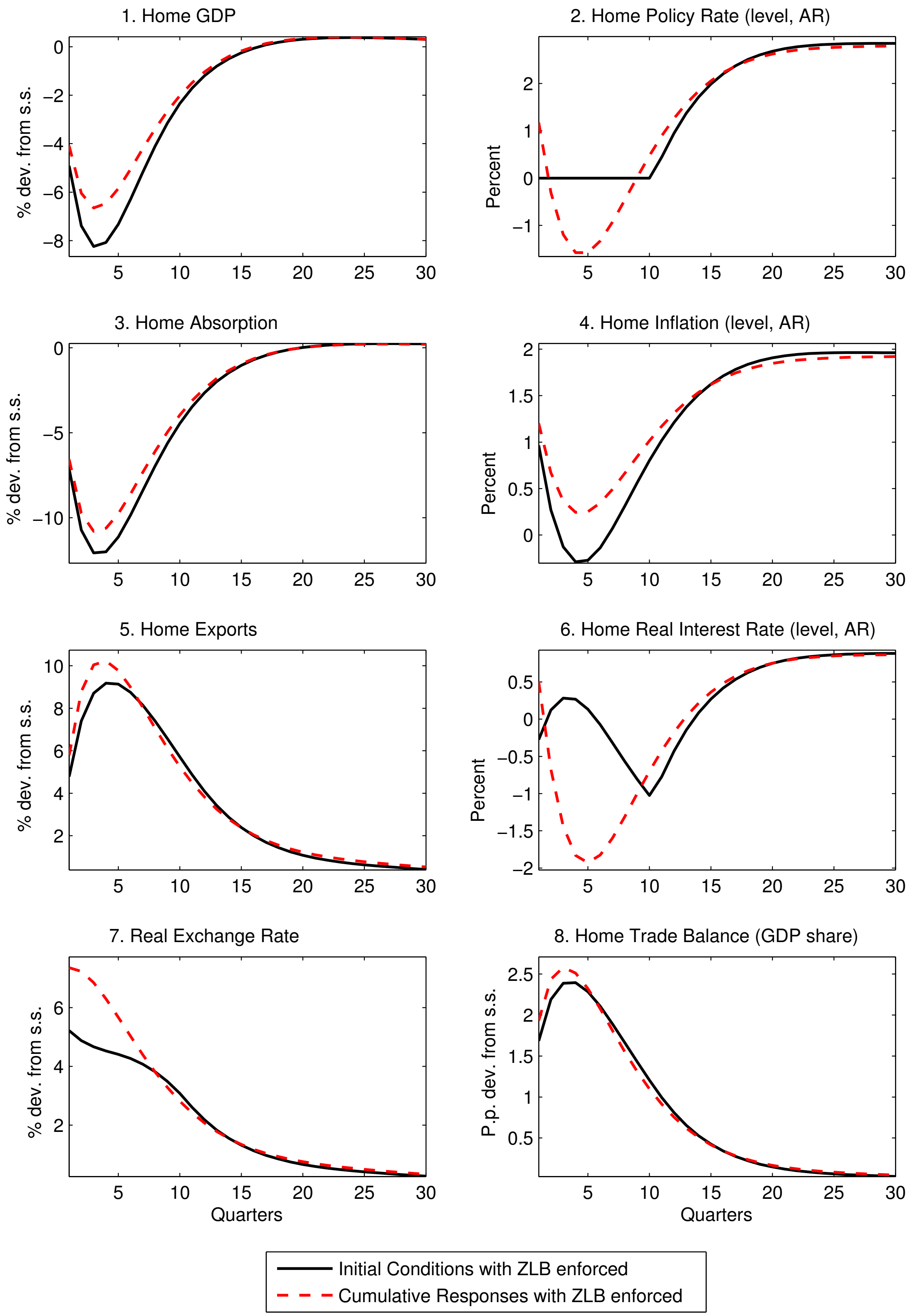


Figure 3: Fall in Foreign Demand (relative to baseline)

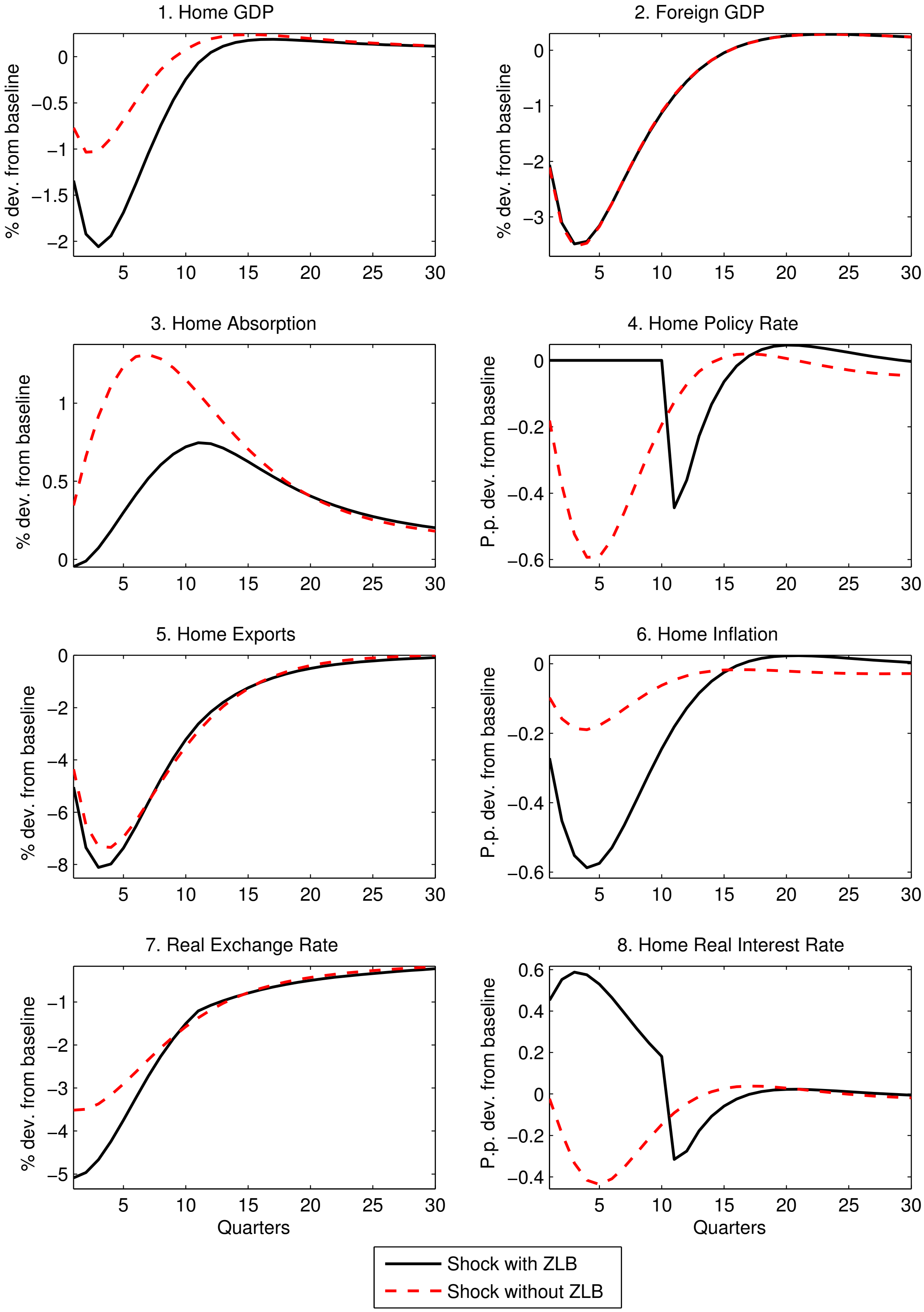


Figure 4: Fall in Foreign Demand ("Secular Stagnation" Calibration)

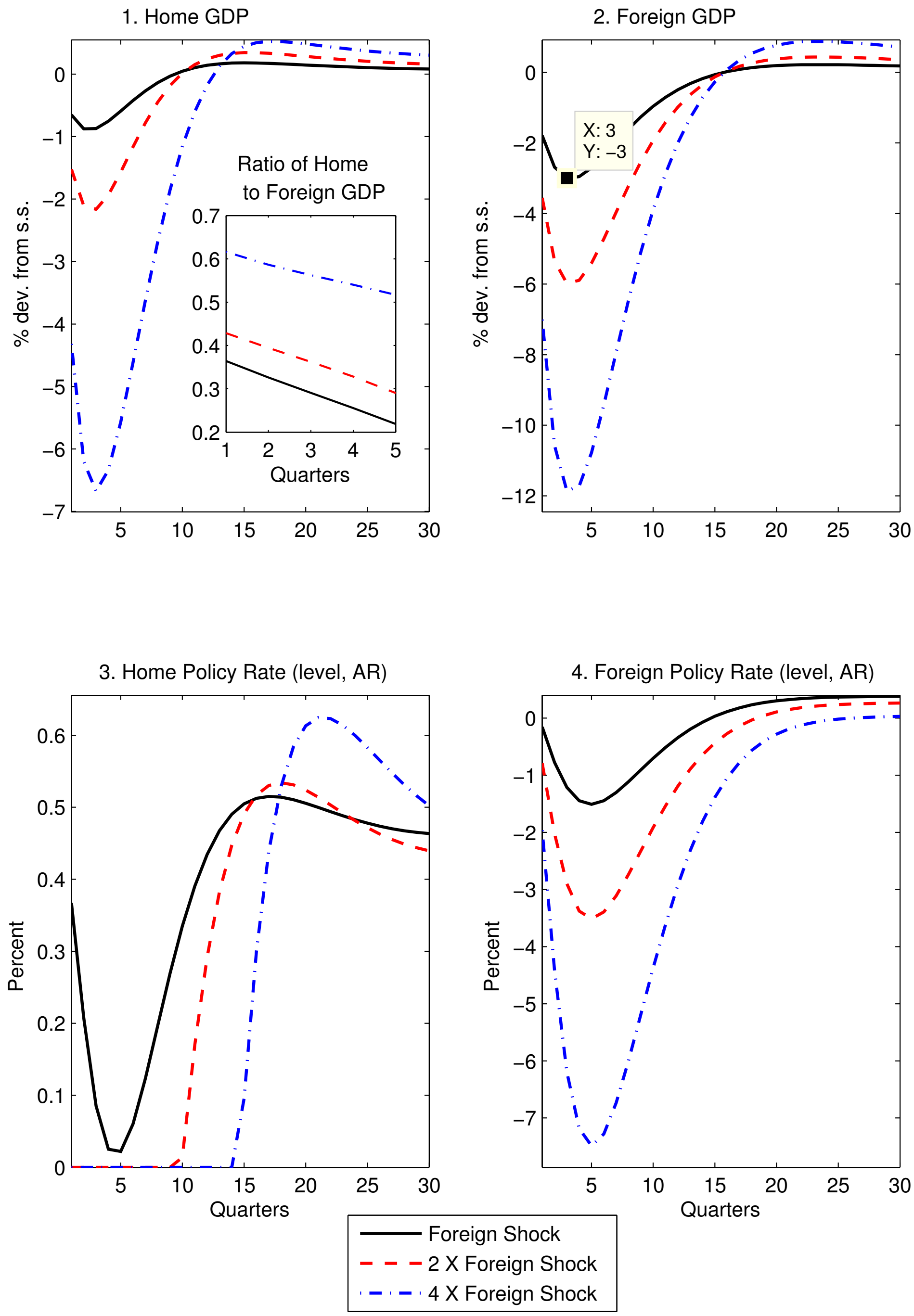


Figure 5: Foreign Demand Shock under Different Levels of Trade Openness)

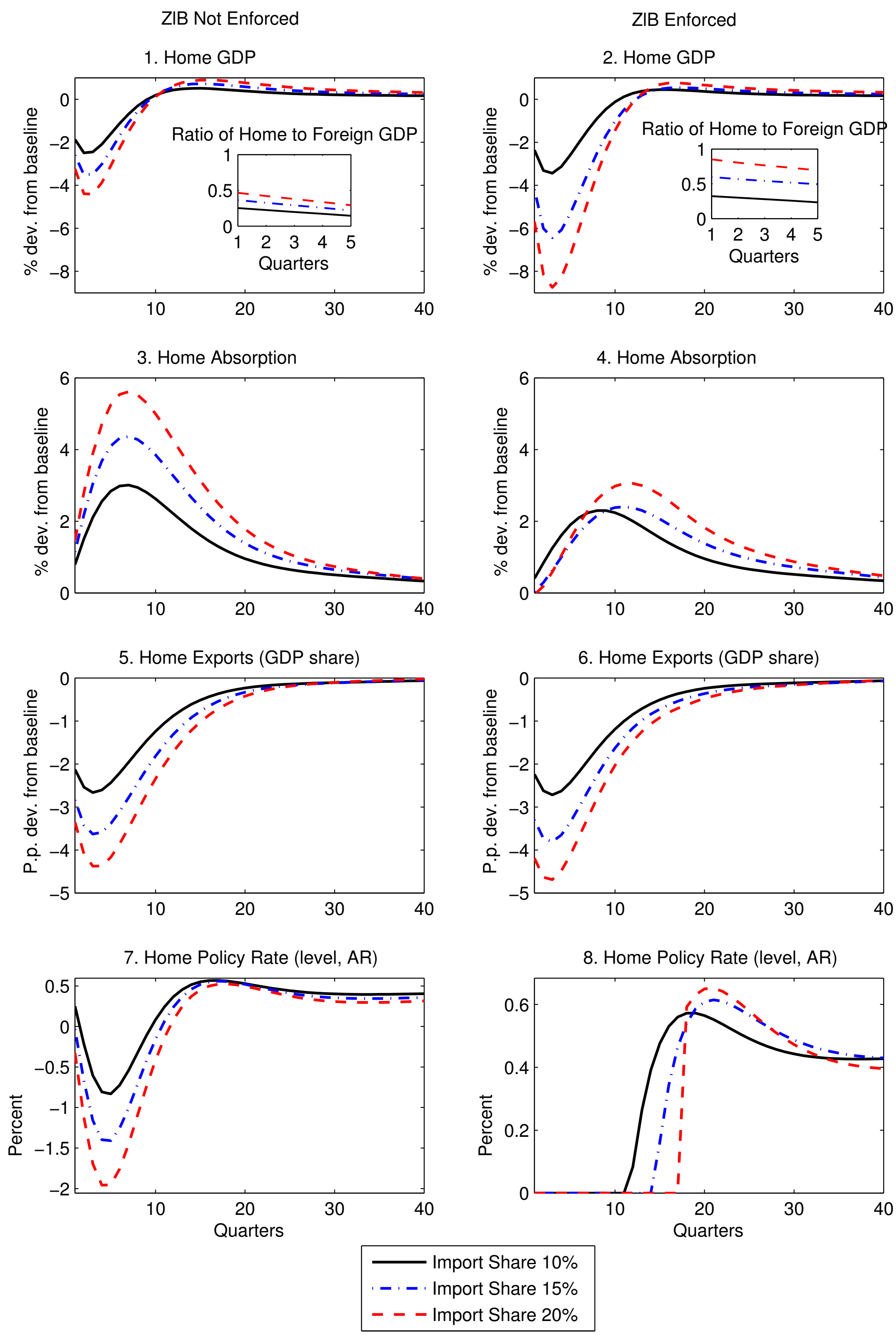


Figure 6: Foreign Demand Shock under Alternative Trade Price Elasticities

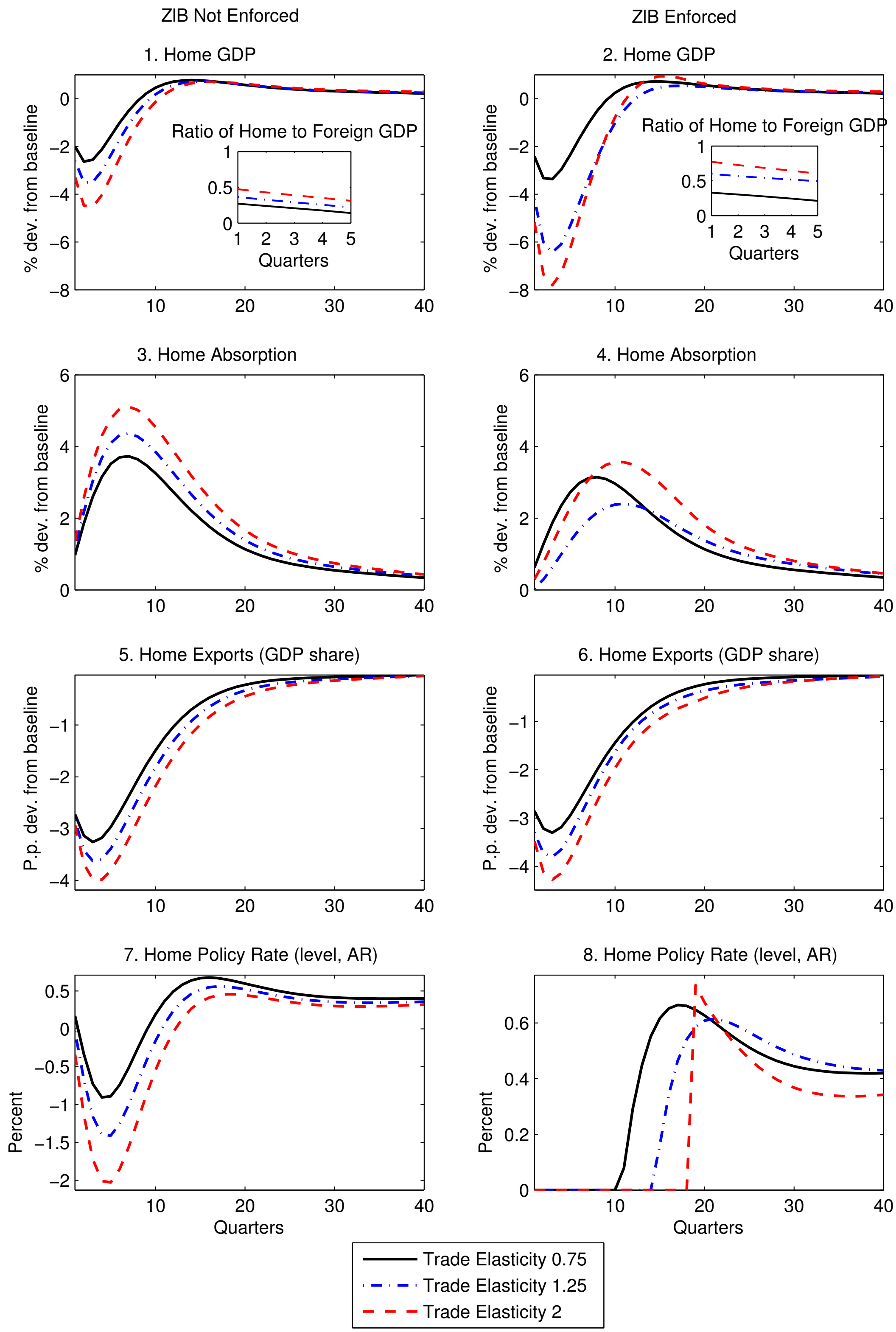
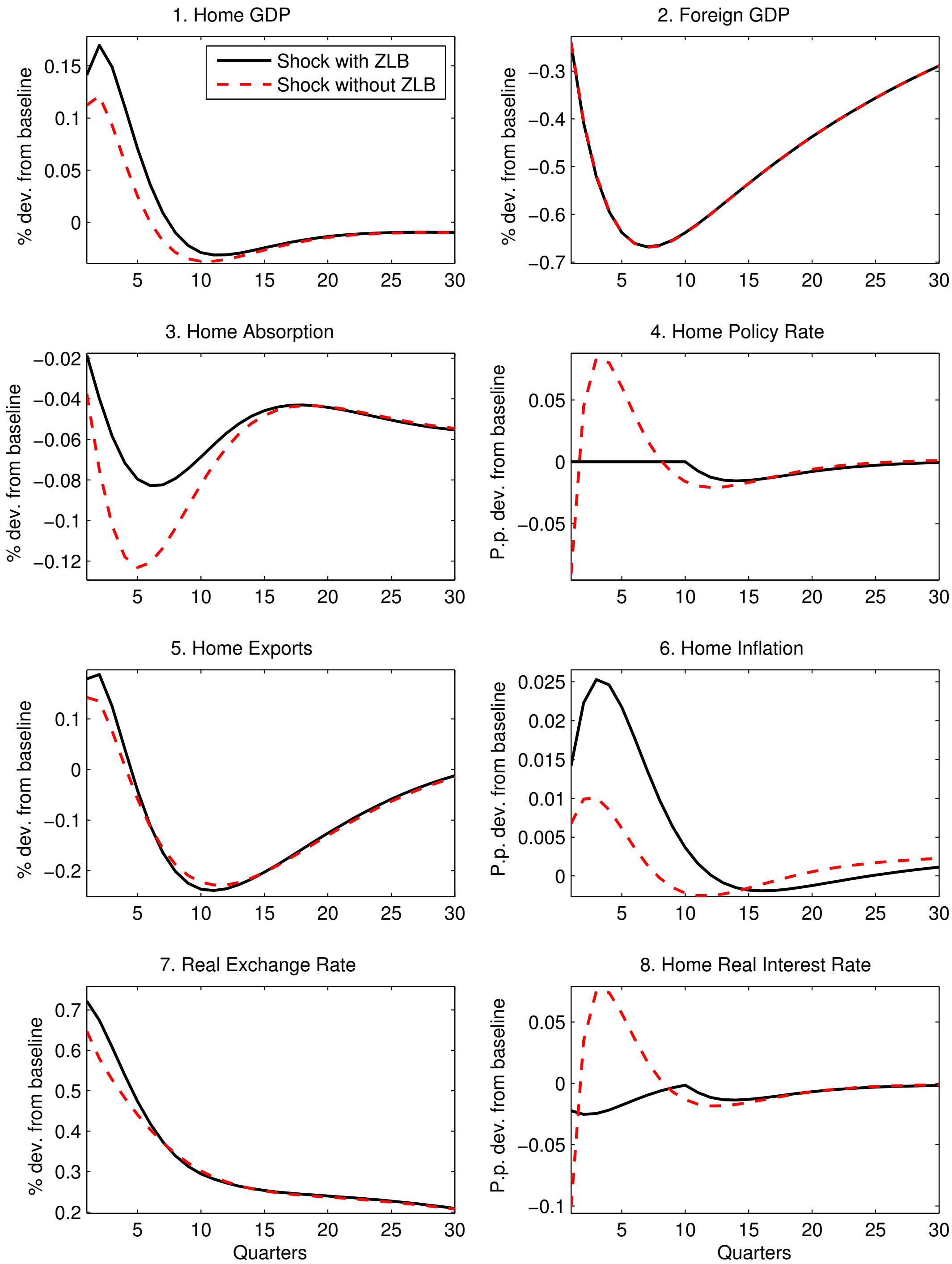


Figure 7: A Foreign Technology Shock



Appendices for Online Publication

A Solution Method

We use a piecewise–linear solution approach to find the equilibrium allocations of the model. This method resolves the problem of computing decision rules that approximate the equilibrium well both away from the zero lower bound and at the bound.

The economy features two regimes: a regime when monetary policy is away from the ZLB, and a regime when monetary policy is constrained by the ZLB. Away from the ZLB, the linearized system of necessary conditions for an equilibrium can be expressed as

$$\mathcal{A}_1 E_t X_{t+1} + \mathcal{A}_0 X_t + \mathcal{A}_{-1} X_{t-1} + \mathcal{B} u_t = 0, \tag{B.1}$$

where $\mathcal{A}_1$, $\mathcal{A}_0$, and $\mathcal{A}_{-1}$ are matrices of coefficients conformable with the vector X collecting the model variables in deviation from the steady state for the regime with binding constraints; and where u is the vector collecting all shock innovations (and $\mathcal{B}$ is the corresponding conformable matrix). Similarly, when the ZLB binds, the linearized system can be expressed as

$$\mathcal{A}_1^* E_t X_{t+1} + \mathcal{A}_0^* X_t + \mathcal{A}_{-1}^* X_{t-1} + \mathcal{B}^* u_t + \mathcal{C}^* = 0, \tag{B.2}$$

where $\mathcal{C}^*$ is a vector of constants. Away from the ZLB, we use standard linear solution methods to express the decision rule for the model as

$$X_t = \mathcal{P} X_{t-1} + \mathcal{Q} u_t. \tag{B.3}$$

At the ZLB, we use a guess-and-verify approach. We shoot back towards the initial conditions, from the first period when the ZLB is guessed not to bind again. For example, if the ZLB binds in t but is guessed not to bind the next period, the decision rule for period t can be expressed, starting from B.2 and using the result that $E_t X_{t+1} = \mathcal{P} X_t$, as:

$$X_t = -\left(\mathcal{A}_1^* \mathcal{P} + \mathcal{A}_0^*\right)^{-1} \left(\mathcal{A}_{-1}^* X_{t-1} + \mathcal{B}^* u_t + \mathcal{C}^*\right). \tag{B.4}$$

We proceed in a similar fashion to compute the allocations for the case when the ZLB is guessed to bind for multiple periods or when the ZLB binds starting in periods beyond t. As shown by equation B.4, the model dynamics when the ZLB binds depend both on the current regime (through the matrices $\mathcal{A}_1^*, \mathcal{A}_0^*$ and $\mathcal{A}_{-1}^*$) and on the expectations of future regimes away from the ZLB (through the matrix $\mathcal{P}$, which is a nonlinear function of the matrices $\mathcal{A}_1$, $\mathcal{A}_0$ and $\mathcal{A}_{-1}$).

For an array of models, Guerrieri and Iacoviello (2015) compare the performance of the piecewise perturbation solution described above against a dynamic programming solution obtained by discretizing the state space over a fine grid. Their results show that the piecewise-linear solution method used here efficiently and quickly computes a solution that closely mimics the global nonlinear solution.